Hanger Force Measurements Using Equivalent Vibration Lengths Determined Through Modal Analysis

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Abstract. Assessment of arch bridges often involves the determination of hanger forces. A common method to determine these forces is through vibration measurements, using the so-called taut-string method. The relation between natural frequency and hanger force is however strongly dependent on the vibrating length of the hanger, as on the connection of the hangers to the arch and the deck. In literature a method is described to determine vibrating lengths based on modal analysis. This paper describes finite element simulations and field tests to validate this extended taut-string analysis. Firstly a hanger is modeled with different boundary conditions, varying from a fully clamped to a fully hinged connection. By using the acceleration data from the simulation at different positions along the hanger, the hanger force is reconstructed with an accuracy of 97% to 100%. Field tests on a bridge with a symmetrical lay-out show that similar vibrating lengths are found for similar hangers, indicating a successful implementation of this extended taut-string analysis, leading to a higher accuracy in the hanger force determination.

Keywords: Hanger force measurements · vibration measurements · taut-string method · vibrating length · modal analysis

1 Introduction

Structural assessment of arch bridges through finite element modelling is key in determining remaining life time [1]. To improve the accuracy of the finite element model, hanger force determination through measurements is highly recommended [2]. Since existing structures rarely have built-in load cells, hanger forces need to be determined differently [3]. A common method to do so is through vibration measurements, using the so-called taut-string method [4]. Using this method, the hanger force can be derived from the natural frequencies of the hanger, making use of an estimation of the stiffness, mass and (vibrating) length of the cable [5].

The taut-string method has been applied in numerous bridge measurements in The Netherlands and in other countries [6]. The downside of applying this straight-forward measurement technique is that some critical parameters, like the bending stiffness, are

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not easy to determine, neither from drawings nor through measurements. On top of this, the relation between natural frequency and hanger force is strongly dependent on the end connection of the hanger [7]. These effects lead to an uncertainty in the resulting hanger forces; an experience shared by the authors based on earlier measurements.

A first literature review has shown multiple experiments to gain additional information on some relevant hanger parameters [8]. One of the most promising techniques is published in [9], describing how a modal analysis leads to additional information on the vibrating length of the hanger. By adding accelerometers to the conventional taut-string method, the accuracy of the hanger force determination can be increased significantly.

This paper focusses on increasing the accuracy of the taut-string method by adding this modal analysis. The authors are asked to perform hanger force measurements at the 'Brug over de Beneden Merwede', a steel arch bridge in The Netherlands with a length of ca. 200 m [8]. A finite element study is firstly performed on one of the hangers of this bridge in order to test the success of the modal analysis as proposed in [9]. Hereafter, the same modal analysis is applied during field tests on the Brug over de Beneden Merwede. This paper describes both the finite element study of a hanger with different boundary conditions, as the effect of the additional modal analysis in the field measurements. Based on the outcome, some recommendations for future measurements are given.

2 Hanger Force Assessment

2.1 Taut-String Method

Determining the hanger force from measured natural frequencies is described in many different papers [10, 11]. The exact relation is dependent on the boundary conditions of the hanger. Equation (1) describes the relation between hanger force F [Newton] and the natural frequency f [Hertz] of mode k for a clamped connection between hanger and bridge, while Eq. (2) lists this relation in case of a hinged connection.

$$f_k = \frac{k}{2L}\sqrt{\frac{F}{\mu}}\left(1 + 2\sqrt{\frac{EI}{FL^2}} + \left(4 + \frac{k^2\pi^2}{2}\right)\frac{EI}{FL^2}\right) \tag{1}$$

$$f_k = \frac{k}{2L}\sqrt{\frac{F}{\mu}}\sqrt{1 + \frac{k^2\pi^2}{L^2}\frac{EI}{F}} \tag{2}$$

In these equations L is the vibrating length of the hanger [m], μ is the mass of the hanger per unit length [kg/m] and EI is the bending stiffness of the hanger [N/m^2].

When all parameters apart from the hanger force and the bending stiffness are known, either from measurements (natural frequencies) or from estimations based on e.g. drawings (mass per unit length, vibrating length), the force in the hanger, as well as the bending stiffness, can be determined by solving the minimization problem as stated in Eq. (3).

$$argmin\left(\sum_{k=1}^{k_{max}} \frac{f_{k,meas} - f_{k,calc}}{f_{k,meas}}, \{F, EI\}\right) \tag{3}$$

In this equation the subscript '*meas*' stands for 'measured' and '*calc*' for calculated (using Eq. (1) or (2)), while k_{max} represents the highest mode measured. This means that by estimating a vibrating length (e.g. hanger length) and mass per unit length, a hanger force and bending stiffness are derived for which the formulas from Eq. (1) or (2) match the measurements best.

