Wake and wind farm modelling

Part B: Wake gust modelling

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Authors

F.J. Verheij (TNO) D. Delaunay (CSTB)

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Intended for

- CEC, DG-XII, att. Dr. G. Caratti Rue de la Loi 200
 B-1049 BRUSSELS, Belgium
- Novem, att. J. 't Hooft
 P.O. Box 8242
 3503 RE Utrecht, The Netherlands
- SEP, att. ir. P. van der Ploeg
 P.O. Box 575
 6800 AN Arnhem, The Netherlands

Summary

The Department of Fluid Dynamics of TNO Environmental and Energy Research has coordinated the CEC JOULE project Wake and Wind Farm Modelling. The project resulted in:

- two modular wind farm codes;
- the gust model WAKEGUST.

The wind farm codes are described in part A of the project report. WAKEGUST is described in this part (part B) of the project report.

WAKEGUST enables wind turbine designers to compute loads on wind turbines in homogeneous terrain, and in a single wake of a wind turbine. WAKEGUST describes the gust amplitude and the gust duration in non-dimensional parameters. The first parameter is normalized by dividing the amplitude with the corresponding rms value, the second parameter is normalized by dividing the duration by the mean gust duration.

WAKEGUST has been based on existing gust models of CSTB (France) and TNO (The Netherlands). These models have been developed for stand-alone wind turbines. The new gust model has been validated using time series of wind measurements collected at several points inside and outside the Dutch experimental wind farm at Sexbierum. The measurements inside the farm have been restricted to single wake cases at distances between 2.5 and 8D behind the upstream wind turbine.

Comparisons between the calculations and the measurements of both the u- and the v-component of the wind speed show very good results. The agreement decreases for increasing values of the gust parameters, i.e. the normalized amplitude and the normalized duration. This is most likely a statistical effect due to the decreasing number of gusts with large amplitude and/or with large duration.

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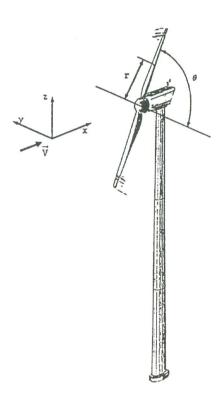
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Nomenclature

System of coordinates



General conventions

- \overline{X} average value of the variable x;
- σ_i rms-value of the wind speed component i (i = u or i = v):
 - the u-, or longitudinal, component of the wind is defined as the 'projection of the instantaneous wind vector in an horizontal plane and in the direction of the mean wind vector';
 - the v-, or lateral, component of the wind is defined as the 'projection of the instantaneous wind vector in an horizontal plane and in the direction perpendicular to the mean wind vector';
- U mean value of the u component;
- V mean wind speed, in formula $V = (u^2 + v^2)^{1/2}$;
- i the index i assigned to a parameter represents the corresponding component of the horizontal wind speed fluctuations (i = u or i = v);

Nomenclature

A_{i}	gust amplitude at rotor centre for component i (one-point amplitude)	(m/s)
$\widetilde{\boldsymbol{A}}_i$	$= A_i/\sigma_i \ normalized \ gust \ amplitude$	(-)
D	rotor diameter	(m)
$F_P(p)$	cumulative distribution function of parameter P , at value $p (= Prob \ (P \le p))$	(-)
$F_{T/P}(t,p)$	conditional cumulative distribution function of T , given $P = p$ at value $T = t$	(-)
$f_P(p)$	probability density function of parameter P, at value p	(-)
$f_{T/P}(t,p)$	conditional probability density function of T , given $P=p$ at value $T=t$	(-)
$f_{T,P}(t,p)$	joint probability density function of parameters T and P, at values $T=t$ and $P=p$	(-)
Н	hub height	(m)
\mathbf{I}_{i}	σ_x/\overline{X} , turbulence intensity	(-)
k_i	coefficient related to $\sigma_{\!Ai}$	(-)
L_{i}	characteristic dimension of the gust for component i	(m)
$^{x}L_{i}$	integral length scale of the turbulence in longitudinal direction for component i	(m)
n	wind fluctuations frequency	(s^{-1})
n_1, n_2	cut-in and cut-off frequencies	(s^{-1})
N	frequency of occurrence of all gusts	(-)
N_j	number of gusts between \boldsymbol{T}_j and \boldsymbol{T}_{j+1} seconds	(-)
S_{ii} (n)	power spectrum of component i	(m^2/s)
$T_{\rm i}$	gust duration for component i	(s)
\tilde{T}_{i}	$(=T_i/\overline{T}_i)$ normalized gust duration	(-)
Z	height	(m)
Z _o	terrain roughness length	(m)
ΔU	difference between two successive 10-minutes average wind speeds	(m/s)
$\Delta\Theta$	difference between two successive 10-minutes	(°)

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1 Introduction

A large and increasing part of wind turbines is operating in an array or in wind farms. Consequently these wind turbines will have a reduced power output and will experience additional fluctuating wind loads. The latter effect will manifest itself as a reduction in fatigue life. In order to design economic and long-lived clusters of wind turbines, engineers need to be able to predict accurately both the overall efficiency of an array and the fatigue life of the machines within it. A good knowledge of the flow, and particularly turbulence properties, within the array therefore is required.

The performed activities in the present JOULE project 'Wake and Wind Farm Modelling' can be seen as a continuation of the previous projects carried out in the CEC concerted actions 'Wake Effects' and 'Gust Modelling'. The aims of this project are:

- improvement and extension of wake models;
- development of an improved modular wind farm code;
- development of a wake gust model.

The project is financed by the CEC DG-XII (56.1%), $SEP^{1)}$ (15.2%), $CRES^{2)}$ (13.1%), $CSTB^{3)}$ (8.4%) and $NOVEM^{4)}$ (7.2%).

The project participants are:

The present project is reported in two parts: part A deals with the first and second aim (*Cleijne '93*), part B deals with the third aim (present report). Both parts can be read separately.

Several activities of the project have been reported in technical notes and progress reports. The essentials are also to be found in one of the parts A or B.

Within the CEC concerted action 'Gust Modelling' two gust models have been developed by CTSB (France) and WISA (Germany) to be used for the calculation of the dynamic loads of single wind turbines exposed to natural wind (*Delaunay & Locatelli '90* and *Nebel & Molly '88*). A third gust model has been developed by TNO (The Netherlands) within the Netherlands Wind Energy Research Programme (*Verheij '91*). This model has been incorporated in the Dutch Handbook on Gust Modelling for Wind Turbine Design (*Verheij et al. '91*). A gust model developed in Sweden (*Bergström '87*) has partly been used as the basis for the CSTB-model.

¹⁾ Dutch Electricity Generating Board

²⁾ Centre for Renewable Energy Sources

³⁾ Centre Scientifique et Technique du Batiment

⁴⁾ Netherlands Company of Energy and Environment

⁵⁾ Netherlands Organization for Applied Scientific Research

⁶⁾ Centre National de la Recherche Scientifique

⁷⁾ G.J. Taylor, consultant

⁸⁾ Universidad Polytecnica de Madrid

The aim of a gust model is to provide wind turbine designers with an engineering tool for calculations of wind turbine loads, especially fatigue loads. The gusts give a discrete description of the wind fluctuations. Important parameters are: the gust amplitude, the gust duration and its spatial dimension. The four above mentioned gust models apply for homogeneous terrain and for undisturbed atmospheric wind. Initially the project participants CSTB and TNO wanted to combine all four models in order to arrive at the final wake gust model. However, the model of Nebel & Molly is based on wind power gradients. Although their report shows some very interesting results, especially for extreme gusts, the basic principles deviates too much from the other three models for further use in the project. Furthermore, the model of Delaunay & Locatelli has been based on that of Bergström. The latter therefore has not been used.

Structure of the report

The contents of this report is structured according to the successive phases of the project part B.

In chapter 2 the evaluation of the TNO and CSTB gust models is briefly described. An extensive description is to be found in a technical note (*Delaunay & Verheij '92*). Common points and problematic differences between the models are shown. An inventory of the weak and/or strong points of each of the models is given, keeping in mind the future extension of the combined model to wake conditions.

Chapter 3 is focused on the new combined gust model WAKEGUST, using the conclusions of the former part. In order to arrive at this wake gust model additional input data was required. To this end data from the Sexbierum wind farm have been used. These data are collected during measuring campaigns being part of the activities of another JOULE project 'Full-scale Measurements in Wind Turbine Arrays' (Cleijne & Hutting '93).

The experimental devices and the data acquisition system of the Sexbierum wind farm as well as the data processing and analyses are described in chapter 4.

The validation of WAKEGUST has been carried out in the final stage of the project. In chapter 5 the results are presented. The evaluation includes the standard deviation of gust amplitudes, the mean gust durations and (conditional) probability functions of normalised gust amplitude and/or durations.

Finally the conclusions are drawn in chapter 6.

2 Evaluation of the TNO and CSTB gust models

2.1 Introduction

The evaluation of the gust models of TNO and CSTB has been performed keeping in mind that a possible combination of these models should describe gusts in wind turbine wakes. Attention has been paid to the basic principles used in the models, the definition of gusts, the gust parameters used to describe the gusts, the input data and the applied analysis on which the models are based and finally, the output of both models.

An extensive description of the evaluation can be found in a technical note (*Delaunay & Verheij '92*). Here the reporting is restricted to the major agreements and the major discrepancies which has led to the framework of the new wake gust model WAKEGUST. In order to provide the reader a better understanding of the report this chapter starts with the definition of gusts.

2.2 Definition of gusts

The wind signal, at a fixed point, is split into a 10-minutes average value and wind speed fluctuations superposed on it. Then the gust is defined as:

The time sequence during which the wind speed differs from this average value.

A new gust is counted each time the mean value is crossed (zero-crossing). The period between two successive zero-crossings is defined as the gust duration T (figure 1).

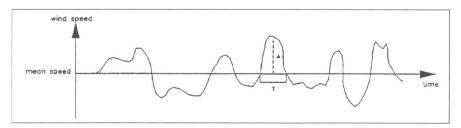


Figure 1 Definition of the gust duration

The gusts can be positive or negative, corresponding to a time sequence with values above respectively below the average value.

For a wind turbine the spatial structure of the wind is of importance. The rotation of the turbine blades influences the response of the machine to turbulence. The rotor effective gust is defined to describe gusts which affect the rotor disc as a whole. Disc averaging is carried out in the frequency domain by integrating the cross-power spectra over the disc area (*Beugeling & Vermeulen '85*). However, the 'rotor effective gusts' are derived from 'single-point gusts' providing statistics on gusts as they are experienced by a fixed anemometer.

2.3 Major agreements

The basic principles of the two models are identical. Both models use:

- well-known statistical descriptions of the atmospheric boundary-layer (turbulence intensity, power spectra, coherence functions, length scales);
- additional statistics obtained via measurements and analysis of long-time wind series.

Moreover, the definition of a gust for a given signal is similar in the two models (zero-crossing gusts) as is the choice of parameters for a single gust, i.e. gust duration and gust amplitude. The data analysis of both the duration and the amplitude (or gust mid-time value) show a quite similar approach.

In both models the outputs of the model are distributions functions of gust parameters, either overall distributions or conditional distributions for given gust durations. Both models describe the gust amplitude and the gust duration in non-dimensional terms.

The two models take into account the spatial effects in a vertical plane, and in particular the statistical distributions on gusts which concern the whole rotor.

2.4 Major discrepancies

The TNO model is dealing with the modulus of the instantaneous wind speed (V), and the wind direction fluctuations (Θ) which can be preferred for some applications. The CSTB model has the advantage to consider the common meteorological description of the wind vector in longitudinal (u) and lateral (v) components which allows:

- to expect possible relations between gust model and the classical theory of the atmospheric boundary layer;
- to give informations on the fluctuations of wind in direction.

TNO and CSTB decided to use the common meteorological description for the analysis of the wake data.

To characterize the magnitude of a gust, the two models define two different parameters: amplitude (CSTB) or mid-time value (TNO). The statistics on these parameters will be different because the gust amplitude is not always reached at the mid-time of the gust. We can be sure that, for a given exceedance probability level, the amplitude will be larger than the mid-time value.

TNO and CSTB decided to use the gust amplitude to characterize the magnitude of the wake gust.

The TNO model considers only gusts which durations are included between 2 and 30 s. The CSTB model takes all the gusts between 1 and 600 s into account. This difference is important for small gusts (between 1 and 2 s) which are numerous but which also have small amplitudes.

TNO and CSTB decided to use the gusts between 1 and 600 s for the analysis and the modelling of the wake gusts.

The normalization of the gust amplitude and the gust duration show slight differences between the CSTB model and the TNO model. This is mainly due to the differences in the data analysis. For the new model WAKEGUST it has been decided to normalize the gust amplitude by the standard deviation of the amplitude and the gust duration by the mean gust duration.

The TNO model considers only rotor averaging effects, by computation of the rotor effective standard deviation of fluctuations. Then the relations obtained by single-point measurements are modified by replacing the single-point standard deviation by this rotor effective standard deviation.

The CSTB model considers gusts with their spatial structure and defines specific parameters for these gusts, i.e. the maximum amplitude on the rotor disc and the characteristic dimension.

TNO and CSTB decided to use the rotor effective gust for the new model.

2.5 Possibilities of the models for extension to wakes

It seems possible to use the basics of the existing gust models of CSTB and TNO for extension in wind turbine wake situations. In order to arrive at a wake gust model the following required conditions seem fulfilled:

- experimental coefficients obtained from a wide range of measurements conditions;
- enough data should be available in order to obtain reliable statistical information;
- few coefficients to fit the gust model with the data;
- a link between the results of the gust model and the well-known functions describing the atmospheric boundary layer, including wind turbine wakes.

The measurement conditions include the characteristics of the site and the upstream wind turbine(s).

3 A new combined gust model WAKEGUST

3.1 Starting conditions

The aim of part B of the project is to give wind turbine designers a gust model which allows to compute loads in homogeneous flat terrains, and also in wind turbine single wakes. To this end information is required on the characteristics of the site, the upwind turbine and the turbulence in the wake. In a later phase the wake gust model should be extended to other cases like double wakes, complex terrain and offshore.

Time series of both the u- and the v-component of the wind have been used to model the wind in terms of gusts.

If there is a demand for gust statistics on modulus of wind speed and on wind direction, the results for the wind component u can be extended to the instantaneous modulus V ($V = (u^2 + v^2)^{1/2}$), and the statistics for the wind direction can be deduced from the statistics of the v-component by the relation $\Theta = \arcsin(v/V)$.

The gust is the sequence of the time series during which the value is different from its mean. So a new gust is counted each time the mean value is crossed. A gust model which uses such a definition is generally called zero-crossing gust model.

Only gusts with duration larger than 1 s have been considered. This lower limit corresponds to common anemometers time response. As a consequence the considered time series, measured at 4 Hz, are filtered at a frequency of 1 Hz.

Two gust parameters have been chosen to describe the different types of gusts:

- duration T defined as the time between two zero-crossings of a time series;
- amplitude A defined as the maximum departure from the mean of a time series. A can be either positive or negative.

These definitions can be applied to each chosen time series (u or v) and to each type of gust (single point or rotor effective). These two gust parameters have been normalized and are defined as:

$$\tilde{A}_{i} = A_{i}/\sigma_{A_{i}} \tag{3-1}$$

$$\widetilde{T}_i = T_i / \overline{T}_i \tag{3-2}$$

in which \overline{T}_i is the mean gust duration and σ_{Ai} is the rms value of the gust amplitudes (i=u or i=v).

The output of the model will give overall distributions of gusts durations and amplitudes, and conditional distributions of the amplitude for a given duration. From these distributions also the conditional distribution of the gust durations for a given gust amplitude can be calculated. All distributions will be given for both longitudinal and lateral components.

The data analysis has been concentrated on single-point gusts. These gusts correspond to a time series measured by one anemometer at a fixed point. It was not

possible to consider rotor effective gusts. It turned out that the amount of stationary time series on all 5 anemometers simultaneously was very small.

