



Discussion

Reply to the note by Li Piani et al.

T. Li Piani ^{a, b, c, *}, J. Weerheijm ^{b, c}, L.J. Sluys ^b^a TU Delft, Stevinweg 1, 2628 CN, Delft, the Netherlands^b TNO, PO Box 45, 2280 AA, Rijswijk, the Netherlands^c NLDA, Faculty of Military Sciences, 1781 CA, Den Helder, the Netherlands

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In 2017, a ballistic phenomenological model was proposed by the authors of Ref. [1] to numerically simulate the experimental depths of small caliber projectiles impacting walls made of adobe. The opportunity for a new model in the field revealed from the observation that two older models recently used by the authors of Ref. [2] shared a linear relationship between the penetration depth P and the impacting velocity v_0 , which was experimentally confirmed by the ballistic tests performed by the authors of Ref. [1] as presented in Ref. [3]. However, these two models substantially differ in the physical hypotheses used to interpret the penetration process in adobe targets. Thus, it was concluded that the process of penetration in Adobe was still unclear and it was plausible to propose a new approach. The new approach in Ref. [1] was consistent with the results of experimental trends observed in previous experimental campaigns in Refs. [4,5], which pioneered adobe as a quasi-brittle material similar to concrete. This hypothesis was lately used by several other authors in the field of adobe (e.g. Ref. [6]).

However, the authors of Ref. [2] now claim that the work of Ref. [1] implicitly criticized their approach and instead claim that the ballistic model in Ref. [1] is not suitable for interpreting the process of penetration into targets of adobe. In particular, the criticism of the authors of Ref. [2] on the approach of authors in Ref. [1] is based on two claims:

- a) *'The resisting force is zero at the beginning ($v = v_0$), and at the end of the penetration process (...) the value of the force is $-\mu\rho TgAPmPv_0$ (...) It is concluded that the fact that the force increases during the deceleration of the projectile is suitable for describing the behavior of a densifying foam upon low-velocity impact but not adobe'.*

In order to counter this claim, a comprehensive background and reasoning of the proposed model is given in this rebuttal. The ballistic phenomenological model in Ref. [1] starts its premises from the well-known so called 'Forrestal model' (Ref. [7]). This was developed in the early '90ties at the Sandia National Laboratories following the investigations at the US Naval weapons laboratory on the penetration process on concrete and soil targets based on ogive-nose projectiles. From these experiments, it was observed that the penetration process of concrete could be distinguished in a crater region ($<2D$, with D the diameter of the impactor) and in a tunnel region ($>2D$), respectively. The corresponding mathematical framework was translated in Ref. [7] in the following equation of motion:

$$-m \frac{d^2x}{dt^2} = R_1 = cx \quad x < 2D \quad (1a)$$

$$-m \frac{d^2x}{dt^2} = R_2 = \pi D^2 (a\tau_0 + BN\rho v^2) \quad x > 2D \quad (1b)$$

Where m is the mass-of the projectile, t is time, x is the horizontal coordinate at time t of the impactor, c is a constant, $a\tau_0$ depends on the shear strength, N is a nose shape factor, ρ is the density of the target, v is the velocity of the impactor at time t and B a compressibility factor. In the Forrestal model, the force at impact location ($x = 0$, $v = v_0$) is, of course, 0. In fact, this model belongs to the so called 'cavity expansion theory'. This family of models shapes the force required to open a crater starting from a 0 mm initial opening, by modelling the resisting forces ahead of the impactor nose. This is indeed the conceptual framework which has been adopted by the authors in Ref. [1], which motivates the use of Eq. (1) in the crater region. In the tunneling region, only a shear Coulomb force has been implemented. For continuity reasons, this corresponds to Eq. (3).

DOI of original article: <https://doi.org/10.1016/j.dt.2021.09.003>.* Corresponding author. TU Delft, Stevinweg 1, 2628 CN, Delft, the Netherlands.
E-mail address: t.liyani@tudelft.nl (T. Li Piani).

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$$-m \frac{d^2 x}{dt^2} = cx \quad (2)$$

$$c = \mu g \rho_t A \quad (3)$$

Where A is the cross-sectional area of the impactor and μ is a friction coefficient. Eqs. (2) and (3) correspond to a setting where the inertial contribution of resistance is neglected and only shear is considered. This heavy simplification in the model developed for adobe was supported by some recent empirical evidence and experimental results. On the one hand, it was observed that a weak correlation was obtained by the authors of Ref. [2,8] in Refs. [9,10], by fitting experimental laboratory tests on adobe with inertial force based phenomenological ballistic models (proportional to V^2). This observation was strengthened by the data of the ballistic test campaign performed by the authors of Ref. [1]. These empirical findings were further supported by some recent experimental evidence emerging from partially confidential projects run in US laboratories, which revealed that in case of ballistic impacts into thick soil-based targets, shear resistance is dominant with respect to compression forces, and assumes the form of a frictional resistance. At this point, it is worthy to stress out that a Coulomb approach does not solely correspond to a 'static' approach, but to a material constitutive law, potentially valid also in a dynamic and impact regime provided strain rate dependencies are taken into account. Ref. [11] is recommended providing thorough background of the resisting mechanisms involved in high velocity penetration in soil based targets. On this basis, the model developed in Ref. [1] corresponds to the Forrestal framework where the inertial term is neglected and a Coulomb shear force is used. This formulation allowed to heavily simplify the math of the model. The used Coulomb friction law has been already extensively used in other phenomenological ballistic models developed for soil-based materials [12]. For frictional drag in soil, the combination of hydrostatic pressure and Coulomb friction gives $R = \mu g \rho_t A x$ as in Eq. (4). Of course, more complex shear laws can be used. For instance, other authors argued that the form of resisting force $F(z)$ should vary from quadratic to constant, owing to the shape of the projectile and of growing crater excavated by its motion. Overall, starting from Eq. (4), the math of the model can be unveiled in the following steps:

$$-m \frac{d^2 x}{dt^2} = cx = \mu g \rho_t A x \quad (4)$$

$$-m v dv = c x dx \quad (5)$$

$$-m \int_{v_0}^{v_f} v dv = c \int_{x_0}^{x_f} x dx \quad (6)$$

$$-m \left[v_f^2 - v_0^2 \right] = c \left[x_f^2 - x_0^2 \right] \quad (7)$$

Given $x_0 = 0$, and when x equals the penetration depth P the corresponding final velocity is 0, these equations evolve into Eqs. (8)–(10). Generalizing k in Eq. (10b):

$$-m \left[-v_0^2 \right] = c \left[x_f^2 \right] \quad (8)$$

$$\frac{m}{c} v_0^2 = P^2 \quad P = \sqrt{\frac{m}{c}} v_0 \quad (9)$$

$$\sqrt{\frac{m}{\mu g \rho_t A}} v_0 = k v_0 \quad (10)$$

$$k \sim \sqrt{\frac{m}{\mu g \rho_t A}} \quad (10b)$$

In the setting of Eq. (10), a linear relationship between v_0 and P emerges with constant k . Alternatively, the kinetic energy of the projectile at a given time during penetration is the total initial kinetic energy subtracted by the dissipated energy via Coulomb friction dislocated between grains and projectile ahead of projectile nose as from Eqs. (11)–(13).

$$-m \left[v_f^2 - v_0^2 \right] = c \left[x_f^2 - x_0^2 \right] \quad (11)$$

$$-m \left[v_f^2 - v_0^2 \right] = c \left[x_f^2 \right] \quad (12)$$

$$-m \left[v_f^2 \right] + m \left[v_0^2 \right] = c \left[x_f^2 \right] \quad (13)$$

$$m \left[v_f^2 \right] = m \left[v_0^2 \right] - c \left[x_f^2 \right] \quad (14)$$

To conclude, the mathematical formulation of the model in Ref. [1] and the physical meaning of resistance R have been explained and confirmed.

Overall, as pointed out by the independent reviewer invited to review the submitted note of Ref. [2], the model of Ref. [1] is correct and consistent with a Poncelet approach.

b) 'The model is not P/D scale-independent.'

The authors in Ref. [2] claim that the model of Ref. [1] is not scale independent and that this contradicts the scale independent relationships of the models used by the authors in Ref. [2], to interpret their experimental data. The authors of Ref. [1] confirm that the model is not linearly scale independent. However, this evidence does not necessarily contradict the physical hypotheses of the model, nor its consistency with the physics of the problem. Claims of scale independence by definition for any ballistic phenomenological model are not definitely supported by physical principles or theories at the current stage. Let us briefly review the elementary models developed over time. The model used since 2011 by the authors starting in Ref. [9] is based on a class of phenomenological models called analytical models (or semi-empirical). Using II Newton's laws, these parametrize the sources of energy dissipation governing penetration into an inertial term (proportional to the square of projectile velocity), a viscous term (proportional to projectile velocity) and a bearing strength term. These terms are presented in Eq. (15) (Ref. [13]). The earliest analytical model is from Robins-Euler in the middle of XVIII century, who assumed a constant resistance over penetration. Integration in order to determine the penetration length of the projectile in the target results in Eq. (16). One of the most widely used models nowadays, especially for sandy and concrete materials, was presented by Poncelet in 1839, who defined the resistance of penetration as the sum of an inertial and bearing strength component as in Eq. (17). Ignoring the bearing strength leads instead to the penetration length in Eq. (18), developed first by Resal. The

approach developed by Resal in 1895 was adopted in 2011 by the authors of Ref. [2] as the initial framework to interpret experimental data of spherical impact tests on semi-infinite adobe targets. In the case of spherical impactors, Eqs. (16)–(18) show a cubic dependence between penetration depth and impactor diameter.

