Beam Model Updating Based on In-Situ Measured Modal Properties of a High-Rise Building

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Abstract. This paper investigates the application of a discrete Timoshenko beam model for estimating structural properties through model updating. It is applied to the bending modes measured on the residential tower "New Orleans" in Rotterdam. A numerical study was conducted to explore the influence of three optimization algorithms (Sequential Least Squares Quadratic Programming (SLSQP), Differential Evolution, and Particle Swarm Optimization) on the model updating. The SLSQP algorithm resulted in the most accurate parameter estimations and was therefore applied to the "New Orleans" tower. A comparison with a previous study using a uniform Euler-Bernoulli beam model indicated that the discrete Timoshenko model better matched the measured modal properties and produced structural property estimates with less variability. This is explained by the ability of the discrete beam to capture local effects more accurately. It is recommended to apply this approach on a structure where shear plays a significant role. A Bayesian framework is recommended for model updating to manage uncertainties and prevent overdetermined issues.

Keywords: Model updating · Timoshenko · Global structural properties · High-rise buildings · Modal properties · Optimization algorithm

1 Introduction

The fundamental natural frequency is a crucial factor in the design of high-rise buildings due to the their sensitivity to wind-induced vibrations. Previous studies, such as [1, 2], have shown that natural frequencies derived from in-situ measurements are often 1.5 to 2 times higher than those estimated during the design phase. These differences lead to an overestimation of the wind-induced vibrations, resulting in overly conservative designs. To improve the accuracy of high-rise building designs, it is important to understand the assumptions that cause the differences.

Parameter identification can be used to detect the causes of mismatches and reduce uncertainty in dynamic models of high-rise buildings. Model updating is a parameter identification technique used to solve inverse problems by adjusting the parameters of a numerical or analytical model to align its modal characteristics with those obtained from measurements.

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When model updating is applied in combination with detailed Finite Element (FE) models, it focuses on the properties of individual structural elements. FE model updating has been successfully applied to 15-story [3] and 26-story [4] shear-core buildings. However, this approach is computationally intensive and complex, particularly when updating large FE models with a substantial number of parameters.

When the focus of the updating is on the global properties of a building, analytical beam models offer a more practical and efficient alternative to FE models. Previous studies have applied model updating to uniform Bernoulli [5] and Timoshenko [6] beam models with translational and rotational stiffness at the base to represent the foundation stiffness. However, these models do not account for variations in stiffness along the building height, leading to inaccuracies in predicting higher modes [5, 6].

This study investigates the use of a discrete beam model in the context of model updating for high-rise buildings. The Timoshenko formulation was adopted to account for the contribution of shear deformations. Different optimisation algorithms – Sequential Least Squares Quadratic Programming (SLSQP) algorithm, Differential Evolution (DE) algorithm, and the Particle Swarm Optimization (PSO) algorithm – were applied in the model updating process. A detailed FE model was used to compute the initial properties for the model updating.

Section 2 explains the discrete Timoshenko beam model, the model updating procedure, and presents the measurement data and model of the "New Orleans" tower. Section 3 discusses the results obtained. Section 4 gives conclusions and recommendations.

2 Methodology

2.1 Beam Model

The analytical beam model applied to approximate the dynamic behaviour of high-rise buildings is a discrete Timoshenko beam model (Fig. 1). The discrete superstructure is modelled with bending stiffnesses EI_x and EI_y, shear stiffnesses kG_xA and kG_yA, and masses ρA per unit length. The foundation includes rotational ($K_{r,x}$ and $K_{r,y}$) and translational springs ($K_{t,x}$ and $K_{t,y}$). The bending stiffness and the rotational stiffness of the foundation around the x-axis are denoted as EI_x and $K_{r,x}$ respectively. The translation stiffness of the foundation in x-direction is denoted as $K_{t,x}$.

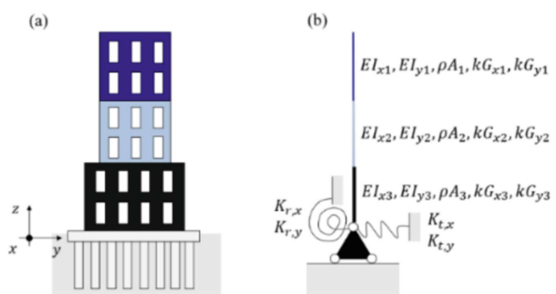

Fig. 1. A schematic representation of (a) a high-rise building and (b) the discrete Timoshenko beam model used to approximate the dynamic behaviour of that building.