3.2 Standard deviation of wake gust amplitudes

The standard deviation (rms-value) of the wake gust amplitudes, σ_{Ai} , are calculated from the standard deviation of the wind speed fluctuations in the wake, σ_i (*Bergström '87* and *Delaunay '91*). The latter is derived from a large number of 10 minutes time series. For both the u- and the v-component of the wind speed (i = u or i = v) the relation is:

$$\sigma_{Ai} = k_i \, \sigma_i \tag{3-3}$$

in which k_i is a constant dependent on the position in the wake. The values for k_i based on the Sexbierum wind data can be found in paragraph 5.2.

The standard deviation of the wind speed fluctuations in a wind turbine wake can be estimated by empirical relations (Wastling & Tindal '91) or more elaborate models (Taylor '93, Crespo et al. '90).

3.3 Mean wake gust duration

Following Rice's theory (*Rice '45*), the mean gust duration can be calculated from:

$$T_{i} = 0.5 \begin{bmatrix} \int_{n_{1}}^{n_{2}} S_{ii}(n) dn \\ \int_{n_{1}}^{n_{1}} S_{ii}(n) dn \end{bmatrix}^{1/2}$$
(3-4)

where the index i is u or v depending on the considered component. In our study $n_1 = 1/300 \text{ Hz}$ and $n_2 = 1 \text{ Hz}$.

To calculate the power spectral density function $S_{ii}(n)$ we use the Von Karman spectral equations (ESDU '85):

$$i = u$$
: $\frac{n S_{uu}(n)}{\sigma_u^2} = \frac{4 \tilde{n}_u}{(1 + 70.7 \tilde{n}_u^2)^{5/6}}$ (3-5)

$$i = v$$
: $\frac{n S_{vv}(n)}{\sigma_v^2} = \frac{1 + 755.2 \tilde{n}_v}{(1 + 283.2 \tilde{n}_v^2)^{11/6}}$ (3-6)

in which \tilde{n}_i is the non-dimensional frequency; $\tilde{n}_i = n \, ^xL_i / V$, n being the frequency, V the mean wind speed and xL_i the integral length scale of the turbulence. The length scale is dependent on the height z and the terrain roughness z_0 (ESDU '75):

$$^{x}L_{u} = 25 z^{0.35} / z_{0}^{0.063}$$
 (3-7)

$${}^{x}L_{v} = 5.1 \ z^{0.48} / z_{0}^{0.086} \tag{3-8}$$

From another JOULE project (*Verheij '93*) it turned out that the Von Karman spectrum also holds in single wake situations if the power spectral density functions are expressed in non-dimensional parameters based on the local values for V, σ_i and xL_i . Therefore the mean gust duration for wake gusts can also be calculated with equation 3-4.

As part of this project a study on turbulent length scales in wind turbine wakes has been carried out (*Delaunay '93*). With the results 'wake' length scales can be estimated.

3.4 Probability density functions of normalized wake gust parameters

The normalized gust parameters \tilde{T}_i and \tilde{A}_i can be described using two statistical functions, i.e. the overall probability density function of \tilde{A}_i and the conditional probability density function of \tilde{T}_i for a given value of \tilde{A}_i .

From these two functions two other statistical functions can be derived, i.e. the overall probability density function of \tilde{T}_i and the conditional probability density function of \tilde{A}_i for a given value of \tilde{T}_i . The expressions concerned are given below. Applying these expressions a wind turbine designer is able to calculate both gust amplitudes and gust durations separately or combined with any desired values (or value intervals).

Overall probability density function of normalized wake gust amplitudes

The frequency distribution of the one-point amplitude $\widetilde{A}_{\rm i}$ can be fitted by a half-gaussian distribution:

$$f_{\tilde{A}_i}(a_i) = \sqrt{(2/\pi)} \exp(-a_i^2/2)$$
 (3-9)

Conditional probability density function of normalized wake gust durations

The conditional probability density distributions of \tilde{T}_i for a given value of the normalized gust amplitude \tilde{A}_i are log-normal and can be described with the equations:

$$ln(\widetilde{T}_i) = ln (0.95\widetilde{A}_i^{qi})$$
(3-10)

$$\sigma(\ln(\tilde{T}_i)) = 0.60 \tag{3-11}$$

The coefficients q_i (i = u or i = v) have been determined from the measured data. The expressions for q_i are given in paragraph 5.4.

Overall probability density function of normalized wake gust durations

If the overall probability density function of $\tilde{A}^{1)}$, $f_{\tilde{A}}$, and the conditional probability density function of \tilde{T} for a given value of \tilde{A} , $f_{\tilde{T}/\tilde{A}}$, have been determined, the joint probability density function of \tilde{T} and \tilde{A} can be calculated by:

$$f_{\tilde{A},\tilde{T}}(a,t) = f_{\tilde{T}/\tilde{A}}(t,a) \cdot f_{\tilde{A}}(a)$$
(3-12)

The overall probability density function of \tilde{T} then can be evaluated by:

$$f_{\tilde{T}}(t) = \int_{0}^{\infty} f_{\tilde{A}, \tilde{T}}(a, t) da$$
 (3-13)

Conditional probability density function of normalized wake gust amplitudes

Using the probability density functions 3-12 and 3-13 of the former section the conditional probability density functions of \tilde{A} for given \tilde{T} can be calculated by:

$$f_{\tilde{A}/\tilde{T}}(a,t) = f_{\tilde{A}/\tilde{T}}(a,t)/f_{\tilde{T}}(t)$$
(3-14)

For a given interval $T_1 < \tilde{T} < T_2$ this function results in the following probability distribution:

$$f_{\tilde{A}/T_{1} < \tilde{T} < T_{2}}(a,t) = \int_{T_{1}}^{T_{2}} f_{\tilde{A},\tilde{T}}(a,t) d\tilde{T} / \int_{T_{1}}^{T_{2}} f_{\tilde{T}}(t) d\tilde{T}$$
(3-15)

3.5 Rotor effective gusts

As already mentioned it was not possible to consider rotor effective gusts due to a restricted amount of simultaneously measured data. If data will become available or information on coherence functions in a wake are available, it is possible to extend WAKEGUST with rotor effective gusts.

The results obtained for single-point gusts can be extended to the rotor effective gusts as follows:

- computation of the effective spectral densities of the components u and v by integration of the cross-power spectra over the rotor disc;
- computation of the effective standard deviations of the components i $(\sigma_{i,eff})$ by integration of the effective spectral densities;

In this section and the section below the authors have skipped the subscription i in the parameters A and T for legibility reasons.

- computation of the effective mean durations (T_{i,eff}) by equation of the Rice formulae (see paragraph 3.3), using effective spectral densities instead of singlepoint spectral densities;
- normalizing rotor-effective gust amplitudes by their standard deviation;
- normalizing rotor-effective gust durations by the effective mean duration.

It is considered that the results obtained for single-point normalized parameters are also valid for normalized parameters of rotor effective gusts.

Estimations of the lateral and the vertical extensions of the gusts, given by the turbulent length scales, can be found in a technical note from CSTB (*Delaunay '93*).

4 Data analysis

4.1 Experimental device and data acquisition

Sexbierum wind farm

The Sexbierum wind farm is located in the Northern part of The Netherlands at approximately 4 km distance of the shore and is surrounded by flat homogeneous terrain, mainly grassland used by farmers for the grazing of cows. According to the Wieringa terrain classification (*Wieringa & Rijkoort '83*), which is close to the Davenport terrain classification (*ESDU '82*), the Sexbierum terrain can be parametrized with roughness length z_0 is about 0.03 to 0.10 m. From the long term data (*Cleijne & Hutting '93*), it seems that $z_0 = 0.05$ m give good results for wind shear and turbulence level at hub height H = 35 m.

The wind farm has a total of 5.4 MW installed capacity, consisting of 18 turbines of 310 kW rated power each. The wind turbines are places in a rectangular grid with inter distances of 5, 8 and 10 rotor diameters. More detailed information on the Sexbierum wind farm and the wind turbines are to be found in (Cleijne & Hutting '93) and (Bulder & Schepers '91) respectively.

Sexbierum wind regime

The prevailing wind direction is South West. The wind climate at hub height is given by Weibull frequency distribution with scale factor $a_{hub} = 8.6$ m/s and shape factor $k_{hub} = 2.1$. The average wind speed at hub height is 7.6 m/s.

The wind shear is calculated according to the logarithmic wall law. The wind speed at height z can be calculated if the wind speed at hub height V_H is known:

$$V(z) = V_{H} \ln(z/z_{0}) / \ln(H/z_{0})$$
(4-1)

The turbulence intensity at height z, I(z) or $\sigma_u/V(z)$, is calculated according to *Beljaars* '87:

$$I(z) = 2.2 \kappa / \ln(z/z_0)$$
 (4-2)

in which $\kappa = 0.4$. For the v-component of the wind the constant is 1.8.

Panofsky & Dutton '84 use the same formula, but with the constants 2.4 and 1.9 for the U- and V-component respectively.

For the surface roughness z_0 = 0.05 m the turbulence intensity at hub height becomes I_H = 0.134.

Instrumentation

The wind measurements have been performed with cup anemometers placed at hub height (35 m) on fixed masts just outside the park and with 3-component propeller anemometers on the mobile masts A, B and C inside the park at heights 47 m (mast B), 35 m (masts A, B and C) and 23 m (mast B). In a later stage two additional cup anemometers have been placed on mast B (heights 41 and 29 m). The sampling rate of the cup anemometers is 1 Hz, for the propeller anemometers this is 4 Hz. Details are given in (Cleijne & Hutting '93).

During a measuring campaign the data of the undisturbed wind condition have been collected with the fixed mast most close to the upstream wind turbine.

Data correction

The measured rms values of wind component fluctuations are corrected following *Teunissen et al.* '85. He shows that for horizontal axis propeller anemometers, with a length response of about 3 m, the variance of longitudinal and lateral fluctuations is underestimated by about 12% for u and 10% for v. These estimations have been obtained at 10 m height on a rural site. The same results have been obtained by CSTB (*Delaunay '90*) with a method of comparison between theoretical and experimental spectra, for measurements at 40 m height on a rural site.

For the Sexbierum measurements this correction means that the estimations of the rms values of horizontal components (σ_u and σ_v) have been multiplied by, respectively the factors 1.06 and 1.05.

4.2 Data processing

In order to perform the gust analysis the raw wind speed data have to be processed into well defined gusts according to the following steps:

- selection of wind directions and wind speeds;
- change of reference frame;
- selection of stationary sequences;
- detect gusts and store relevant gust parameters;
- group gusts as a function of the measurement position.

As we are interested in gusts in wind turbine wake situations the last step not only concerns the height z, but also the longitudinal distance behind the upstream turbine x and the lateral distance from the wake centre line y.

For the data processing computer programs of both TNO and CSTB have been applied.

Selection of wind directions and wind speeds

For each measuring campaign only the wind speed data from wind directions ± 30 degrees from the wake centre line have been selected. The range of wind directions is given in table 1 further on.

Only data during which the reference mean wind speed at hub height was higher than 6 m/s are taken into consideration. For lower wind speeds the wind turbine is out of operation. If the upstream turbine was not operating for any other reason the data were not taken into consideration too.

Change of reference frame

The campaigns consist of time series ranging from several minutes to almost 3 hours. The series have been split into sequences of 10 minutes. In order to gain more gusts (and improve the statistics) the remaining part of a series has also been used if the duration exceeds 5 minutes.

For each sequence the original wind data have been translated into the wind components u (in the direction of the mean wind speed vector) and v (in the horizontal direction perpendicular to the mean wind speed vector).

Furthermore, according to the starting points mentioned in chapter 3, the data sampled at 4 Hz have been filtered at 1 Hz by averaging every 4 data points.

Selection of stationary sequences

The stationarity of the sequences is determined by the upwind undisturbed measurements for both the u and v series. So the wake flow is considered to be stationary if the upwind flow is defined as stationary.

The stationarity of the u and v series is determined using the variation of the functions $F_U(u)$ and $F_V(v)$. These functions are least square fittings of a second-order polynomial on the u and v series. An example is given in figure 2 (to be found at the end of this report). The series is defined stationary if the difference between the maximum and the minimum value of these functions is smaller than 10% of the mean wind speed and at the same time smaller than the rms value of the considered component.

The upstream wind turbine should not show fluctuations in yaw angle, thrust coefficient or rotational speed during the sequence. If one of these parameters vary more than 10% the sequence was skipped.

In table 1 an overview of the selected data series for the three measurements campaigns is given. During campaign numbers 1 and 3 the three mobile masts were erected next to each other, while during campaign 2 the masts were erected in line at 2.5D (mast B), 5.5D (mast A) and 8D (mast C) behind turbine T16 (see also the former paragraph).

Table 1 Overview of the selected sequences

Campaign	Number of sequences	Total time (minutes)	Wind speed range (m/s)	Wind direction range (degrees)
1: 4D	35	325	6.2 - 14.0	164 - 204
2: 2.5; 5.5; 8D	74	610	6.0 - 14.6	195 - 257
3: 7D	21	160	8.0 - 13.5	260 - 280

Gust detection and storage

From the selected sequences gusts have been detected according to the definition given in chapters 2 and 3. For all sequences the following parameters have been stored per anemometer (for both u- and v-component, i = u or i = v):

- the mean and the rms values (σ_i) of the wind speed;
- the mean gust duration $(\overline{T}_i)^{(1)}$;
- the rms value of the gust amplitudes (σ_{Ai}) ;
- the minimum and maximum value.

For all gusts also the gust duration T_i and the gust amplitude A_i have been stored. In a later stage these parameters have been normalized by the mean gust duration \overline{T}_i or the rms value of the gust amplitude σ_{Ai} respectively. The notations for the normalized parameters are \widetilde{T}_i and \widetilde{A}_i .

Sequence grouping

The selected sequences have been grouped together based on the position of the anemometer in the wake. The non-dimensional parameters which define this position are the ratios x/X_N and y/b; where x is the longitudinal distance from the upstream turbine, X_N is the near wake length, y is the lateral distance from the centre line of the wake and b is the wake width.

The near wake length and the wake width are computed applying the TNO model FARMS. The relevant formulae can be found in appendix A.

The thrust coefficient C_T is computed from the measured undisturbed mean wind speed at hub height, using the $C_T - \lambda$ and $\omega - U$ graphs of the Sexbierum WPS-30 wind-turbine (Bulder & Schepers '91).

We have defined 12 groups with data measured at hub-height, 8 groups with data measured at H - 0.4D and 8 groups with data measured at H + 0.4D. The relevant characteristics are given in table 2.

The subdivision in groups has been made such that the wake conditions were expected to be more or less similar regarding wind speed deficit and added turbulence. The gusts characteristics should therefore be comparable within one group.

If the power spectral densities S_{uu} and S_{vv} of the fluctuations of u and v components are known, these mean values can be estimated from Rice's theory (*Rice '45*, *Delaunay & Locatelli '90*).