2.2 Determining Vibrating Length

Equations (1) and (2) underline the necessity of an accurate estimation of the vibrating length, since a change in length acts quadratically with respect to the resulting hanger force. As mentioned in [12], the vibrating length is not (necessarily) equal to the hanger length, but defined as "the distance between the two modal nodes close to the cable ends". By adding more measurement locations, information on the mode shape ratios as proposed in [9] can be gathered, from which the vibrating length L_k can be determined following the minimization problem as stated in Eq. (4), with a_k being the amplitude coefficient of mode k:

$$argmin\left(\sum_{k=1}^{k_{max}}\sum_{j=1}^{n}\left\{a_k cosin\left[k\pi\frac{x_j-d_k}{L_k}-\widehat{\phi_{jk}}\right]\right\}^2, \{d_k, a_k, L_k\}\right) \tag{4}$$

The mode shapes corresponding to each natural frequency are found to be in the form of sinusoidal functions, assuming the origin at one end of the hanger. Assuming no information on the boundary conditions is present, an origin shift towards the middle of the hanger is proposed in [13]. In Eq. (4), x_j is thus the location of accelerometer j with respect to the middle of the hanger [m] ($-\frac{L}{2} \leq x_j \leq \frac{L}{2}$.). With this coordinate transformation, assuming symmetric boundary conditions, the even mode shapes remain sine functions whereas the odd mode shapes become cosine functions. Hence the term *cosin*, representing a cosine-function for all odd modes and a sine-function for all even modes.

The value $\widehat{\phi_{jk}}$ in Eq. (4) denotes the mode shape ratio of accelerometer j for mode k, determined through Eq. (5).

$$\widehat{\phi}_{jk} \approx \frac{Y(x_j, \omega_k)}{Y(x_i, \omega_k)} \tag{5}$$

In this equation, $Y(x_j, \omega_k)$ is the Fourier-transform of the measured acceleration. The value $\widehat{\phi_{jk}}$ is then obtained by dividing the Fourier-transform by a common denominator $Y(x_i, \omega_k)$, which can be the result of any one of the accelerometers. Finally, in order to include asymmetry of the hanger, for instance in case of a different connection to deck and arch, the parameter d_k is introduced, representing an additional origin shift [m].

The minimization problem is now written as an optimization with three unknowns (d_k, a_k and L_k), leading to a vibrating length (per mode). This vibrating length, representing an equivalent hinged hanger, can then be used in combination with Eq. (2) to determine the hanger force.

3 Simulations

3.1 Model Description

A finite element simulation in Abaqus is set up to assess the accuracy of the hanger force determination using the taut-string method in combination with the modal analysis as described in the previous section. With the prospect of the field test, a hanger from the Brug over de Beneden Merwede is modeled. Given the fact that hanger force determination is hardest for short cables [14], the shortest cable of the bridge is modelled. Multiple simulations with different boundary conditions are conducted.

In Abaqus a hanger is modeled as a beam supported at both ends, according to the parameters as listed in Table 1. The design load of 660 kN is applied to the hanger by means of an elongation of 9.42 mm. The ends of the cable are fixed using rotational springs with different stiffnesses, to simulate a variety of boundary conditions ranging from a hinged to a clamped connection.

Table 1. Relevant parameters of the hanger modelled in Abaqus.

Parameter	Modeled value
Density	7850 kg/m^3
Length	14.32 m
Diameter	78 mm
E-modulus	210 GPa
Poison ratio	0.3
Spring stiffness	$[0, 10^3, 10^4, 10^5, \infty]$

3.2 Data-Analysis

Two different types of analysis are performed on this model: frequency analysis and explicit dynamic analysis. The first analysis is conducted with the aim of finding the natural frequencies of the system, by letting Abaqus solve the eigenvalue problem for an undamped finite element model. The second analysis is a time dependent analysis, in which the hanger is excited by an external force, perpendicular to the longitudinal axis.