The natural frequencies and mode shapes of this model are obtained by solving a system of second-order partial differential equations:

$$EI\frac{\partial^2\varphi(z,t)}{\delta z^2}+kGA\left(\frac{\partial w(z,t)}{\delta z}-\varphi(z,t)\right)-\rho I\frac{\partial^2\varphi(z,t)}{\delta t^2}=0 \tag{1}$$

$$kGA\left(\frac{\partial^2 w(z,t)}{\delta z^2}-\frac{\partial\varphi(z,t)}{\partial z}\right)-\rho A\frac{\partial^2 w(z,t)}{\delta t^2}=0 \tag{2}$$

In which $w(z,t)$ represent the displacement and $\varphi(z,t)$ the rotation of the beam, both dependent on the height z and time t. Using separation of variables, i.e. $w(z,t) = W(z)T(t)$ and $\varphi(z,t) = \Phi(z)T(t)$, a general solution is obtained, dependent on the value of the natural frequency ω.

If $\omega < \sqrt{kGA/\rho I}$:

$$\tilde{W}(\tilde{z}) = A_1\sin(p\tilde{z}) + A_2\cos(p\tilde{z}) + A_3\sinh(q\tilde{z}) + A_4\cosh(q\tilde{z}) \tag{3}$$

$$\tilde{\Phi}(\tilde{z}) = B_1\sin(p\tilde{z}) + B_2\cos(p\tilde{z}) + B_3\sinh(q\tilde{z}) + B_4\cosh(q\tilde{z}) \tag{4}$$

If $\omega > \sqrt{kGA/\rho I}$:

$$\tilde{W}(\tilde{z}) = C_1\sin(p\tilde{z}) + C_2\cos(p\tilde{z}) + C_3\sin(q\tilde{z}) + C_4\cos(q\tilde{z}) \tag{5}$$

$$\tilde{\Phi}(\tilde{z}) = D_1\sin(p\tilde{z}) + D_2\cos(p\tilde{z}) + D_3\sin(q\tilde{z}) + D_4\cos(q\tilde{z}) \tag{6}$$

In which $\omega = \sqrt{kGA/\rho I}$ is the transition frequency, $\tilde{W}(\tilde{z}) = W(z)/L$ and $\tilde{z} = z/L$, and coefficients p, q, A_{1-4}, B_{1-4}, C_{1-4}, D_{1-4} are dependent on the applied boundary and initial conditions. The boundary conditions for this model in x- and y-direction are:

$$\frac{kGA}{L}\left(\tilde{\Phi}(\tilde{z})-\frac{d\tilde{W}(\tilde{z})}{d\tilde{z}}\right)+K_t\tilde{W}(\tilde{z}) = 0 \text{ for } z=0 \tag{7}$$

$$\frac{EI}{L}\frac{d\tilde{\Phi}(\tilde{z})}{d\tilde{z}}-K_r\tilde{\Phi}(\tilde{z}) = 0 \text{ for } z=0 \tag{8}$$

$$\frac{d\tilde{\Phi}(\tilde{z})}{d\tilde{z}} = 0 \text{ for } z=L \tag{9}$$

$$\tilde{\Phi}(\tilde{z})-\frac{d\tilde{W}(\tilde{z})}{d\tilde{z}} = 0 \text{ for } \mathrm{z}=\mathrm{L} \tag{10}$$

The continuity conditions in x- and y-direction are:

$$\tilde{W}_l(\tilde{z}) = \tilde{W}_r(\tilde{z}) \tag{11}$$

$$\frac{d\tilde{W}_l(\tilde{z})}{d\tilde{z}} = \frac{d\tilde{W}_r(\tilde{z})}{d\tilde{z}} \tag{12}$$

$$\frac{d\tilde{\Phi}_l(\tilde{z})}{d\tilde{z}} = \frac{EI_r}{EI_l}\frac{d\tilde{\Phi}_r(\tilde{z})}{d\tilde{z}} \tag{13}$$

$$\tilde{\Phi}_l(\tilde{z}) - \frac{d\tilde{W}_l(\tilde{z})}{d\tilde{z}} = \frac{kGA_r}{kGA_l}\left(\tilde{\Phi}_r(\tilde{z}) - \frac{d\tilde{W}_r(\tilde{z})}{d\tilde{z}}\right) \tag{14}$$

Here, l denotes the left side of the continuity condition, and r denotes the right side of the continuity condition. More information of the derivation of the discrete Timoshenko beam model can be found in [7].