Table 2 Characteristics of the groups used for the gust analysis

Group number	Height	x/X _N	y/b	Number of sequences	Total time (minutes)
1	Н	1.1	0.0-0.7	21	150
2	Н	1.0	0.8-1.2	34	290
3	Н	0.8	1.1-1.5	16	135
4	Н	1.8	0.0-0.6	27	240
5	Н	1.8	0.6-0.9	12	105
6	Н	1.5	1.3-2.0	20	200
7	Н	2.5	0.0-0.6	13	100
8	Н	2.4	0.6-0.9	8	55
9	Н	2.3	1.1-1.6	29	240
10	Н	2.8	0.0-0.6	25	170
11	Н	2.8-3.5	1.1-1.7	24	205
12	Н	3.5	0.7-1.1	8	60
21	H-0.4D	1.1	0.0-0.7	23	170
22	H-0.4D	1.0	0.8-1.2	34	290
23	H-0.4D	0.8	1.1-1.5	16	135
24	H-0.4D	1.8	0.0-0.6	19	160
25	H-0.4D	1.8	0.6-0.9	5	45
26	H-0.4D	1.5	1.1-1.6	14	135
27	H-0.4D	2.8	0.0-0.6	10	75
28	H-0.4D	2.8	1.1-1.7	5	40
31	H+0.4D	1.1	0.0-0.7	21	155
32	H+0.4D	1.0	0.8-1.2	34	290
33	H+0.4D	0.8	1.1-1.5	16	135
34	H+0.4D	1.8	0.0-0.6	18	155
35	H+0.4D	1.8	0.6-0.9	5	45
36	H+0.4D	1.5	1.1-1.6	14	135
37	H+0.4D	2.8	0.0-0.6	8	55
38	H+0.4D	2.8	1.1-1.7	6	45

5 Results of the measurements and validation of WAKEGUST

5.1 Methodology

Following the model description made in chapter 3, a number of tasks have to be completed in order to validate the new gust model WAKEGUST. Some of these tasks are part of the data processing which is described in the former chapter. The other tasks are described in this chapter and concern:

- verification of equation 3-3, i.e. $\sigma_{Ai} = k_i \sigma_i$. The verification can be found in par. 5.2;
- verification of the following hypotheses:
 - the overall probability density functions of \tilde{A}_u and \tilde{A}_v are half-gaussian distributions (par. 5.3);
 - the conditional probability density distributions of \tilde{T}_u and \tilde{T}_v for given \tilde{A}_u and \tilde{A}_v are log-normal and can be described with the equations 3-12 and 3-13, i.e. $\ln(\tilde{T}_i) = \ln(0.95\tilde{A}_i^{qi})$ and $\sigma(\ln(\tilde{T}_i)) = 0.60$ respectively. The verification can be found in par. 5.4;
 - if the above mentioned distributions are known the overall probability density functions of \tilde{T}_u and \tilde{T}_v as well as the conditional probability density functions of \tilde{A}_u and \tilde{A}_v , for given \tilde{T}_u and \tilde{T}_v can be derived (par. 5.5).

The coefficients k_i and q_i (i = u or i = v) have to be determined from the measured data. To this end we have applied fitting procedures.

5.2 Standard deviations of wake gust amplitudes

The relations between the standard deviations of the (wake) gust amplitudes and of the corresponding wind component (equation 3-3) are shown in figures $3^{1)}$ for all 28 groups (the left-hand side of the graphs concern the u-component, the right-hand side the v-component). From these figures it is clear that the linear relations are still valid in wind turbine wakes. The values of the fitted coefficients k_u and k_v are given in table 3.

The figures 3 to 7, referred to in this chapter, are to be found at the end of this report.

Table 3 Values of the coefficients k_u and k_v for each group

Group number	Height	x/X _N	y/b	k _u	k _v
1	Н	1.1	0.0-0.7	1.11	1.24
2	Н	1.0	0.8-1.2	1.11	1.23
3	Н	0.8	1.1-1.5	1.04	1.19
4	Н	1.8	0.0-0.6	1.09	1.20
5	Н	1.8	0.6-0.9	1.10	1.18
6	Н	1.5	1.3-2.0	1.00	1.13
7	Н	2.5	0.0-0.6	1.05	1.20
8	Н	2.4	0.6-0.9	1.02	1.17
9	Н	2.3	1.1-1.6	1.03	1.14
10	H	2.8	0.0-0.6	1.01	1.12
11	Н	2.8-3.5	1.1-1.7	1.03	1.12
12	Н	3.5	0.7-1.1	1.03	1.16
21	H-0.4D	1.1	0.0-0.7	1.11	1.21
22	H-0.4D	1.0	0.8-1.2	1.08	1.20
23	H-0.4D	0.8	1.1-1.5	1.04	1.15
24	H-0.4D	1.8	0.0-0.6	1.08	1.18
25	H-0.4D	1.8	0.6-0.9	1.06	1.17
26	H-0.4D	1.5	1.1-1.6	0.97	1.13
27	H-0.4D	2.8	0.0-0.6	1.03	1.06
28	H-0.4D	2.8	1.1-1.7	1.02	1.10
31	H+0.4D	1.1	0.0-0.7	1.12	1.23
32	H+0.4D	1.0	0.8-1.2	1.08	1.22
33	H+0.4D	0.8	1.1-1.5	1.01	1.17
34	H+0.4D	1.8	0.0-0.6	1.07	1.21
35	H+0.4D	1.8	0.6-0.9	1.07	1.22
36	H+0.4D	1.5	1.1-1.6	1.00	1.13
37	H+0.4D	2.8	0.0-0.6	1.03	1.08
38	H+0.4D	2.8	1.1-1.7	1.01	1.11

From these values the following simple rules have been derived (for $x/X_N > 1$):

- inside the wake (y/b < 1):

$$k_u = 1.00 + 0.05 (3 - x/X_N)$$
 (5-1)

$$k_v = 1.13 + 0.05 (3 - x/X_N)$$
 (5-2)

- outside the wake (y/b > 1):

$$k_u = 1.00$$
 and $k_v = 1.15$. (5-3)

At distances larger than $x/X_N = 3 \ (\approx 7D)$ the value x/X_N in equations 5-1 and 5-2 should be set to 3.

These equations satisfy the measured values with a mean error of 2%. All the measured values of k_u and k_v are predicted within 4%, except for k_v in 3 cases:

- group 3: k_v (measured) = 1.19 and k_v (formula) = 1.13;
- group 27: k_v (measured) = 1.06 and k_v (formula) = 1.14;
- group 37: k_v (measured) = 1.08 and k_v (formula) = 1.14.

In the first case (group 3) we are in the near wake ($x/X_N = 0.8$). It appears that even if we are theoretically 'outside the wake' the structure of v-gusts is disturbed by the wake.

For the second and third cases (groups 27 and 37), which correspond to heights different from hub height, these low values should be put into perspective. The number of sequences in these groups is rather small.

The values of $k_u = 1.00$ and $k_v = 1.15$ obtained for undisturbed homogeneous terrain (*Delaunay & Locatelli '90*) are found again for locations outside the wake (equation 5-3).

5.3 Overall probability density functions of normalized wake gust amplitudes

The overall probability density function of the normalized amplitudes of both u- and v-gusts can be considered as half-gaussian distributions. In figures 4 it is shown that this holds up to the 99% level (cumulative frequency). This feature is found to be independent on the location in the wake.

5.4 Conditional probability density functions of normalized wake gust durations

The figures 5 show that the conditional probability density functions of (wake) gust durations can be considered as log-normal functions. These conditional cumulative probability density functions are compared with cumulative frequency distributions for three intervals of \tilde{A} : 0.3 < \tilde{A} < 0.6, 0.75 < \tilde{A} < 1.25 and 1.5 < \tilde{A} < 2.5.

The conditional cumulative probability density function for these intervals is obtained by integration of the (distributive) function $f_{\tilde{T}/\tilde{A}}$.

Comparison of the measured data and these functions has shown that the value $\sigma(\ln(\widetilde{T}_u)) = \sigma(\ln(\widetilde{T}_v)) = 0.6$ used for the undisturbed gust model, also holds at positions in wind turbine wakes. The values of q_u and q_v which allow a good fitting of the curves can be given by:

— wake centre region (y/b < 0.7):

$$q_u = 1.42 - 0.12 (3.5 - x/X_N)$$
 (5-4)

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$$q_v = 1.10 - 0.20 (3.5 - x/X_N)$$
 (5-5)

— wake edge region (0.7 < y/b < 1.1):

$$q_u = 1.42$$
 (5-6)

$$q_v = 1.10 - 0.20 (3.5 - x/X_N)$$
 (5-7)

- outside the wake (1.1 < y/b < 2):

$$q_u = 1.42 \text{ and } q_v = 1.10$$
 (5-8)

It can be noted that, at larger distances from the axis, the wake effects have a larger influence on the v-gusts than on the u-gusts. At the largest distances measured here (x/ $X_N \approx 3.5$) the values of $q_u = 1.42$ (and also $k_u = 1.00$) used for the 'undisturbed' gust model are found again. For the v-component, however, the values of $q_v = 1.10$ (and $k_v = 1.13$) are a little different from the undisturbed gust model. It seems that the wake is larger for lateral wind speeds than for longitudinal wind speeds.

The fact that the values of q_u and q_v for wake gusts are smaller than for undisturbed gusts, shows that the gust amplitudes and gust durations are less correlated in a wake. Large amplitude gusts with short durations appear more often in wakes than in undisturbed cases, especially at the wake edge in the near wake. This is possibly due to the tip vortices.

5.5 Overall probability density functions of normalized wake gust durations

The overall probability density function of the normalized gust durations \tilde{T} has been calculated, by integrating the joint probability density function of the normalized gust amplitudes and gusts (see paragraph 3.4). The figures 6 show the calculated cumulated frequency distribution of \tilde{T} , compared with the measured ones. The agreement is quite good up to the 99% level.

5.6 Conditional probability density function of normalized wake gust amplitudes

Using the probability density functions mentioned in the former paragraph the conditional probability density functions of \tilde{A} for given \tilde{T} has been calculated for three intervals: $0.3 < \tilde{T} < 0.6, 0.75 < \tilde{T} < 1.25$ and $1.5 < \tilde{T} < 2.5$ (see also equations 3-12 to 3-14). The computations of this distribution have been compared with the probability distribution of the measured (wake) gusts. The results are shown in figures 7. Except in particular cases, corresponding to positions in the wake edges, the model predicts the value of \tilde{A} well. This holds for all given gust duration intervals and for probability levels up to about 90 to 95% (cumulative frequency). The error is smaller than 20% of σ_u or σ_v for the high values of \tilde{A} , and much smaller for the low values. Besides the number of gusts with high amplitudes is relatively small which increases the inaccurancy of the high values.

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6 Conclusions

WAKEGUST is a new gust model. The main gust parameters, i.e. the gust amplitude and the gust duration are described in statistical terms. The gust amplitude and the gust duration have been defined in non-dimensional units. WAKEGUST enables wind turbine designers to compute loads on wind turbines in homogeneous terrain, and in a single wake of a wind turbine.

With the new gust model WAKEGUST the following distributions of the main gust parameters can be calculated:

- the overall probability density function of the normalized gust amplitudes and ditto for the normalized gust durations;
- the conditional probability density function of the normalized gust duration for a given value (or interval) of the normalized gust amplitude and visa versa.

The validation of WAKEGUST has been carried out using time series of wind measurements collected at several points inside and outside the Dutch experimental wind farm at Sexbierum. The measurements inside the farm have been restricted to single wake cases at distances between 2.5 and 8D behind the upstream wind turbine. Comparisons between the calculations and the measurements show that the results found from measurements outside wind turbine wakes are equal to results found in earlier studies. Besides the formulae to model the 'undisturbed' gusts also holds for 'wake gusts' if the gust parameters are written as non-dimensional parameters. The only adjustments to be made concern the position in the wind turbine wake in the description of two 'constants'. The agreement between the calculations and the wake measurements is very good. Only for large gust amplitudes in combination with large gust durations the agreement decreases with increasing parameter values. The number of measured gusts with large parameter values, however, is low. This is most possibly the reason for the diminishing agreement.

From the analysis of the wake gust measurements it seems that the wake is somewhat larger for lateral wind speeds (component perpendicular to the wind direction) than for longitudinal ones (component along the wind direction). This conclusion is based on the fact that, at larger distances from the wake axis, the wake effects have a larger influence on the v-gusts (lateral) than on the u-gusts (longitudinal).

It also turned out that the gust amplitudes and the gust durations are less correlated in the wake than outside the wake.

The calculation of rotor-effective gusts (gusts experienced by the turbine rotor as a whole) can be performed if information on the coherence function in wake situations is known. This requires a large number of measurements. Besides, these measurements should be carried out at a number of positions in the plane perpendicular to the mean wind direction simultaneously.

It is recommended to analyse gusts measured in double wake situations and overlapping wakes. This addition then would enable wind turbine designers to calculate loads in the most relevant wind farm cases.

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8 Authentication

Name and address of the principal

- CEC, DG-XII, att. Dr. G. Caratti Rue de la Loi 200 B-1049 BRUSSELS, Belgium
- Novem, att. J. 't Hooft
 P.O. Box 8242
 3503 RE Utrecht, The Netherlands
- SEP, att. ir. P. van der Ploeg P.O. Box 575 6800 AN Arnhem, The Netherlands

Names and functions of the cooperators J.H.A.M. van der Brugh - research engineer

Names of establishments to which part of the research was put out to contract CSTB, D. Delaunay

Date upon which, or period in which, the research took place September, 1990 - September, 1993

Signature

F.J. Verheij research coordinator

Approved by

N.J. Duijm

section manager wind engineering

Appendix A Calculation of the near wake length and the wake width

The near wake length and the wake width are computed applying the TNO model FARMS (Vermeulen & Builtjes '81, Vermeulen & Vijge '81 and Luken '88). With this model the yearly averaged energy production of wind farms in flat, homogeneous terrain can be estimated. For the wake gust modelling only the calculation of the near wake length X_N , in most cases $2 < X_N/D < 3$, is of importance. The relevant formulae are given in this appendix.

In FARMS the wake is divided into four sections:

- I. The 'potential core' section;
- II. The near wake section;
- III. The far wake section;
- IV. The final section.

The size of a section is described in terms of the wake width and the distance to the turbine. Both width and distance are related to the expanded rotor diameter D_0 :

$$D_0 = D \{(m+1)/2\}^{0.5}$$
(A-1)

in which D is the rotor diameter and m is a thrust parameter

$$m = (1 - C_T)^{-0.5}$$
 (A-2)

In FARMS the half-wake widths are calculated. Per definition:

$$R_0 = 0.5 D_0$$
 (A-3)

Here, information will only be given on sections I and II.

I. 'Potential core' section

In the first section a uniform velocity distribution is assumed behind the rotor disc. The centre line velocity deficit in this section is calculated according to the actuator disc theory as.

The section ends at X_H , the position where the potential core is fully eroded. The flow inside and outside the potential core mixes due to a turbulent exchange of momentum, i.e. ambient turbulence (index α), rotor induced turbulence (index λ) and shear induced turbulence (index m).

The length of the first section X_H is calculated by:

$$X_{H} = R_{0} / \{ (dr/dx)_{\lambda,\alpha}^{2} + (dr/dx)_{m}^{2} \}^{0.5}$$
 (A-4)

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The mixing of the flow due to the ambient and the rotor induced turbulence $(dr/dx)_{\lambda,\alpha}$ is found by Luken and is related to the ambient turbulence level α and the tip speed ratio λ :

$$(dr/dx)_{\lambda,\alpha} = \{ 7.50.10^{-2} \alpha^{0.21} + 4.57.10^{-4} \lambda^{(0.71\alpha + 2.62)} \}^{0.5}$$
 (A-5)

The mixing of the flow due to the shear is related to the thrust parameter m:

$$(dr/dx)_{m} = \{ (1-m)(1.49+m)^{0.5} \} / \{ 9.76 (1+m) \}$$
 (A-6)

The half-wake width R_H at position X_H is calculated as follows:

$$R_{\rm H} = R_0 (0.134 + 0.124 \text{ m})^{-0.5}$$
(A-7)

II. Near wake section

The near wake section begins at X_H . This section ends at position X_N , the near wake length. In FARMS the wake is taken constant over this section. Among other things this means that equation A-7 of section I also holds for section II.