$$-\frac{mdv}{dt} = Av^2 + Bv + C \quad (15)$$

$$P = \frac{mv^2}{2C} \text{ Robins} \quad (16)$$

$$P = \frac{m}{2A} \ln \left(1 + \frac{Av^2}{C} \right) \text{ Poncelet} \quad (17)$$

$$P = \frac{m}{A} \ln \left(1 + \frac{Av^2}{B} \right) \text{ Resal} \quad (18)$$

$$\frac{P}{D} \sim \frac{\rho_p}{B\rho_t} v_o \quad (19)$$

However, by fitting the model in Eq. (18) with the values of penetration depths P from experiments, the authors of Ref. [2] revealed a weak correlation with the inertial term ($A0$), with respect to the viscous one (B). Thus, the general Eq. (15) could be reformulated by considering only B and excluding A and C . This led to a simplified equation for P shown in Eq. (19). The resulting model yields that given a same velocity and mass of the projectile, P is linearly dependent of the diameter of the impacting sphere, namely the model is geometrically independent. The authors of Ref. [2] claim that this linear relationship fits well with their experimental data and these reviewers do not have reasons to doubt on this statement. Further experimental reference is indeed needed to clarify all the unknowns which characterized so called ‘not-engineered materials’ as adobe, especially against high velocity impacts with respect to simplified models and inherent uncertainties caused by basic physical hypotheses. This wish for new tests was also reported in the publication of the authors of Ref. [1]. For this reason, real shooting tests using wide ranges of impactors shapes and impacting velocities are still needed. Utilizing the public data of an in-field small caliber shooting tests as in Ref. [3], a large scatter when using the hypothesis of linear scale independence was observed. This happens because, not only the diameter, but the shape of the projectile exerts a dominant influence on the overall impact response. Also the Forrestal model based on ogive impactors, is not solely scale independent, but is equipped with a nose shape factor. Also factor k in Ref. [1] depends on a series of variables including the friction coefficient and the nose shape factor. If being scale independent is not a definite requirement yet (at the current state of the art), most of literature agree ballistic phenomenological model parameters shall not being dimensional dependent. This was the claim of the model in Ref. [1], whose model is consistently implemented. In fact, the origin of the model in Ref. [1] does not refer to the independence of scale, but to the independence of dimension (which are different concepts). Dimensional dependent variables expose to controversy and debate on the physical interpretation of the parameters used to calibrate any model with respect to experimental data. For instance, from Eq. (15), only C has a unit of measure directly referred to the physical resistance the term incorporates. A is proportional to $[\text{kg/m}]$ and B is proportional to

$[\text{kg/s}]$. Physically interpreting these parameters and possibly curing the dimensional dependence of many other empirical models is a task scientific works are currently focusing on (Ref. [14]). Instead, the parameter k of the model of the authors in Ref. [1] is consistent with a cavity expansion theory and may assume also a physical interpretation of amplification of dynamic effects (DIF), as well as Eq. (10) is dimensionally independent. The overall framework of the authors in Ref. [1] lies on interpreting adobe as a quasi-brittle material, following the results of previous experimental evidence followed by further experimental and numerical validations in the static as well as in the dynamic regime. Overall, the model in Ref. [1] represents an alternative vision on a series of previous models and approaches whose physical consistency has not been fully assessed yet. For instance, the material parameters A and B in Eqs. (16)–(19) may be related to dimensionless drag coefficients in aero and fluid dynamics by employing momentum transfer and Newton’s third law. In this sense, A can be related to the Newton quadratic drag force model, while B is linked to the Stokes’ drag law for cylindrical bodies as in Eq. (19). Translated to solids, the first term represents the ‘dynamic pressure’ determined by the inertia of the material in front of the projectile. The second term used by the authors of Ref. [2] to interpolate experimental data on adobe, may correspond to the shear-resistance of the target material activated along the projectile. On the one hand, treating adobe targets as Stoke’s fluids and thus considering only the contribution to resistance of the shear layers activated along the penetrating projectile, implies neglecting the effect of adobe ahead of the impactors. On the other hand, the same authors of Ref. [2] also used a different model directly coming from satellite impacts which is based on a shock wave approach. Both are plausible, neither are definitely correct nor consistent in their physical interpretations of ballistic impact on adobe. In fact, the models as shown in this submission have been already previously published. In the current stages, all of these models including in Ref. [1] are simplistic interpretations of the physical reality. The set of hypotheses at the basis of all the analytical models following Eq. (15) is in itself far from being physically consistent with real tests and actual reality. Hypothesizing a rectilinear trajectory with no deviations, intact projectile at the end of the test as well as constant density of the target are hypotheses which can not be (hardly) met in any ballistic tests. Thus, any of the ballistic phenomenological models should not be considered to assess the physics of adobe, nor published in scientific journals?

Overall, these reviewers feel sorry if Ref. [1] might have appeared to implicitly criticized previous papers. This was not in the intention of the authors. On the other hand, as pointed out by the independent reviewer, also in the view of these reviewers the model of Ref. [1] is plausible and instead it is self-evident that the elucubrations and interpretations as emerging from the submitted note unfortunately appear to be out of context, inconsistent, and ultimately, wrong. However, these reviewers hope that the provided comments will be of any help to authors of the note to refine and update the message of the submitted paper in the revision process. In fact, these reviewers are in favour of the publication of the note, because the option of a double publication will enhance and consolidate the reasoning on the premises, content and interpretation of the model in Ref. [1]. Scientific debate is always welcome to promote knowledge and progress.

Declaration of competing interest

No conflict of interest.

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