2.2 Model Setup and Measurement Data of the "New Orleans" Tower

The discrete beam model approach is applied on the residential tower "New Orleans" in Rotterdam, shown in Fig. 3a. The discrete Timoshenko beam model used to approximate the dynamic behaviour of the "New Orleans" is created using the Finite Element (FE) model of the building. This FE model (Fig. 2) is divided into five segments (0 till IV). The division is based on the height segments of the building, which have the same floor plan in the FE model. The initial structural property values applied in the beam model are listed in Table 1. The floor plans were used to determine the cross-sectional area A and the second moment of area I, while the FE model was used to determine the segment length L, Young's modulus E, and density ρ. A detailed explanation of the approach is found in [7].

Vibrations were measured using acceleration sensors installed on the 15^{th}, 34^{th} and 40^{th} floor, as shown in Fig. 3b, 3c and 3d. The measured modal properties are listed in Table 2. The natural frequencies and mode shapes were obtained using the Frequency Domain Decomposition technique, as described in [1].

2.3 Model Updating Procedure and Settings

Model updating is performed using the objective function:

$$J = w_f \sum_{i=1}^{N} \frac{|f_{n,i} - \hat{f}_{n,i}|}{\hat{f}_{n,i}} + w_\phi \sum_{i=1}^{N} \left(1 - MAC(\phi_i, \hat{\phi}_i)\right) \tag{15}$$

In which index i denotes the mode considered, $f_{n,i}$ and ϕ_i denote respectively the estimated natural frequencies and mode shapes, and $\hat{f}_{n,i}$ and $\hat{\phi}_i$ the measured natural frequencies and mode shapes. To quantify the mode shape mismatch, the Modal Assurance Criterion (MAC) is used:

$$MAC(\phi_i, \hat{\phi}_i) = \frac{|\phi_i \cdot \hat{\phi}_i|^2}{(\phi_i \hat{\phi}_i)(\phi_i \cdot \hat{\phi}_i)} \tag{16}$$

The MAC returns a value between 0 and 1, indicating no or complete similarity between the two mode shapes. The weight factors w_f or w_ϕ determine the error contributions of the terms in the objective function. Both are set to 1, in correspondence with [5, 6].