Finally, the parameter of interest for this study, the near wake length X_N , can be calculated as follows:

$$X_N = \{ C1.(1 - C2) / C2.(1 - C1) \} X_H$$
 (A-8)

in which the 'constants' C1 and C2 are:

$$C1 = (0.214 + 0.144 \text{ m})^{-0.5}$$

$$C2 = (0.134 + 0.124 \text{ m})^{-0.5}$$

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Figure 2Example of u and v series fitted by a second-order polynomial function

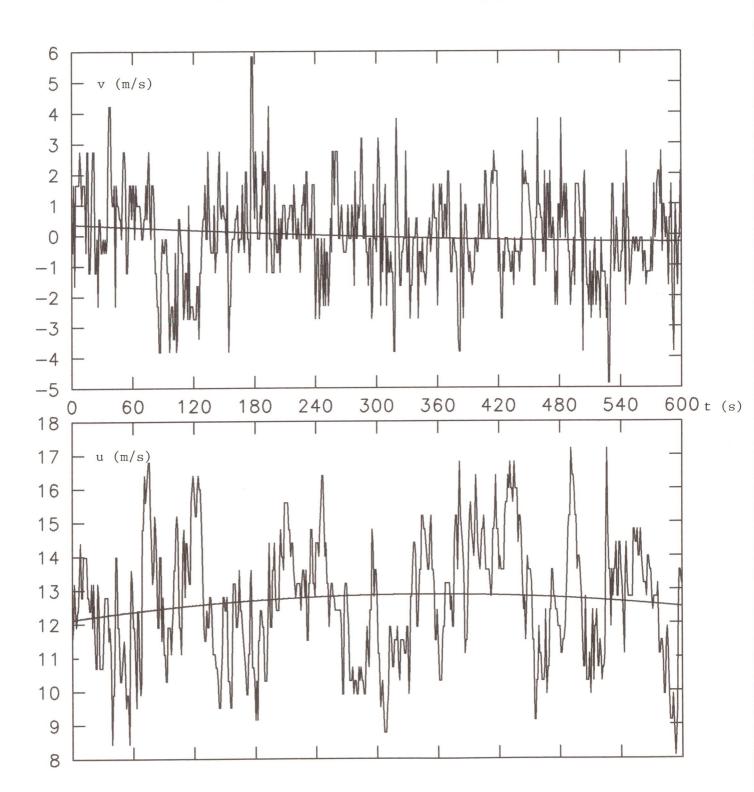
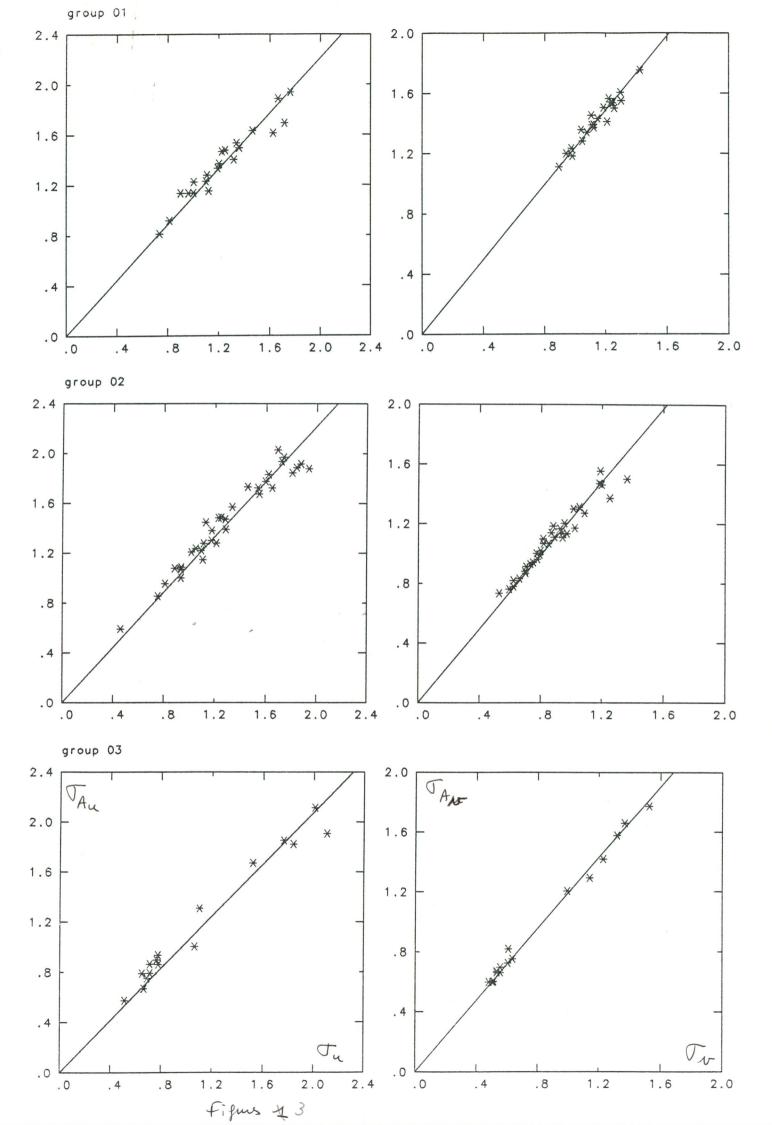
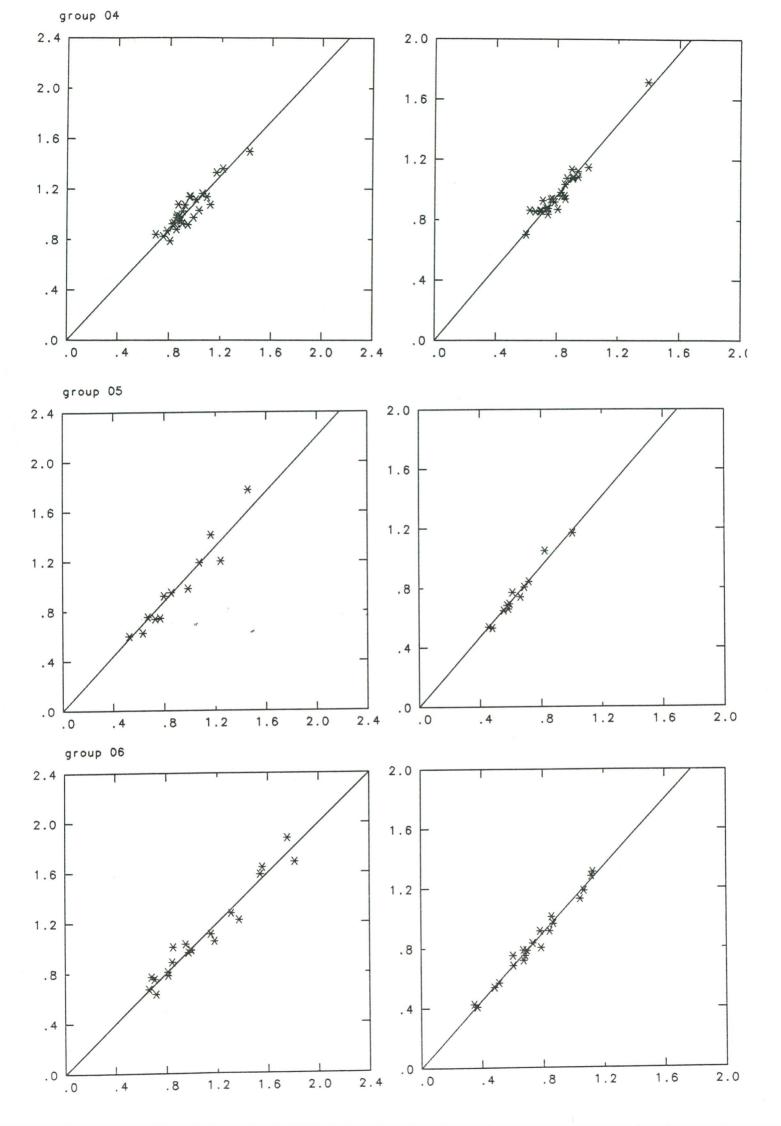
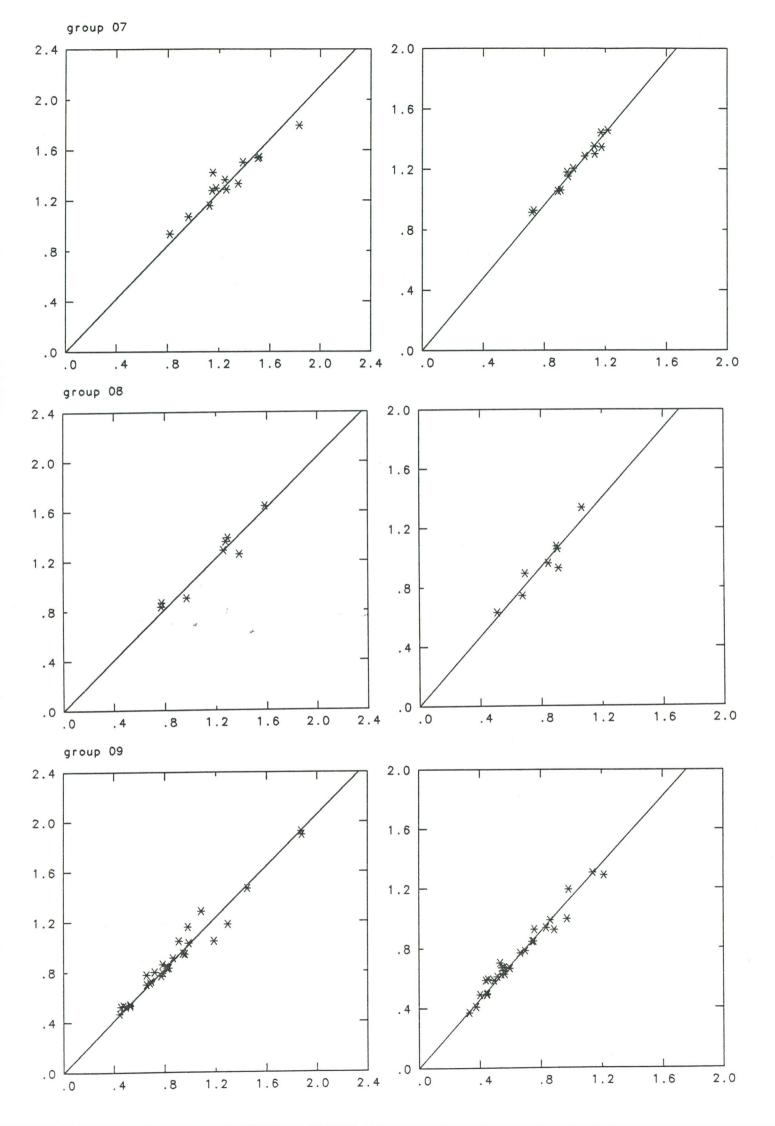


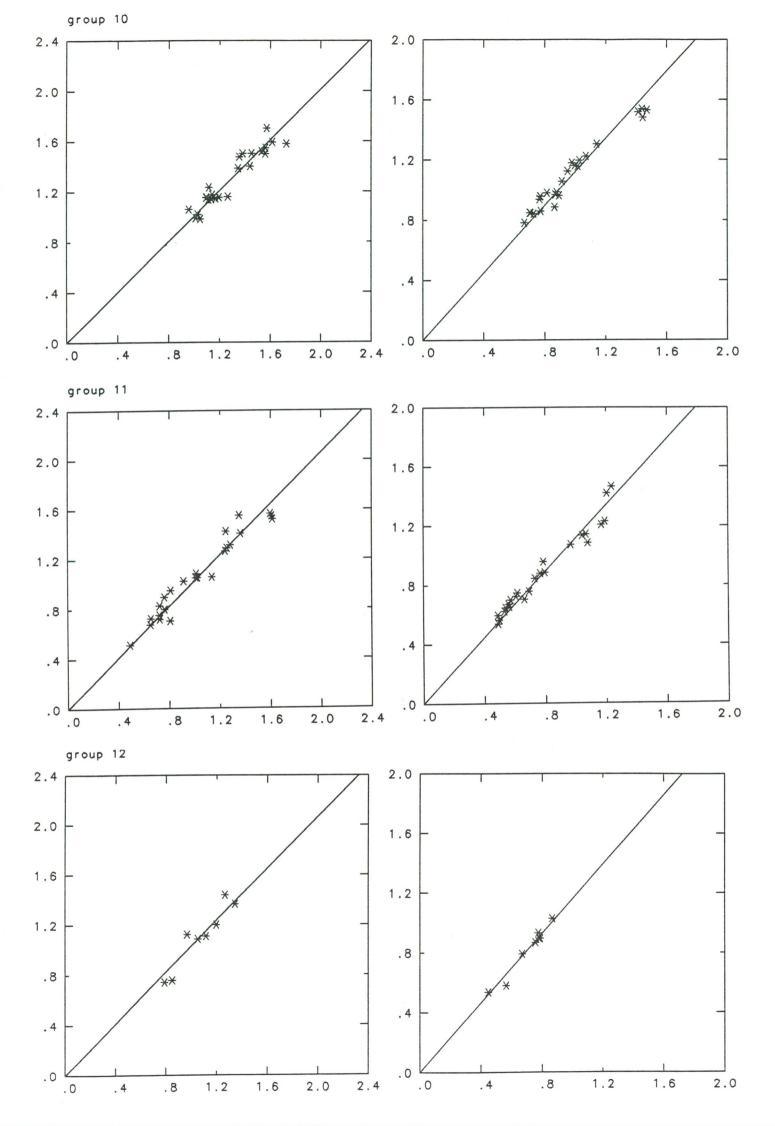
Figure 3

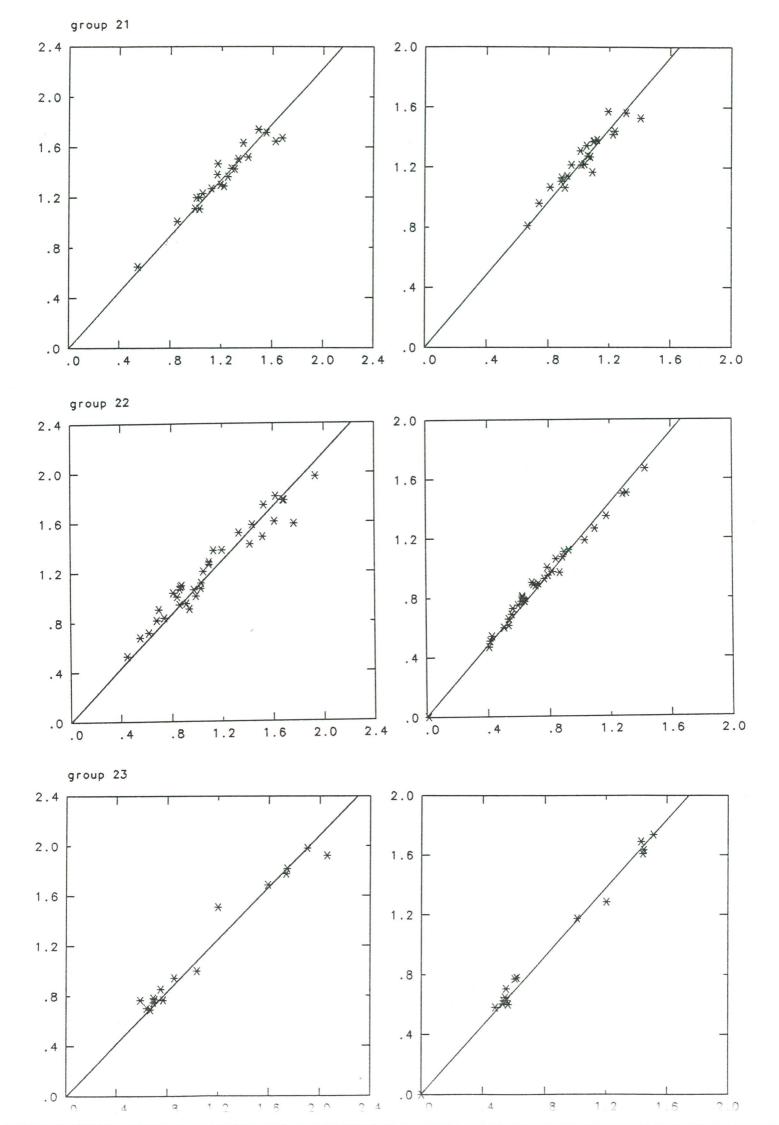
The relation between the standard deviation of the wake gust amplitude as a function of the corresponding standard deviation measured in the undisturbed situation (wind component u on the left side of the figures, wind component v on the right side of the figures). The slope of the line denotes the coefficient k_u and k_v respectively. The relation has been drawn for all individual groups mentioned in table 3.