In the field tests described in the next section, the sensor locations as prescribed in [9] are used. For that reason, these locations are also used in the finite element model. The accelerations at these four locations ($x = [-5.66, -4.16, -2.66, 6.16]$) during one minute of vibration are exported from Abaqus, as well as the natural frequencies following from the Abaqus frequency solver.

3.3 Results

Results from the simulations are shown in Fig. 1. This figure shows the mode shapes of the first (top) and second mode (bottom) of simulations with a fully hinged (left) and fully clamped (right) boundary condition.

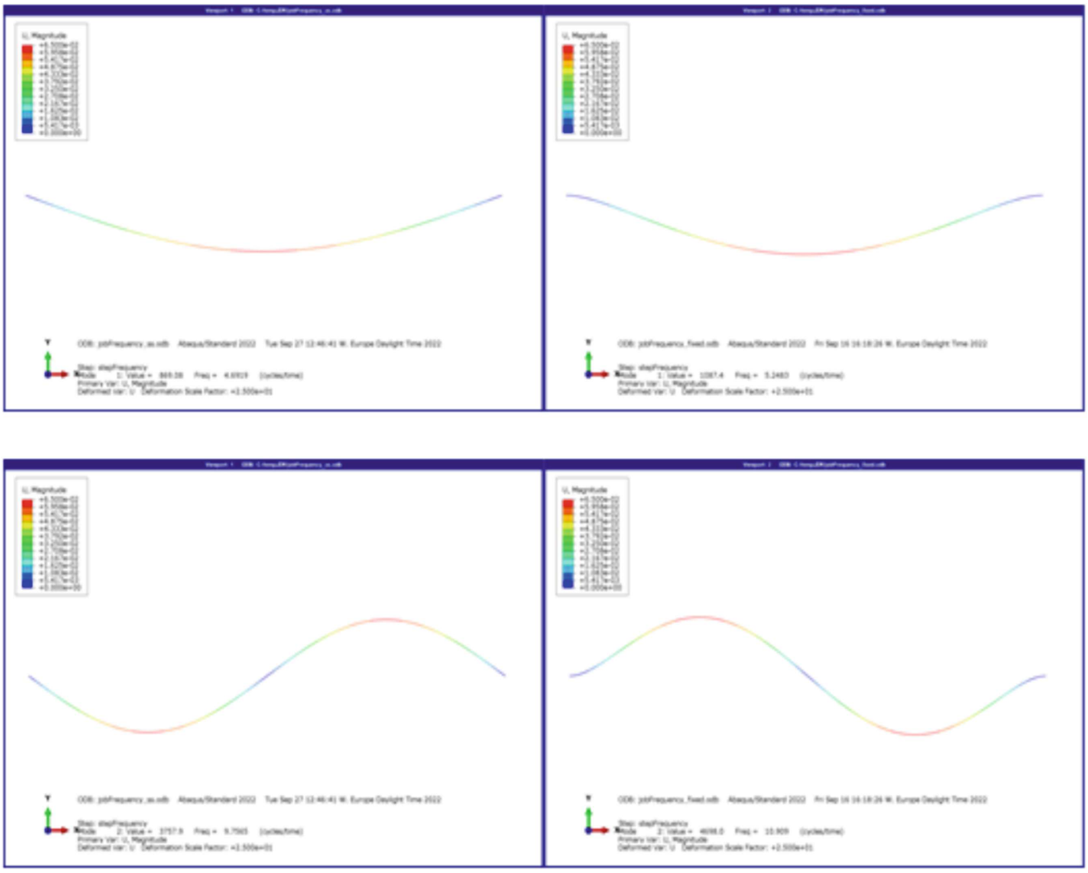

Fig. 1. First (top) and second (bottom) mode shape of the modeled hanger, using a fully hinged (left) and fully clamped (right) boundary condition.

Using the built-in matlab fourier transform function ‘fft’, a fourier transform of the acceleration signal is made, from which both the mode shapes and the natural frequencies are derived. Since the natural frequency is also retrieved from the abaqus frequency solver, Table 2 firstly shows the comparison of these frequencies for the simulation with the fully clamped hanger. it can be seen that the timestep Δt in the dynamic analysis is of influence on the resulting natural frequency. If this timestep is taken sufficiently small, results from both analysis are almost identical.

For all different boundary conditions, the resulting natural frequencies of the first six modes are listed in Table 3. It can be seen that for each mode the intermediate spring stiffnesses lead to natural frequencies that are indeed in between the frequencies found for the cases with simply supported and fixed connections.