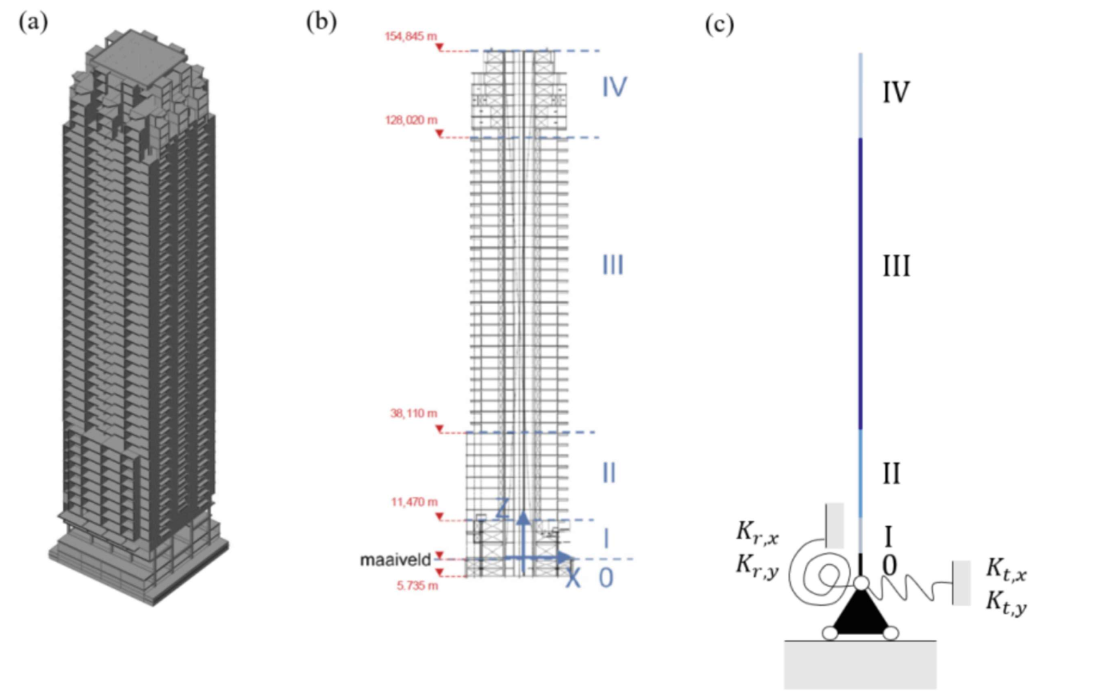

Fig. 2. Pictures of (a) the FE model of the "New Orleans", (b) the 5 segments defined for the "New Orleans" model, and (c) the five-segment Timoshenko beam model.

Table 1. The structural property values obtained with the floor plans and FE model of the "New Orleans".

Segment	0	I	II	III	IV
L[m]	5.735	11.47	26.640	89.91	26.825
A[m^2]	784	784	784	784	784
I_x[m^4]	1444	1500	1393	1389	1293
I_y[m^4]	1532	1548	2796	2324	760
ρ[kg/m^3]	1933	495	486	440	383
E[N/m^2]	$38.2{\cdot}10^9$	$38.2{\cdot}10^9$	$38.2{\cdot}10^9$	$38.2{\cdot}10^9$	$32.8{\cdot}10^9$
υ[-]	0.2	0.2	0.2	0.2	0.2
G[N/m^2]	$15.9{\cdot}10^9$	$15.9{\cdot}10^9$	$15.9{\cdot}10^9$	$15.9{\cdot}10^9$	$13.7{\cdot}10^9$
k[-]	0.85	0.85	0.85	0.85	0.85
Boundary					
$K_{r,x}$[Nm/rad]	$2.625{\cdot}10^{12}$	$K_{t,x}$[N/m]	$4{\cdot}10^9$		
$K_{r,y}$[Nm/rad]	$2.380{\cdot}10^{12}$	$K_{r,y}$[N/m]	$19{\cdot}109$		

The model updating procedure starts with an initial selection of input parameter values, uniformly sampled of a value range corresponding to their initial structural property values in Table 1, scaled by a factor of ten. These initial values result in a mismatch between the reference modal properties $\hat{f}_{n,i}$ and $\hat{\phi}_i$ and the estimated modal properties

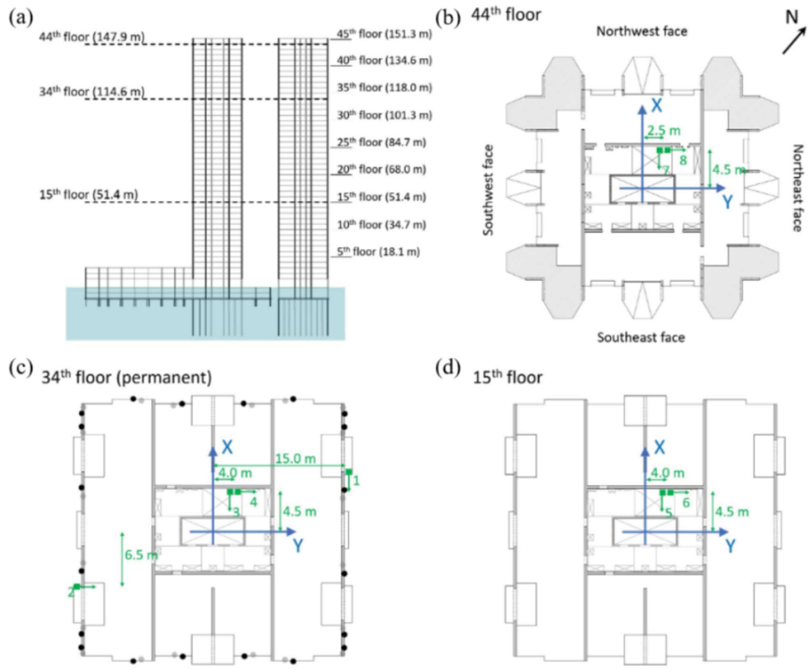

Fig. 3. Pictures of (a) the "New Orleans" and the positions of the acceleration sensors (green squares) on (b) the 44th floor, (c) the 34th floor, and (d) the 15th floor. The green arrows indicate the directions of the acceleration sensors. Taken from [1].