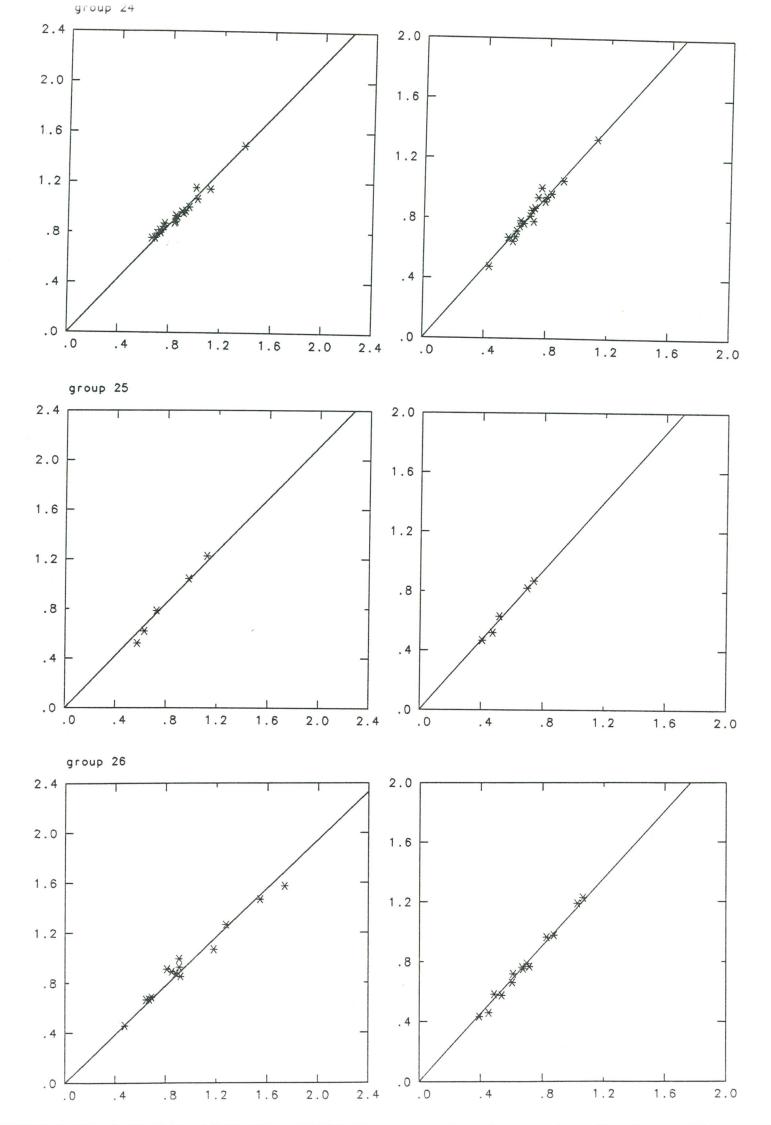


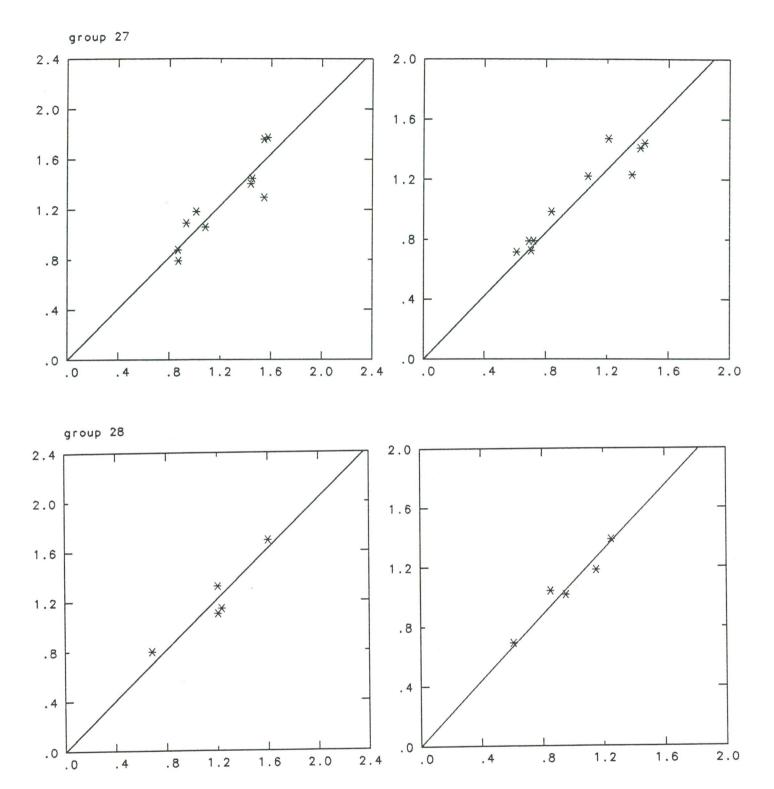


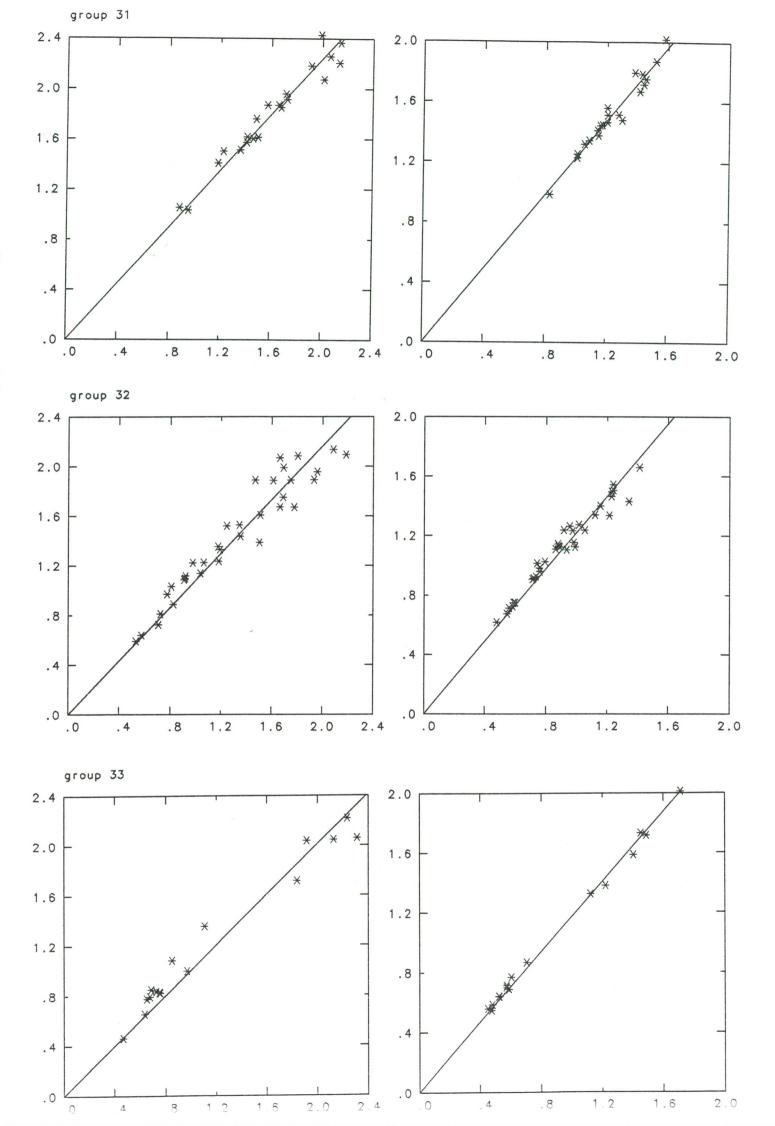


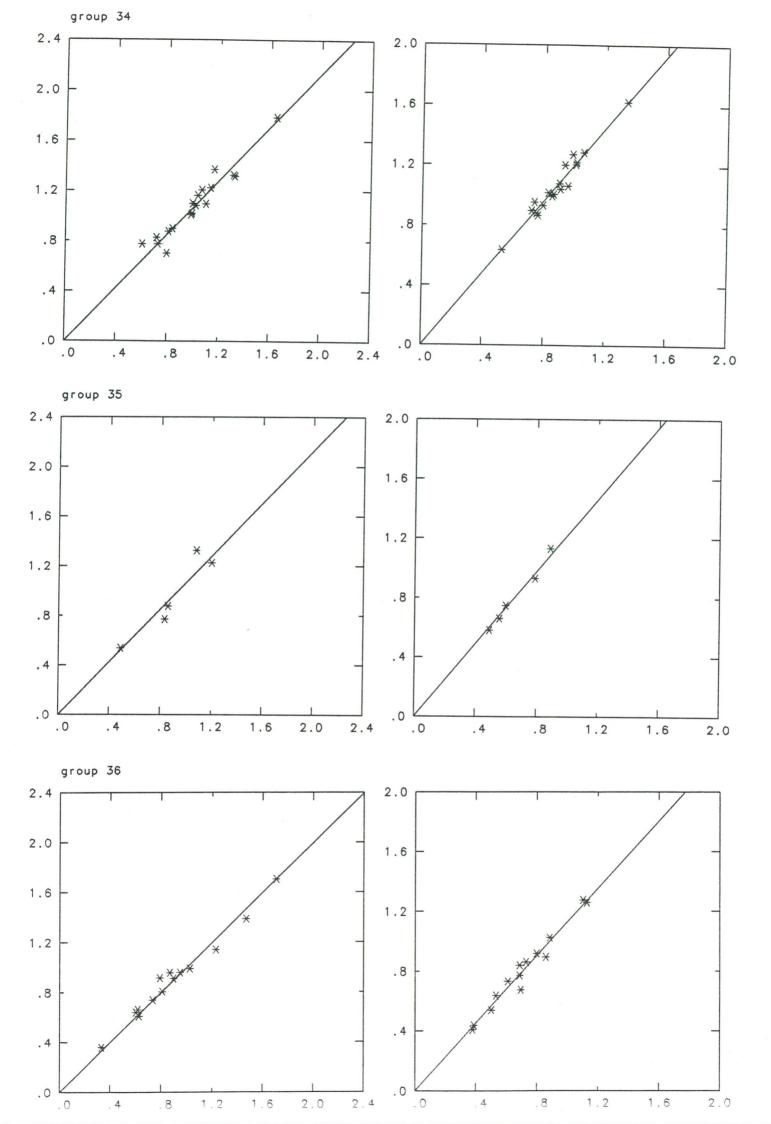


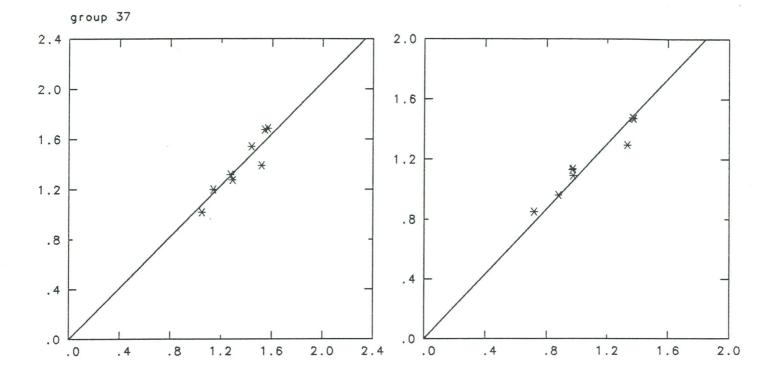


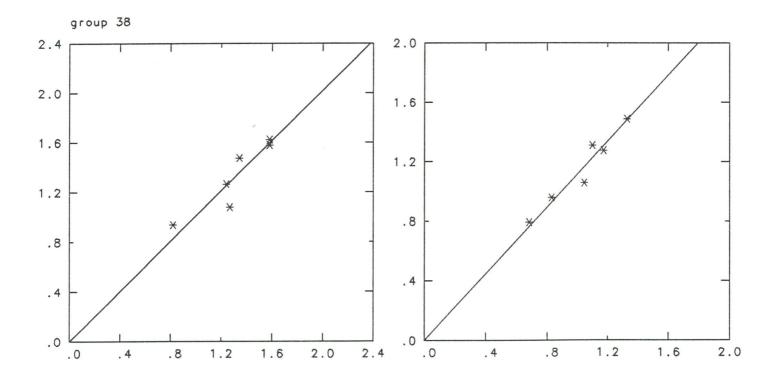










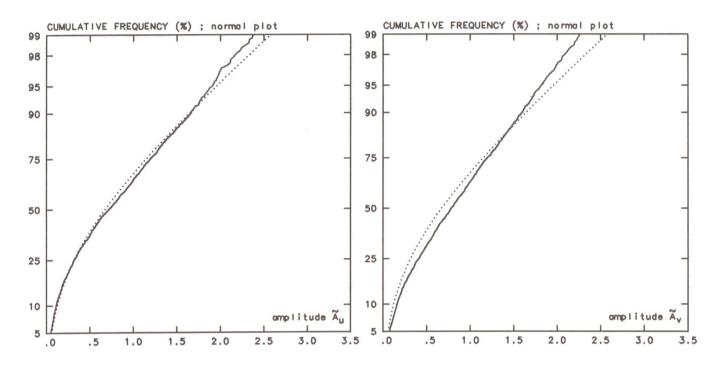


Wake and wind farm modelling Part B: Wake gust modelling

Figure 4

The cumulative frequency (in %) of the overall probability density function of the normalized gust amplitudes (wind component u on the left side of the figures, wind component v on the right side of the figures). The solid line represents the measured gusts, the dashed line represents a half-gaussian distribution given in equation 3-9. The relation has been drawn for all individual groups mentioned in table 3.

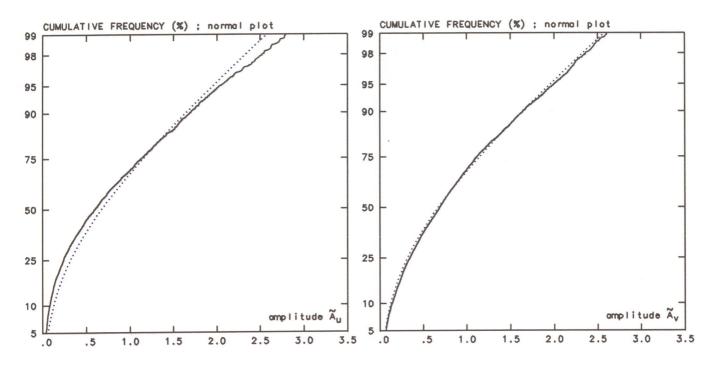
group 01



DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES

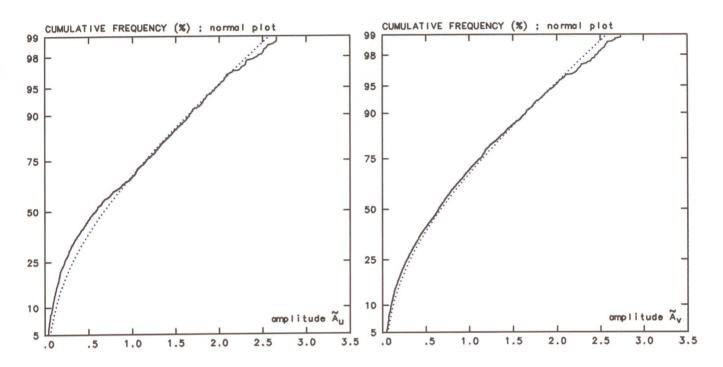
Figures 24

group 02



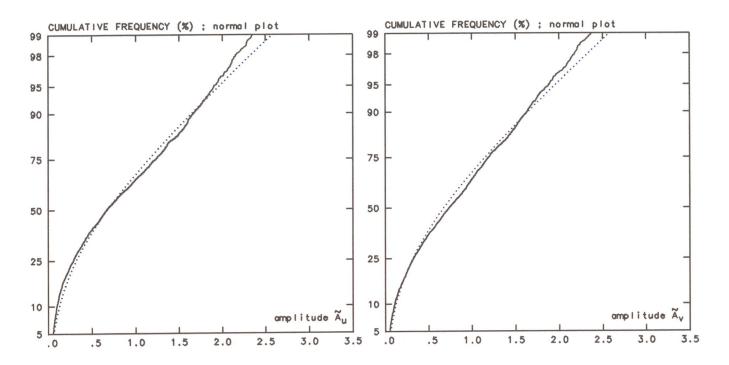
DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES

