Tight and Compact MILP Models for Storage Operation and Unit Commitment

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Abstract

To achieve climate goals by 2050, accurate energy system optimization (MIP) models are needed to help decision-makers make investment plans. To increase accuracy, a high resolution in the temporal and spatial dimensions is needed, as well as many details on the operational capabilities of energy generators. However, this results in large-scale models that do not scale well. Thus, researchers often seek the right trade-off between computational tractability and accuracy. Here, we present a tighter formulation for optimal storage operation and investment problems, including reserves, along with the methodology we used to obtain it, based on the work of [2]. Additionally, we present some preliminary work aiming to provide tight and compact unit commitment models with different levels of detail. These models can be included in large-scale energy system optimization models to increase model accuracy while keeping the models computationally tractable.

CCS Concepts

• **Mathematics of computing** → **Mixed discrete-continuous optimization**; • **Applied computing** → *Physical sciences and engineering*.

Keywords

Energy storage systems, unit commitment, mixed-integer linear programming (MIP), linear programming (LP), energy system optimization model (ESOM), convex hull, tight formulation, optimal investments, reserves, ramping

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1 Introduction

1.1 Motivation

Due to climate change, our energy system needs to be decarbonized by 2050. To achieve these climate goals, accurate energy system optimization (Linear Programming) models are needed to help decision-makers make investment plans. An illustration of the scope of such a model is given in Figure 1. To increase accuracy, a high resolution in the temporal and spatial dimensions is needed, as well as many details on the operational capabilities of energy generators. However, this results in large-scale models, of which the optimal solution cannot be obtained within any meaningful computing time, not even by supercomputers using the best possible solvers. Thus, researchers often seek the right trade-off between computational tractability and accuracy.

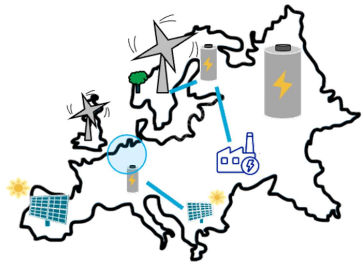

Figure 1: An illustration of the model scope: investments in different energy generators, converters, lines, and storage assets in different countries in Europe.

In this work, we focus on including operational models for storage units and energy generators (unit commitment models) in large-scale models. Renewable energy systems need to be implemented on a large scale. However, renewable energy generation fluctuates, due to its intrinsic weather dependency. Storage systems have become a promising solution to complement this fluctuating production, by storing energy when there is an energy surplus, and discharging when there is a deficit. Additionally, existing thermal generators can further complement this fluctuating production, which can be accurately modeled with unit commitment models. Therefore, we want to include these operational models in large-scale investment models.

However, binary variables are needed to correctly model storage operation, storage reserves, and unit commitment constraints. This results in Mixed-Integer Programming (MIP) models, which generally do not scale well. Relaxing these formulations results in LP models, which can be solved much faster in practice. However, LP-relaxations can give solutions that cannot be implemented in practice, for example because simultaneous charging and discharging would be needed to execute it. Thus, accurate but scalable MIP models for these operational problems are needed.

1.2 Background and related work

To address these scalability issues, much research has been done on tightening unit commitment models [3, 4, 6]. The tightness of a formulation increases if the LP relaxation is closer to the convex hull of the MIP model. The convex hull of an MIP model is a set of constraints that form the tightest possible LP formulation of the solutions to this model. Every vertex of the convex hull is a feasible solution to the MIP model. This is illustrated in Figure 2. For more background information on general LP and MIP theory, see [7]. In general, we cannot expect to explicitly generate the convex hull of an NP-hard problem, even if we allow for constraint types that contain exponentially many constraints [5], but improving the tightness could still speed up the solving time of the model. Therefore, it is important to consider the trade-off between model tightness and model size.

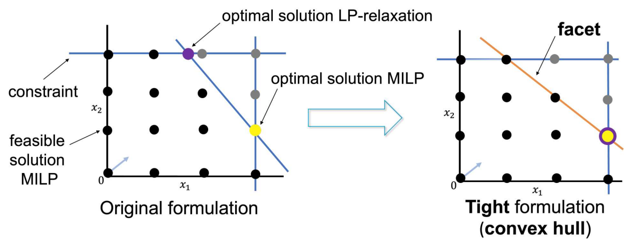

Figure 2: A schematic view of the solution space of a MIP model and its convex hull.

1.3 Contribution

In this ongoing work, we present tighter formulations for optimal storage operation and investments problems, including reserves, based on the work of [2]. Furthermore, we present different tight unit commitment models, with different levels of detail, that can be incorporated in large-scale investment models.

2 Storage operation

2.1 Methodology

For storage operation problems in one time period, we are able to find the convex hull by exploiting the disjunctive nature of the MIP. Balas [1] showed how the convex hull of any disjunctive problem can be obtained. This results in a convex hull formulation in a higher dimension than the original disjunctive problem, but it can be projected onto the original dimension. The obtained projected formulation is still a convex hull [7, Section 9.2.3]. Thus, it follows that the obtained formulation describes the convex hull of the original disjunctive problem. A schematic overview of this methodology is given in Figure 3.

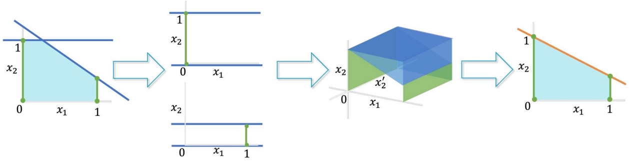

Figure 3: A schematic view of methodology to obtain the convex hull of disjunctive problems.

2.2 Preliminary Results

In ongoing work [2], we obtain the convex hull for the optimal storage operation problem including reserves, as well as the optimal storage investment problem.

3 Unit Commitment

3.1 Methodology

In the ongoing work on unit commitment models, we consider the tightest constraints known in literature of unit commitment models (minimum and maximum generation, (start-up and shut-down) ramps, minimum up and down times, etc.). We provide a clear overview of the results, and fill some gaps.

3.2 Preliminary Results

We provide different tight unit commitment models with different levels of details. The modeler can select the desired level of detail for every generator in the investment problem, resulting in a more accurate model that can be solved faster than when including fully detailed unit commitment models for all generators.

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