Table 2. The measured natural frequencies and modal displacements.

Mode	1	2	3	4	5
Dominant direction	X	Y	Y	X	Y
Natural frequency [Hz]	0.282	0.291	1.332	1.527	2.771
Height 1 (51.4 m)	−0.30	−0.29	−0.90	0.89	−0.73
Height 2 (114.6 m)	−0.71	−0.78	0.19	−0.23	0.83
Height 3 (147.9 m)	−1.00	−1.00	1.00	−1.00	−1.00

$f_{n,i}$ and ϕ_i, quantified with Eq. (15). Based on the value and the gradient of this function, the structural properties are updated iteratively by the chosen algorithm, reducing the mismatch between the reference and estimated modal properties. The updating process is ended when the gradient of the cost function drops below a limit value of 10^{-6}.

The updated parameters are K_r, K_t, E and I. Parameters A and ρ are excluded, as including the mass would over-determine the optimization problem. It is assumed that the mass is predicted more accurately than the stiffness in the design of high-rise buildings and is therefore not included in the model updating. Based on a sensitivity study, it is determined that the influence of changing kG within the considered value range is minimal in both x- and y-direction. This outcome can be explained by the fact that the "New Orleans" tower is expected to be primarily bending-dominant, based on its structural characteristics. Therefore, parameter kG is also excluded.

It is assumed that the ratios of the initial structural properties values in Table 1 between the segments are well estimated. By maintaining these ratios, only one set of parameters of the superstructure needs to be updated, thereby reducing the total number of parameters that needs to be updated.

Due to the non-linear nature of the cost function, the obtained solution is most likely a local minimum. A different initial selection of structural properties will result in a different solution. Therefore, the model updating is repeated for various initial selections of the structural properties.

3 Results and Discussion

3.1 Sensitivity Study on Numerical Case

To investigate the influence of the applied algorithm and to select the most suitable algorithm for model updating of the "New Orleans" tower, a sensitivity study was conducted using a numerical case.

The numerical case was created by applying random scalars to the initial values of the chosen updating parameters (Table 1) to generate another model with known structural properties and modal properties. The scalars applied are listed in Table 3. This model modal properties were selected as measured modal properties for the study.

Different amounts of initial solutions were considered, and for each set, the solution with the lowest cost function after model updating was retained. The optimization algorithms applied were the Sequential Least Squares Quadratic Programming (SLSQP) algorithm, the Differential Evolution (DE) algorithm, and the Particle Swarm Optimization (PSO) algorithm. SLSQP is an individual-based optimization algorithm that iteratively refines one initial solution at a time to minimize Eq. (15). In contrast, DE and PSO are population-based algorithms that evaluate multiple solutions simultaneously in each iteration. Further details on the algorithms and their differences can be found in [8]. The results for EI are shown in Fig. 4 as ratios relative to their initial values in Table 1. The differences between the algorithms are minimal. Despite using only a small number of initial solutions, the SLSQP algorithm was able to find the global minimum. This indicates that the solution space is well-conditioned, as individual-based algorithms are generally more prone to getting trapped in local minima. As the SLSQP algorithm achieved the most accurate estimation for all parameters, this algorithm is chosen for the "New Orleans" tower.

3.2 Model Updating "New Orleans"

The model updating procedure uses 400 initial solutions and follows the approach as described in [6]. Compared to [6], the lowest octile of the solutions is retained rather than the lowest quantile, as the discrete Timoshenko beam model is expected to result in a more complex solution space for identifying the global minimum than the uniform Euler-Bernoulli beam model.

Table 4 gives the estimated and measured natural frequencies and the Modal Assurance Criterion (MAC) values for the mode shapes. Case 1 gives the results of the model

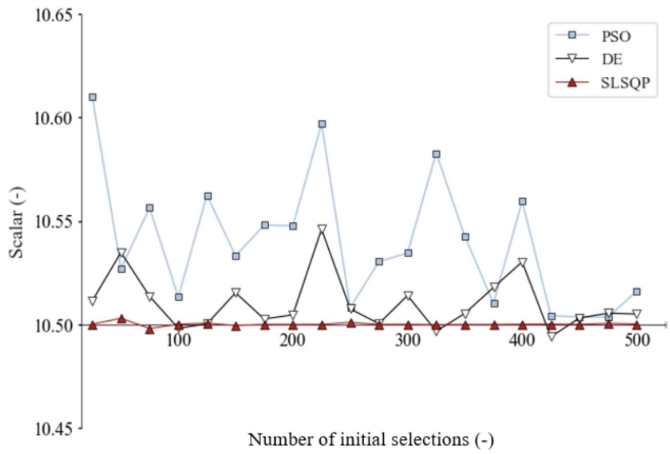

Fig. 4. The influence of different algorithms on the results of the product EI.