Using Eq. (4) in combination with the accelerations following from the Abaqus simulations, a vibrating length can be determined for each simulated hanger. Together with the mass per unit length from Table 1 and the natural frequencies from Table 3, Eq. (2) is used to determine the hanger force (and bending stiffness) for each simulation.

Results are listed in Table 4, where it can be seen that that the vibrating length decreases with stiffening of the support, being in line with physics. Note that in theory one would expect a value of $d = 0$ for all cases due to the modeled symmetry; for unknown reasons the resulting value of d is slightly off. For the case of a fully hinged connection, the calculated force is (almost) equal to the applied load of 660 kN. For all other cases the force is slightly deviating, up to 3% for the extreme case of a fully clamped connection.

Table 2. Derived natural frequencies from the Abaqus frequency solver and based on the dynamic analysis, using different timesteps for the first six modes of the hanger.

Mode	Abaqus frequency solver	Dynamic, $\Delta t = 0.003$	Dynamic, $\Delta t = 0.002$	Dynamic, $\Delta t = 0.001$	Dynamic, $\Delta t = 0.0005$
1	5.25	5.25	5.25	5.25	5.256
2	10.912	10.833	10.9	10.933	10.929
3	17.33	17.183	17.267	17.333	17.349
4	24.765	24.283	24.57	24.733	24.769
5	33.378	32.233	32.867	33.267	33.371
6	43.284	40.85	42.15	43	43.228

Table 3. Natural frequency [Hz] per mode for different boundary conditions.

Mode	Simply supported	$K = 10^3$	$K = 10^4$	$K = 10^5$	Fixed support
1	4.69	4.69	4.70	4.78	5.25
2	9.76	9.76	9.78	9.92	10.91
3	15.53	15.53	15.55	15.76	17.33
4	22.25	22.25	22.28	22.54	24.77
5	30.12	30.12	30.16	30.46	33.38
6	39.23	39.26	39.30	39.63	43.28

Table 4. Results (vibrating length and hanger force) per simulated case.

Spring stiffness	Vibrating length [m]	Force [kN]	a_k [–]	d [m]
0 (Simply supported)	14.35	659	3.065	–0.015
10^3	14.37	664	3.065	–0.015
10^4	14.37	664	3.065	–0.025
10^5	14.13	645	3.266	0.005
∞ (Fixed support)	13.15	640	4.774	0.055

4 Field Test

4.1 Test Description

Hanger force measurements are performed on all hangers of the Brug over de Beneden Merwede (Fig. 2). This bridge consists of 28 hangers, all having the same characteristics and dimensions, apart from the full cable length which ranges from 14.32 m to 30.87 m. Since the bridge is symmetric in both length and width, each hanger length occurs at four locations along the bridge.

Fig. 2. Photo of the Brug over de Beneden Merwede [15].

Accelerations are recorded using wireless accelerometers (Lord Microstrain, G-Link 2000), positioned at four locations along the hanger based on the dimension of [9]: 3.4, 4.4 and 5.9 m above deck and 1.0 m below the arch (Fig. 3). The anti-vandalism covers which are normally present along the first three meter of the hangers are removed prior to the measurements. Excitation is performed using a lashing strap attached to the barrier at the deck of the bridge. Five consecutive free vibrations of one minute are analyzed per hanger. During the measurements there was no traffic present at the bridge. More details on the execution of the measurements can be found in [8].

4.2 Results

The measured accelerations of each hanger are evaluated to determine natural frequencies and vibrating lengths. Data was sampled with a rate of 256 Hz. All five consecutive vibrations per hanger are used. The followed procedure is similar to the procedure followed in the Abaqus simulations: using the built-in Matlab function 'fft' a Fast Fourier Transform is made to derive both mode shapes and natural frequencies at all four locations of the hanger. An extended description of the data-analysis can be found in [8].