group 03



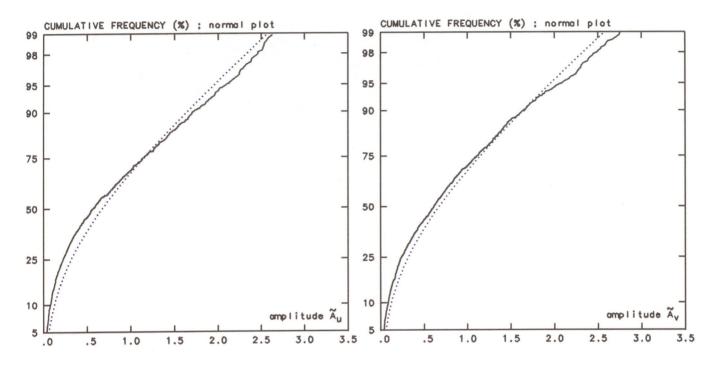
DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES

group 04



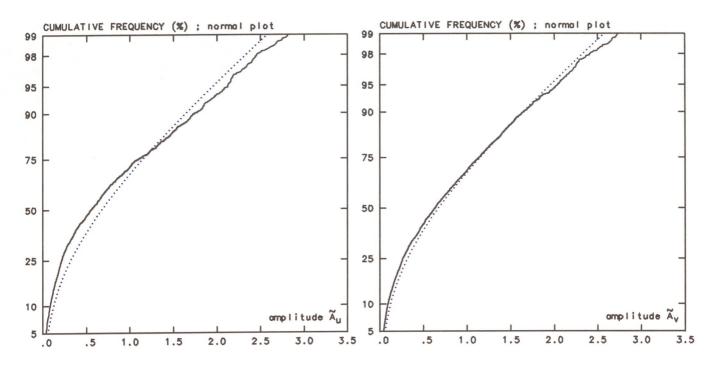
DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES

group 05



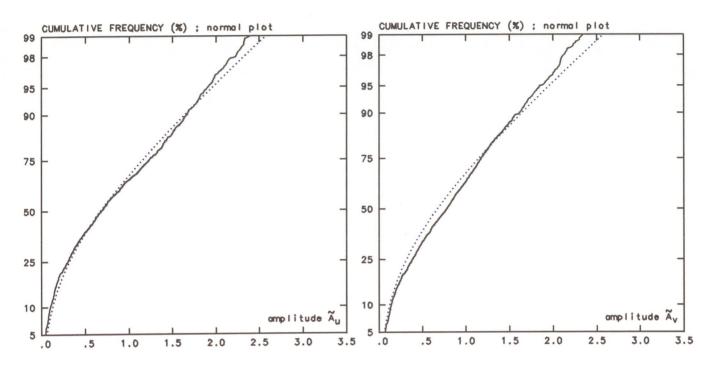
DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES

group 06



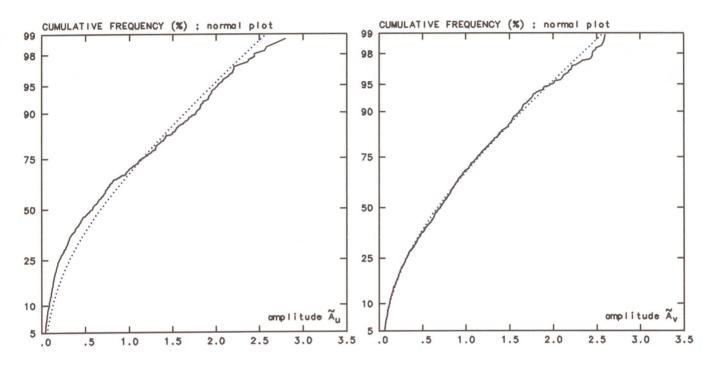
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group 07



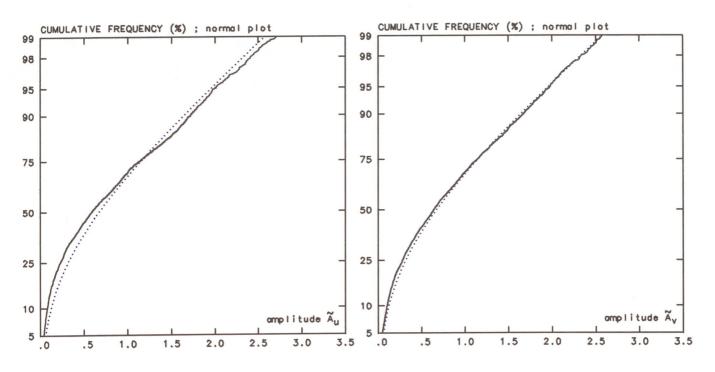
DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES

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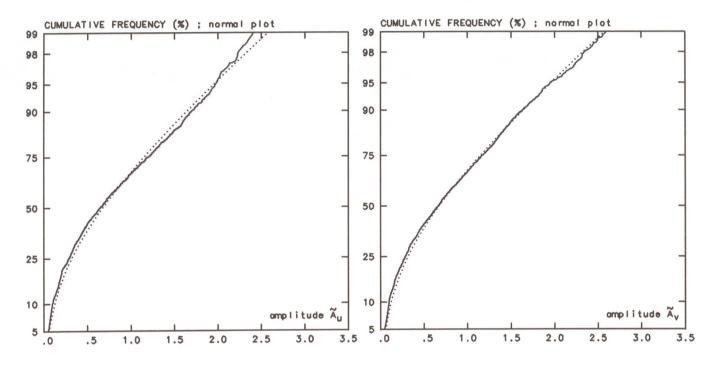
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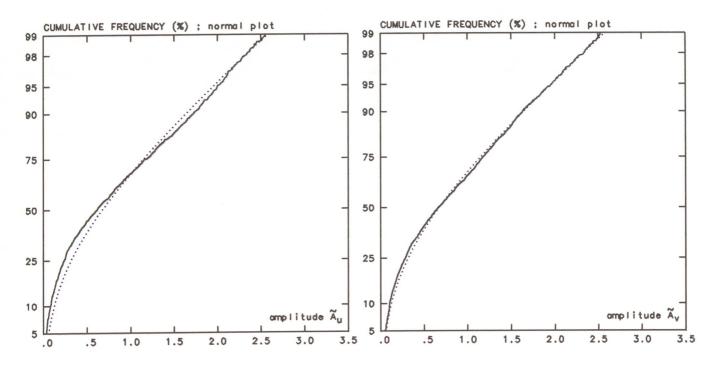
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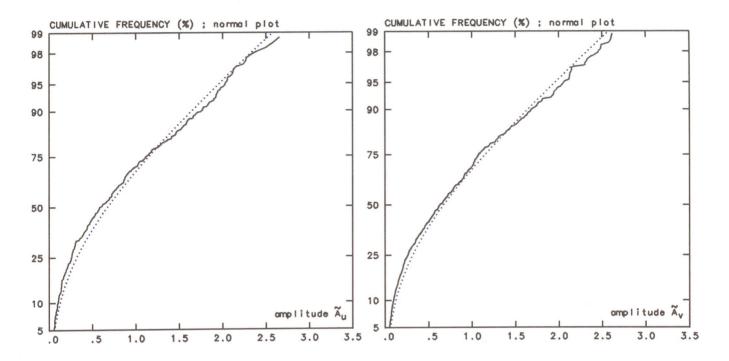
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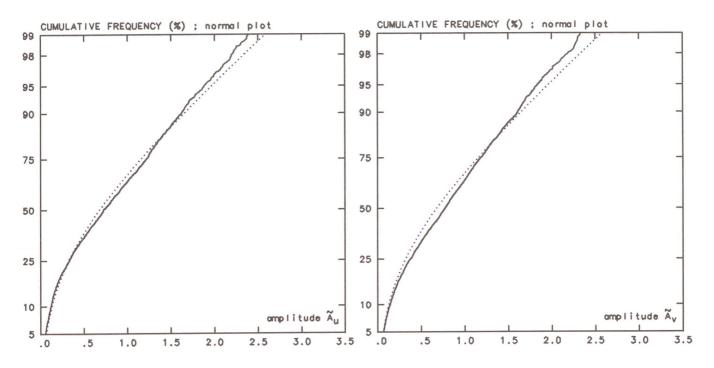
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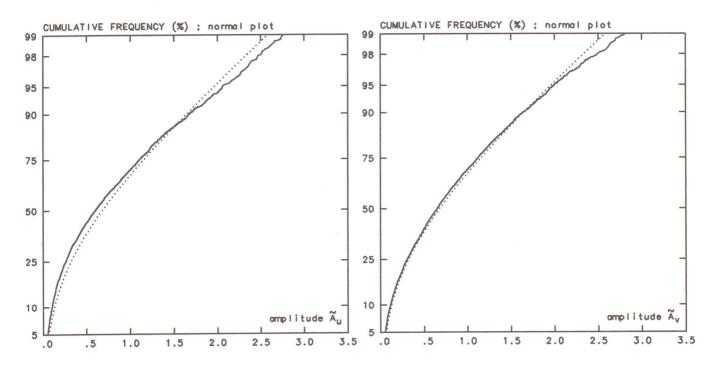
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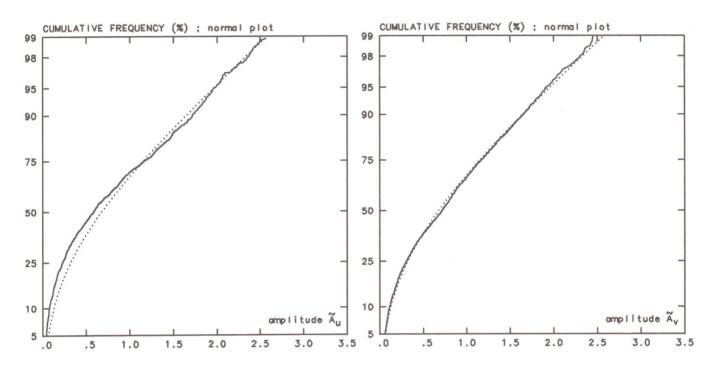
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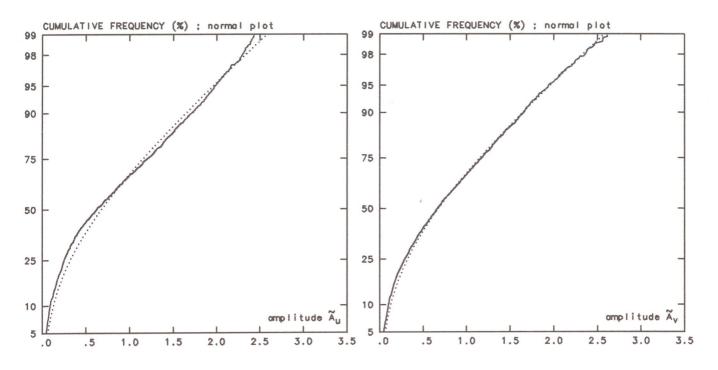
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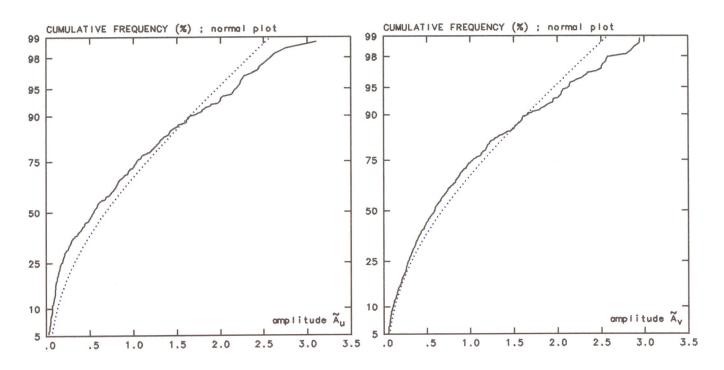
group 24



DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES

----- measurements mode!

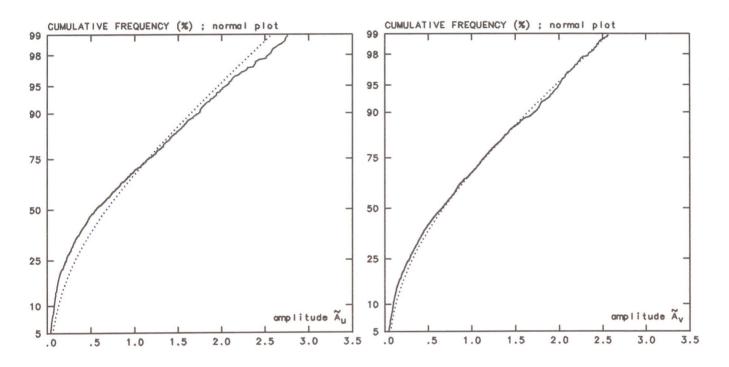
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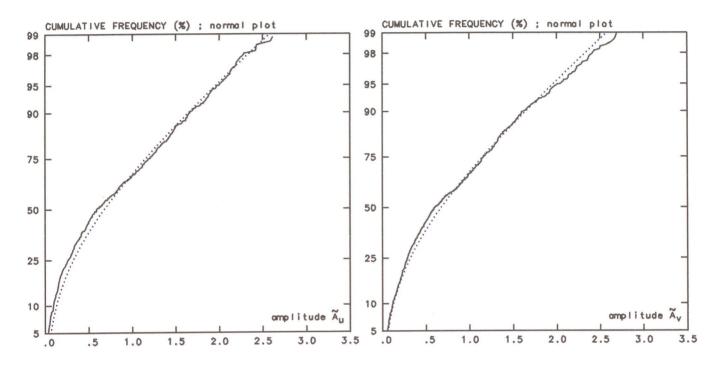
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group 26



DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES

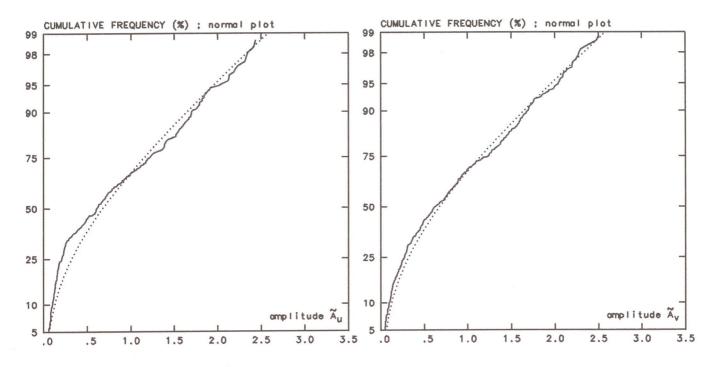
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DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES

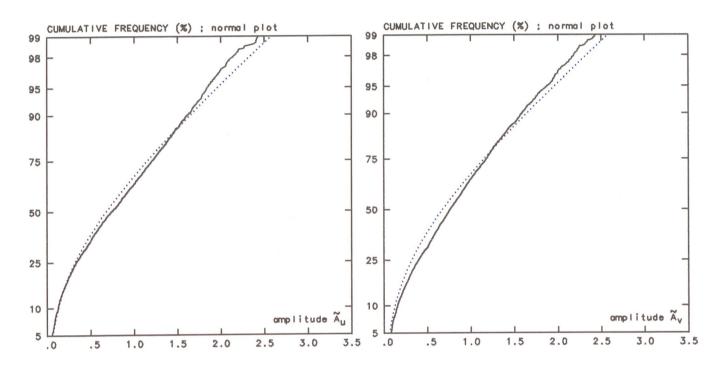
----- measurements model

group 28



DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES

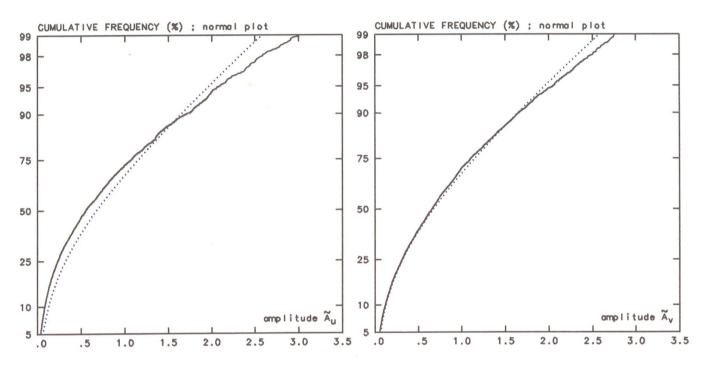
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DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES

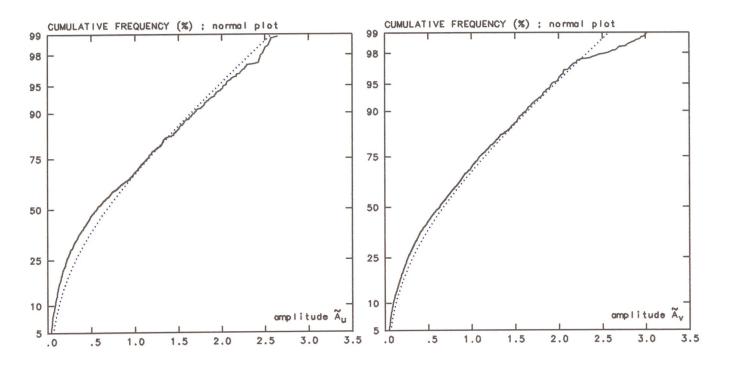
----- measurements model

group 32



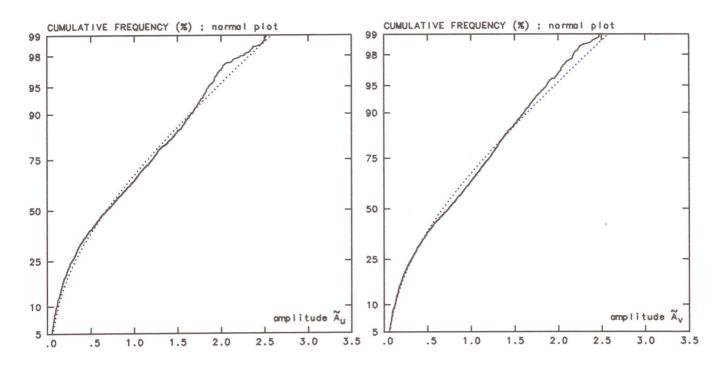
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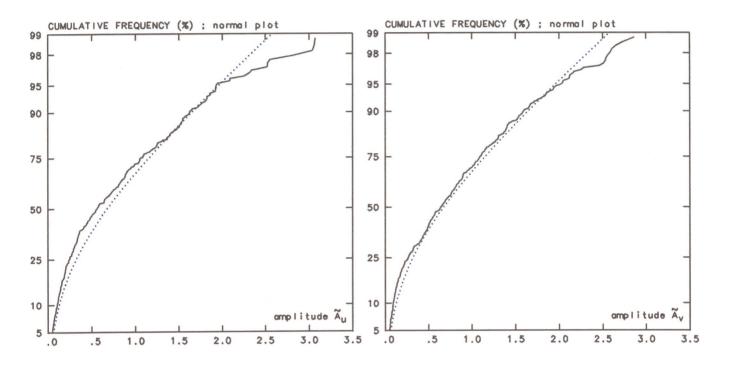
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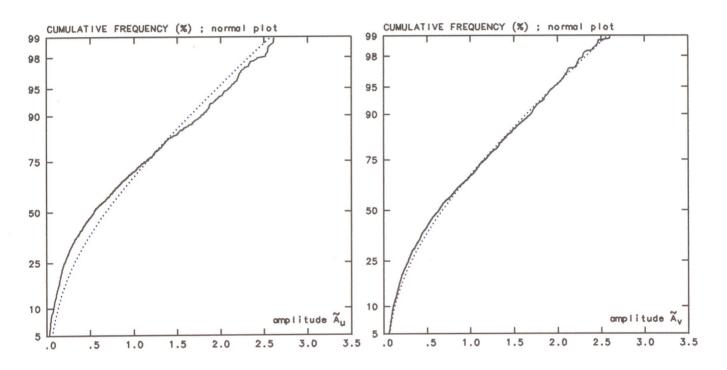
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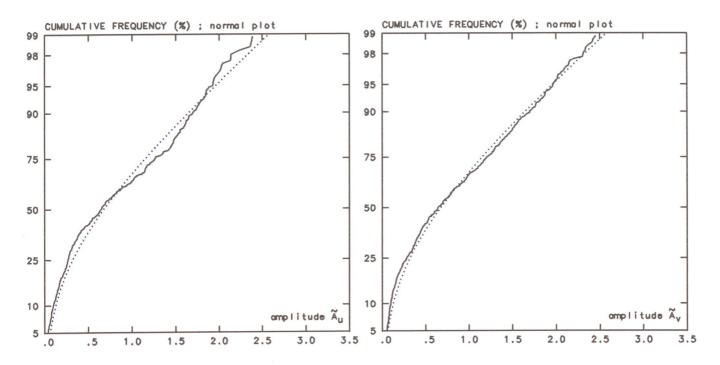
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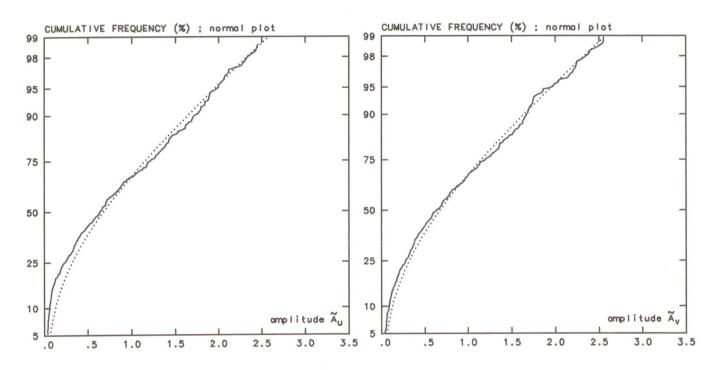
DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES

group 37



DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES

group 38



DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES

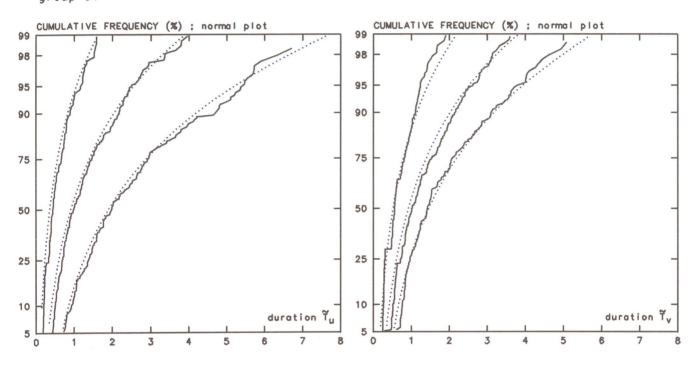
Wake and wind farm modelling Part B: Wake gust modelling

Figure 5

The cumulative frequency (in %) of the conditional probability density function of the normalized gust durations for given normalized gust amplitudes (wind component u on the left side of the figures, wind component v on the right side of the figures). The solid line represents the measured gusts, the dashed line represents a log-normal distribution given in equations 3-10 an 3-11.

The relation has been drawn for all individual groups mentioned in table 3.

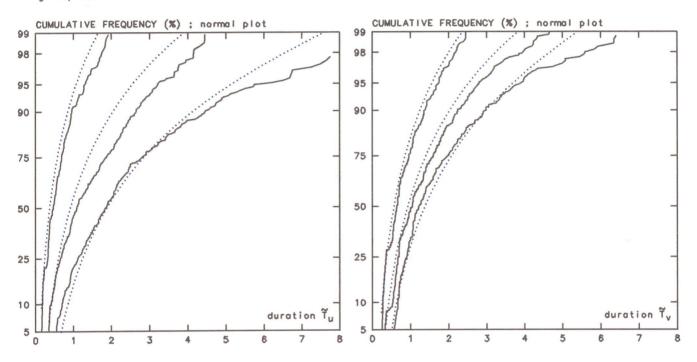
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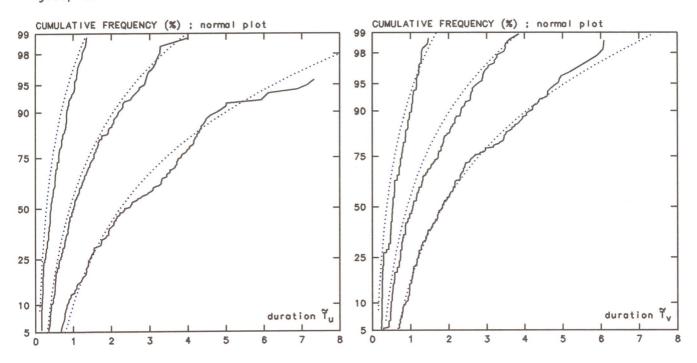
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST DURATIONS FOR GIVEN NORMALIZED ONE-POINT AMPLITUDE

Figures \$ 5

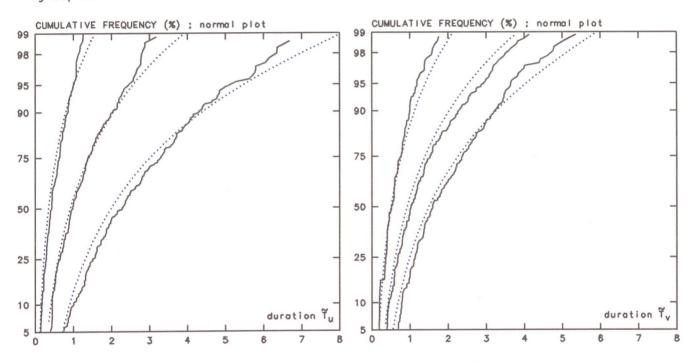
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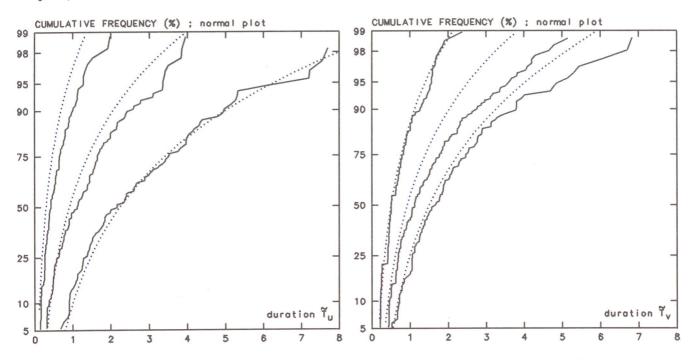
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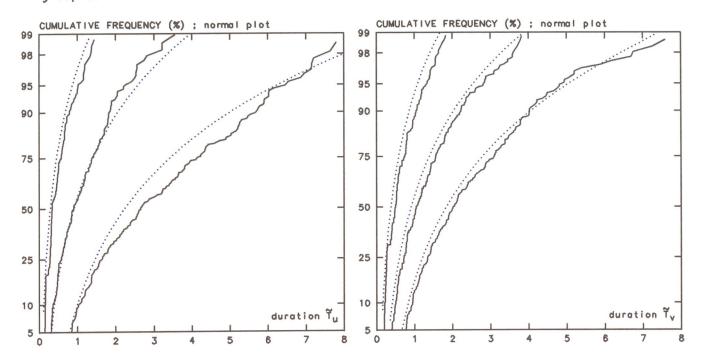
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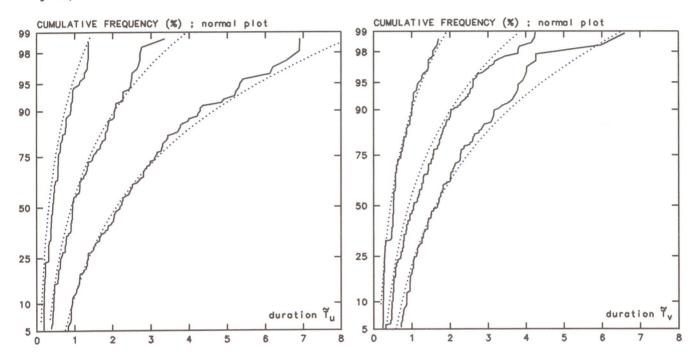
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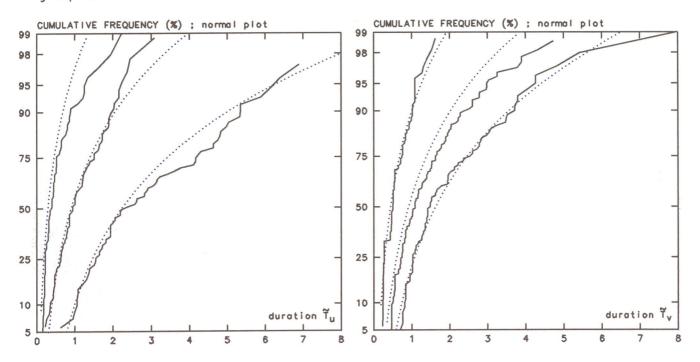
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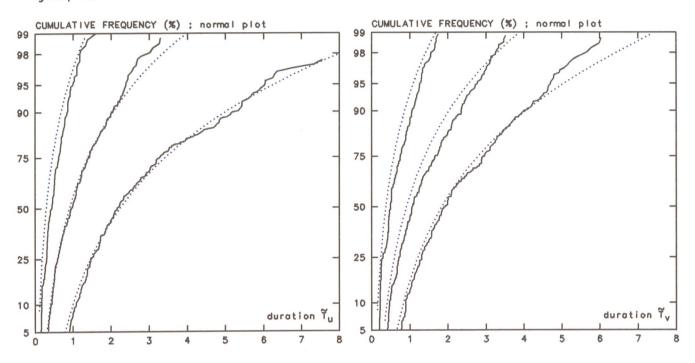
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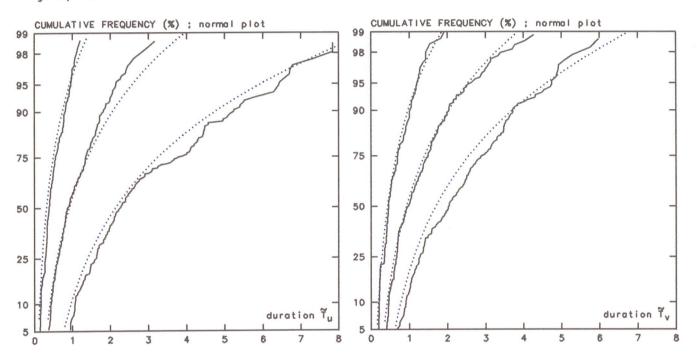
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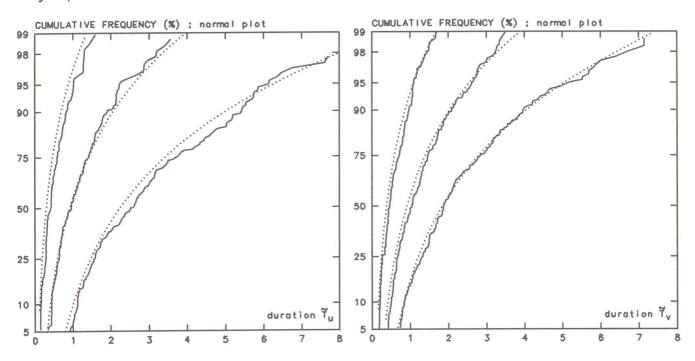
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group 10