Table 3. The random scalar applied on the initial structural properties values of Table 1.

	K_r	K_t	E	I
Scalar [-]	0.4	9.0	7.0	1.5

updating using the two lowest bending modes in both x- and y-directions, and Case 2 the results of the model updating using two bending modes in the x-direction and three bending modes in the y-direction. The results of [6] are included for comparison.

Compared to the uniform Euler-Bernoulli beam model applied in [6], improvements in matching the measured modal properties are observed with the discrete Timoshenko beam model for Case 1 and especially Case 2. For Case 2, discrepancies exist between the estimated and measured first natural frequency in the y-direction, which were also observed for the uniform Euler-Bernoulli beam in [6]. The error in the third mode is significantly reduced compared to [6]. The discrete beam model captures local effects more accurately, which are more important for higher bending modes. This could explain the improvements observed in the third bending mode. The inclusion of shear in the beam model may also contribute to the improved estimation, as it also becomes more influential at higher bending modes. Since Case 2 did not predict the modal properties as well as Case 1, only the structural property values obtained with the discrete Timoshenko beam model for Case 1 are considered here.

Table 5 list the estimated structural property values in the x- and y-directions obtained through model updating for Case 1. For the estimates per parameter, the Coefficient of Variation (CV) is calculated. The CV shows the relative variability of the estimates:

$$CV = \left(\frac{\sigma}{\mu}\right) \times 100\% \tag{17}$$

In which σ is the standard deviation and μ is the mean. The results of [6] are also included.

Table 4. The measured and estimated natural frequencies and MAC values between the measured and estimated mode shapes.

Results	Natural frequency [Hz]				MAC [-]	
Mode	Design	Measured	Case 1 [Δ_m %]	Case 2 [Δ_m %]	Measured vs. Case 1	Measured vs. Case 2
1 (x)	0.200	0.282	0.282 (0%)	0.282 (0%)	0.998	0.998
2 (x)	1.020	1.527	1.527 (0%)	1.527 (0%)	0.999	0.999
1 (y)	0.230	0.291	0.291 (0%)	0.241 (−17.2%)	0.999	0.995
2 (y)	2.140	1.332	1.332 (0%)	1.332 (0%)	1.000	0.963
3 (y)	2.440	2.771	-	2.773 (+0.1%)	-	0.872
Results of [6]						
1 (x)			0.282 (0%)	0.286 (+1.4%)	0.998	0.976
2 (x)			1.516 (−0.7%)	1.548 (+1.4%)	0.997	0.955
1 (y)			0.291 (0%)	0.253 (−13.1%)	0.997	0.992
2 (y)			1.332 (0%)	1.333 (+0.1%)	0.999	0.985
3 (y)			-	3.111 (+12.3%)	-	0.781

Compared to the uniform Euler-Bernoulli beam model applied in [6], the discrete Timoshenko beam model shows improvements in the Coefficient of Variations, indicating more stable outcomes in the model-updating process, regardless of the chosen initialization. The significant reduction in the variability of $K_{r,x}$ shows the influence of the discrete beam approach in estimating this parameter. The results of [6] showed differences in the median values between $K_{r,x}$ and $K_{r,y}$, despite the square building planform and the almost uniform pile plan below the tower. With the discrete Timoshenko beam model, the median values of the rotational stiffnesses are more comparable, better aligning with the expected structural symmetry.

The bending stiffness $EI_y = 9.70 \cdot 10^{13}$ in [6] was larger than the bending stiffness $EI_x = 6.51 \cdot 10^{13}$, which was surprising given that the first natural frequency in the x-direction was smaller than in the y-direction. The larger first natural frequency in y-direction was stated in [6] to be the result of a larger rotational stiffness around the x-axis ($K_{r,x}$). Model updating with the Timoshenko beam model shows segments (0, I and IV) where the bending stiffness EI_x is greater than EI_y, which aligns more closely with expectations. The influence of the rotational stiffness $K_{r,x}$ on the natural frequency remains, as $K_{r,x}$ is still larger than $K_{r,y}$.