To investigate the success of deriving a vibrating length per hanger, use is made of the symmetry of the bridge. It is assumed that all hangers have similar boundary conditions

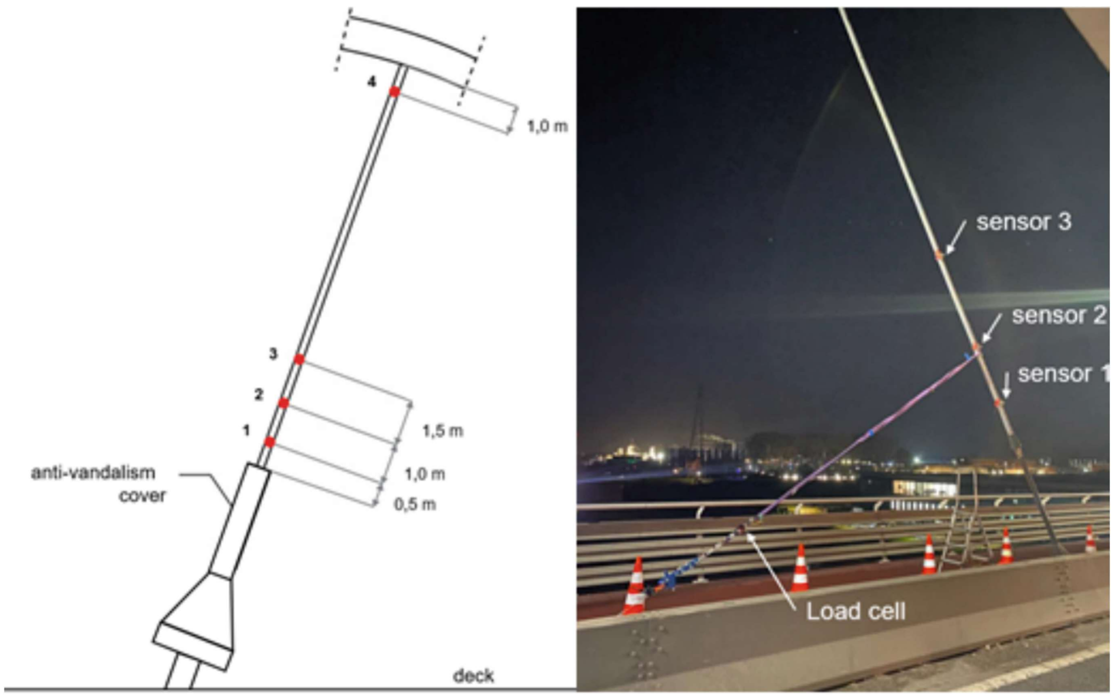

Fig. 3. Schematic depiction of the four accelerometers along the length of the hanger (left). Picture during the measurement showing three of the four sensors placed on the hanger, as well as the lashing strap used for excitation (right) [8].

at their connection with deck and arch, as is to be expected based on drawings. This assumption means that each pair of four hangers that have a similar design length must have a similar vibrating length. Table 5 lists the vibrating lengths determined in the measurement of all hangers per pair of four hangers with equal length.

Table 5. Hanger length according to design and vibrating length according to measurements, for all 28 hangers of the bridge, grouped per pair of four.

Hangerpair	Design length [m]	Vibrating length [m]	Standard deviation
1	14.32	13.29, 13.30, 13.40 and 13.58	0.13
2	22.25	21.08, 21.09, 21.04 and 21.21	0.07
3	22.23	21.17, 21.26, 21.09 and 21.09	0.08
4	27.97	27.78, 26.74, 27.13 and 26.98	0.45
5	27.89	27.02, 26.94, 26.97 and 26.95	0.14
6	30.31	29.74, 29.96, 29.92 and 30.19	0.19
7	30.87	29.91, 29.87, 29.80 and 29.91	0.05

The resulting vibrating lengths from Table 5 show that all hangers with equal design length have similar vibrating lengths. On top of this, all vibrating lengths are between 0.60 and 1.25 m less than the design length of the hanger, which is in line with the physical understanding of this vibrating length (partial clamping of 30 to 63 cm at each side of the hanger). Individual differences in vibrating length between the four hangers within one pair can either be explained by small differences in clamping conditions between the four hangers or by inaccurate positioning of the accelerometers, which is done manually for each hanger consecutively, without aiding instruments. Unfortunately, since the exact force in the hangers is unknown, no analysis can be performed to quantify the improvement of accuracy in hanger force determination by adding this modal analysis.

5 Conclusions

This paper describes the application of modal analysis to derive the vibrating length of hangers, necessary to accurately determine hanger forces based on acceleration measurements. The finite element simulations show that the hanger force can be reconstructed for different types of boundary conditions within an accuracy of 97% or more. The same methodology is applied in a field test. Although no reference hanger forces are known and verification is therefore not possible, it is shown that for similar hangers, vibration lengths are found with very little scatter. For future measurements, it is advised to develop aiding tools for a more accurate positioning of the accelerometers.

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