Table 5. Results of the model updating procedure using 2 bending modes in x- and y-direction. For each property, the median and the coefficient of variation are computed.

Property	Median	Median of [6]	CV	CV of [6]
$EI_{0,x}$[Nm^2]	$1.21 \cdot 10^{14}$	$6.51 \cdot 10^{13}$	5.06%	7.9%
$EI_{0,y}$[Nm^2]	$1.16 \cdot 10^{14}$	$9.70 \cdot 10^{13}$	15.45%	7.0%
$EI_{I,x}$[Nm^2]	$1.25 \cdot 10^{14}$	$6.51 \cdot 10^{13}$	5.06%	7.9%
$EI_{I,y}$[N/m^2]	$1.17 \cdot 10^{14}$	$9.70 \cdot 10^{13}$	15.45%	7.0%
$EI_{II,x}$[Nm^2]	$1.16 \cdot 10^{14}$	$6.51 \cdot 10^{13}$	5.06%	7.9%
$EI_{II,y}$[Nm^2]	$2.13 \cdot 10^{14}$	$9.70 \cdot 10^{13}$	15.45%	7.0%
$EI_{III,x}$[Nm^2]	$1.16 \cdot 10^{14}$	$6.51 \cdot 10^{13}$	5.06%	7.9%
$EI_{III,y}$[Nm^2]	$1.76 \cdot 10^{14}$	$9.70 \cdot 10^{13}$	15.45%	7.0%
$EI_{IV,x}$[Nm^2]	$1.08 \cdot 10^{14}$	$6.51 \cdot 10^{13}$	5.06%	7.9%
$EI_{IV,y}$[Nm^2]	$5.78 \cdot 10^{13}$	$9.70 \cdot 10^{13}$	15.45%	7.0%
$K_{t,x}$[N/m]	$2.80 \cdot 10^{9}$	$2.20 \cdot 10^{9}$	2.27%	5.9%
$K_{t,y}$[N/m]	$5.70 \cdot 10^{9}$	$2.85 \cdot 10^{9}$	5.87%	5.4%
$K_{r,x}$[Nm/rad]	$2.89 \cdot 10^{12}$	$1.11 \cdot 10^{13}$	4.54%	46.9%
$K_{r,y}$[Nm/rad]	$2.14 \cdot 10^{12}$	$3.07 \cdot 10^{12}$	6.91%	5.8%

4 Conclusion

The modal properties obtained through model updating of the discrete Timoshenko beam model better match those obtained with a uniform Euler-Bernoulli beam in previous work. The discrete Timoshenko beam model showed a reduction in error when estimating the third bending mode. The discrete beam approach captures local effects more accurately. The inclusion of shear in the beam model may also contribute to the improved estimation.

The results for the structural properties indicate that most of the properties obtained through model updating of the discrete Timoshenko beam model show less variability than those obtained with the uniform Euler-Bernoulli beam model. For parameter $K_{r,y}$, the variability significantly decreased, showing the influence of the discrete beam approach in estimating this parameter. The increase in variability of EI_y suggests that the maintained ratios of the initial bending stiffness values in the y-direction do not accurately reflect the true stiffness distribution in that direction, limiting the accuracy of the results obtained.

It is expected that the "New Orleans" tower is primarily bending-dominant in behaviour. A sensitivity study showed that the influence of parameter kG was minimal. To assess the impact of shear, it is recommended to test this approach on a structure where shear plays a significant role. For this case study, sufficient modal data should be available, as an additional parameter needs to be updated.

Fixing the ratios between the discrete segments gives weight to the initial structural property values, which need to be accurate for a reliable estimation of the structural properties. Therefore, it is recommended to allow some flexibility in these ratios, acknowledging that they may not be entirely accurate. Exploring model updating without keeping these ratios constant would be valuable. However, given the large number of parameters involved, this approach could potentially lead to an overdetermined problem. Applying a Bayesian framework to the model updating could help address this issue as it allows for regularization of the problem, as well as allowing for an appropriate consideration of uncertainties related to the measurements, modal analysis and model.

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