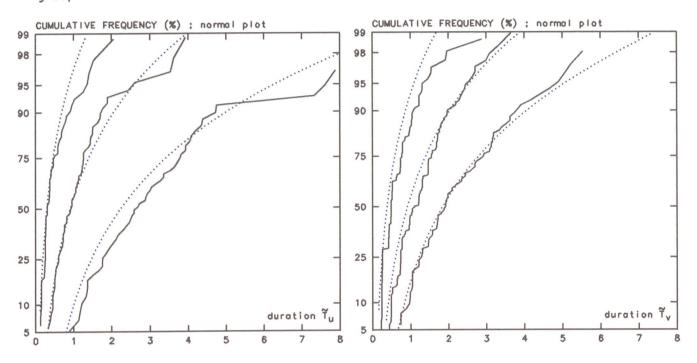


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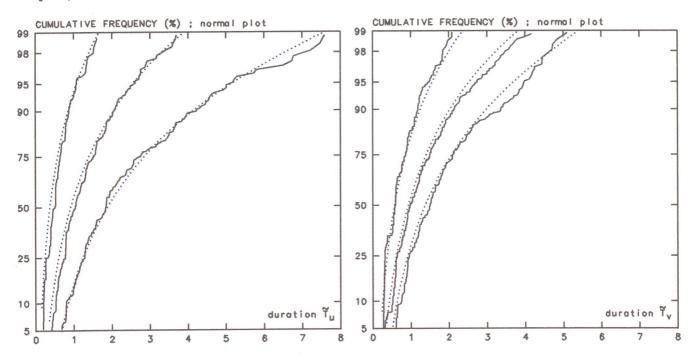


CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST DURATIONS FOR GIVEN NORMALIZED ONE-POINT AMPLITUDE

group 12

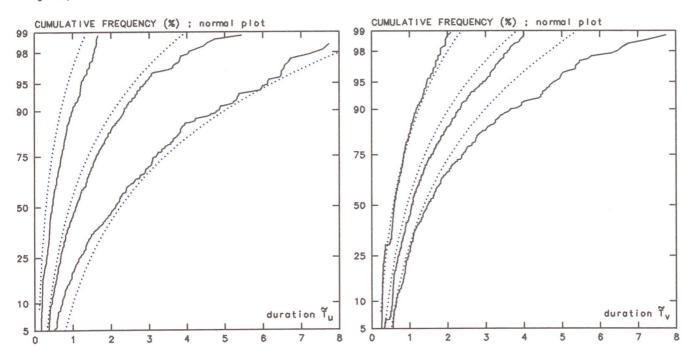


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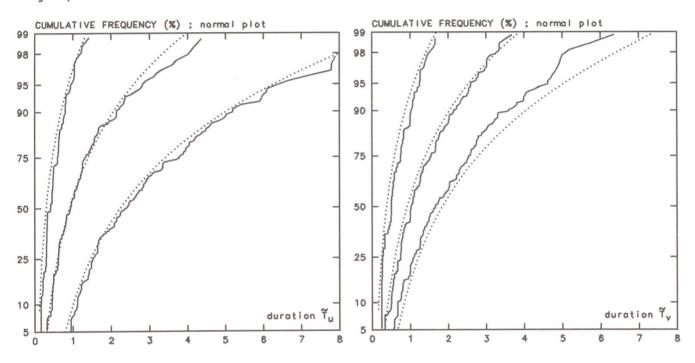


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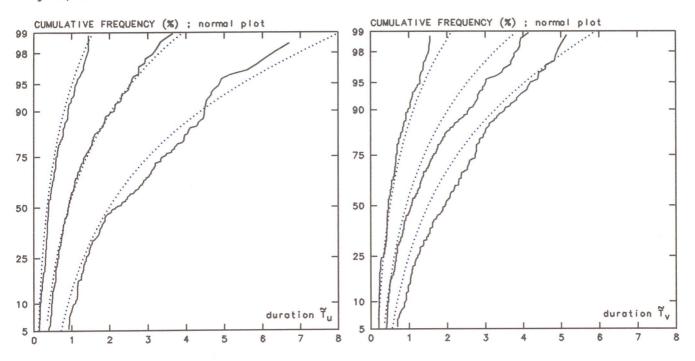
group 22



group 23

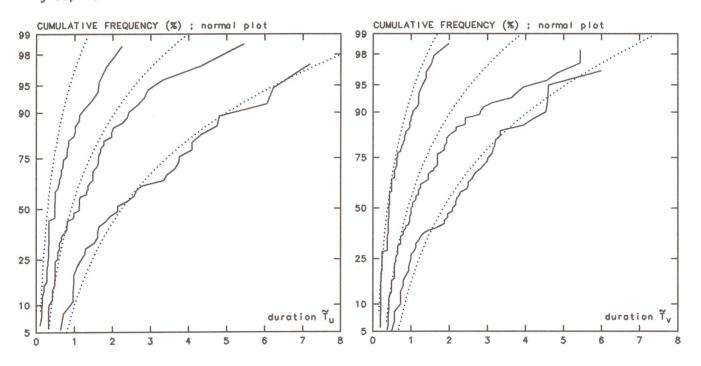


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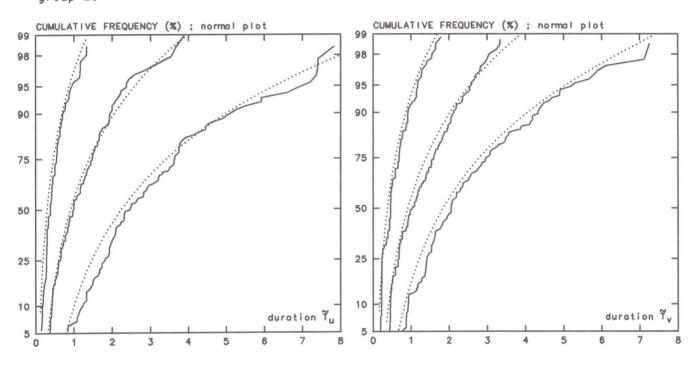


----- measurements model

group 25

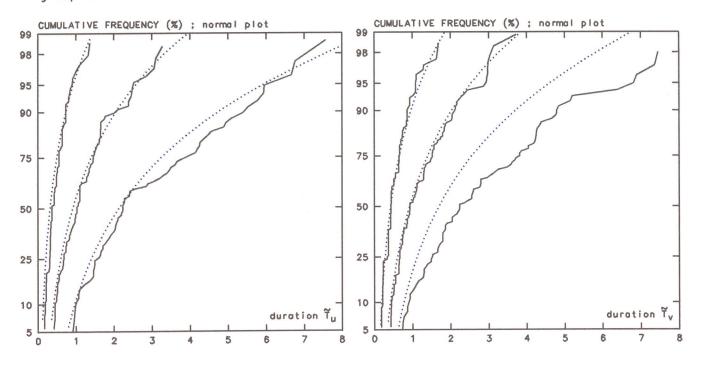


group 26

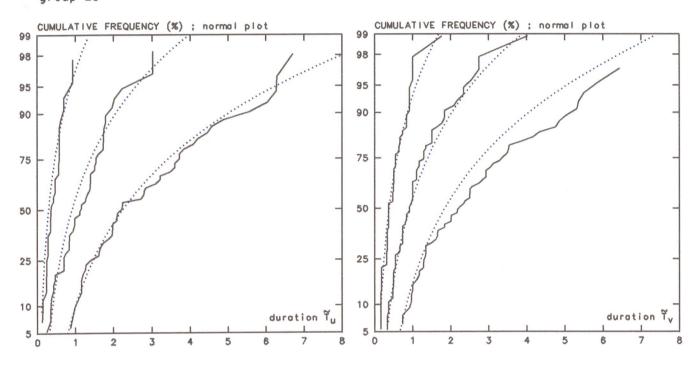


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group 27

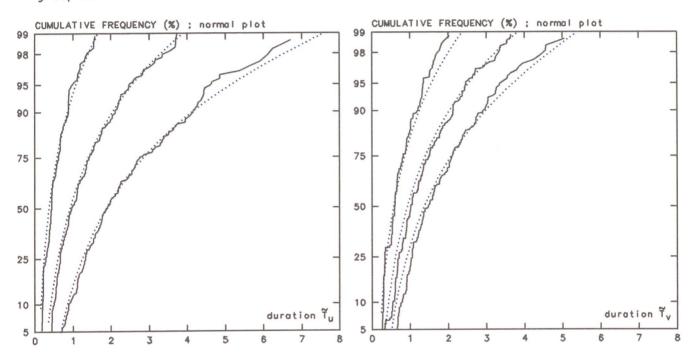


group 28

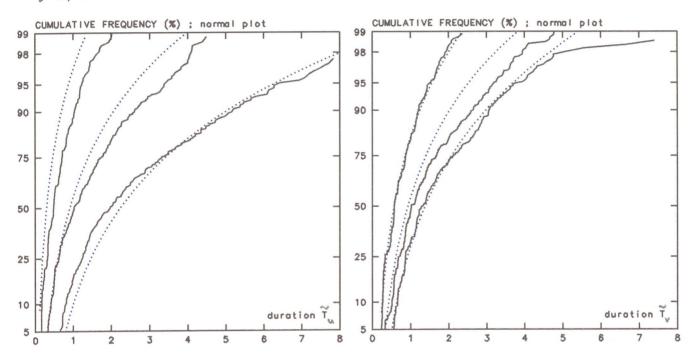


CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST DURATIONS FOR GIVEN NORMALIZED ONE-POINT AMPLITUDE

group 31

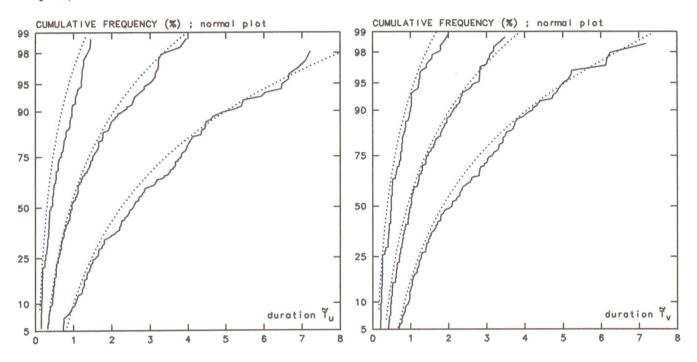


group 32

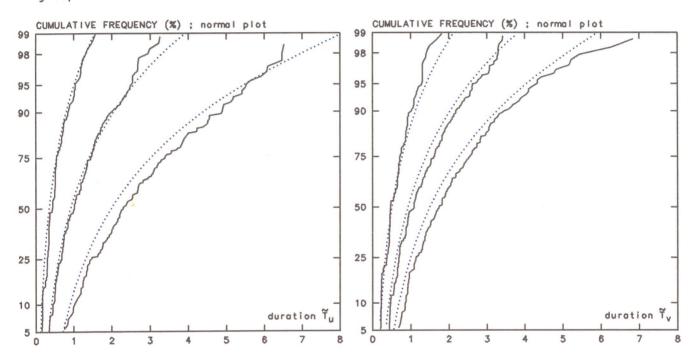


CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST DURATI

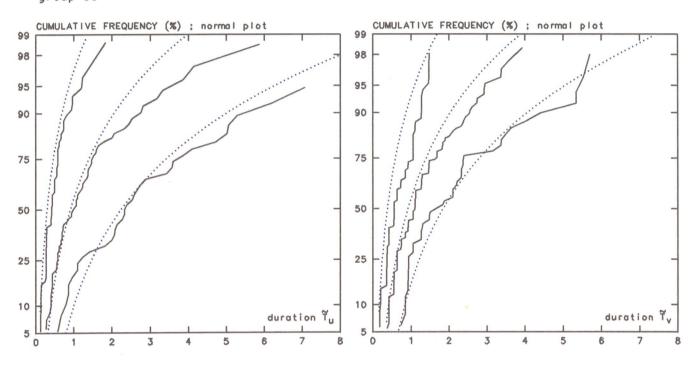
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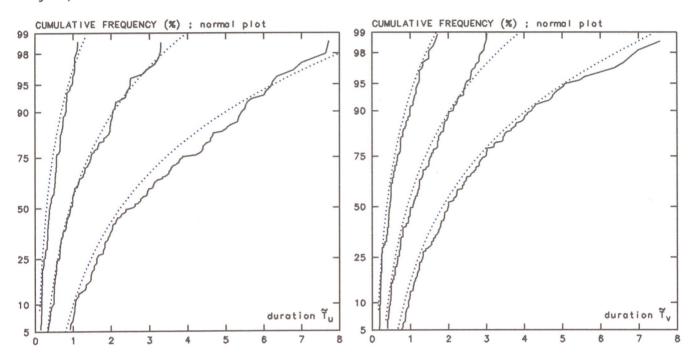
group 34



group 35

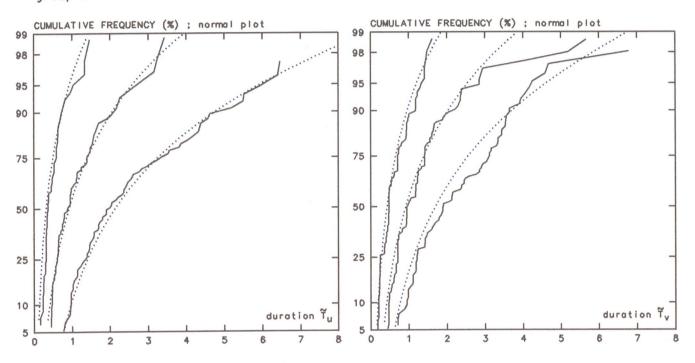


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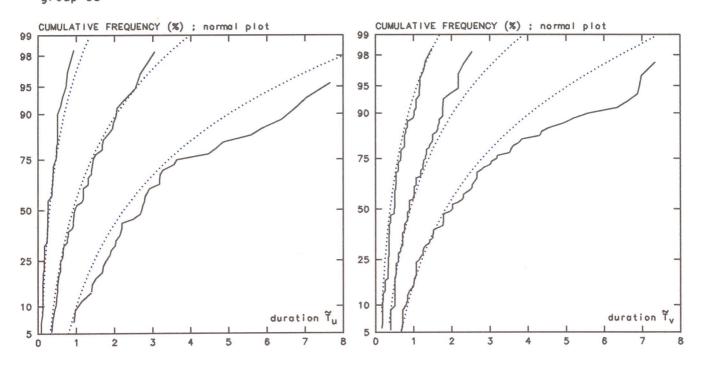
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST DURATIONS FOR GIVEN NORMALIZED ONE-POINT AMPLITUDE

group 37



CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST DURATIONS FOR GIVEN NORMALIZED ONE-POINT AMPLITUDE

group 38



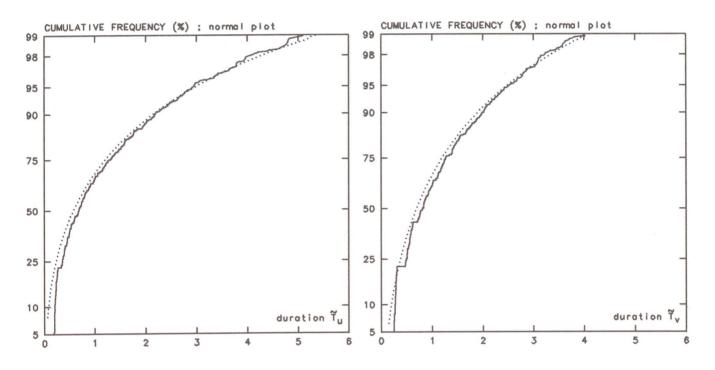
Wake and wind farm modelling Part B: Wake gust modelling

Figure 6

The cumulative frequency (in %) of the overall probability density function of the normalized gust durations (wind component u on the left side of the figures, wind component v on the right side of the figures). The solid line represents the measured gusts, the dashed line represents a distribution function based on the functions mentioned at figures 4 and 5.

The relation has been drawn for all individual groups mentioned in table 3.

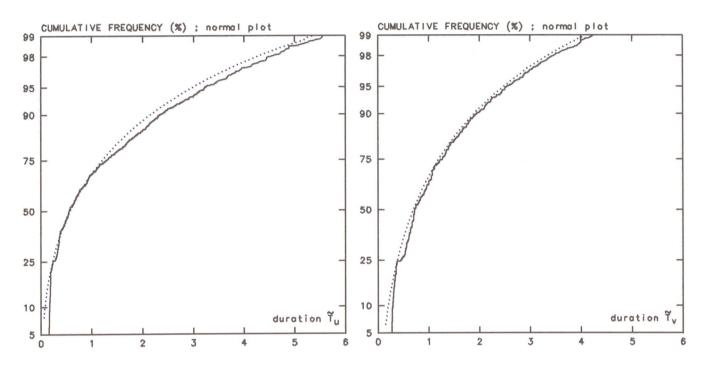
group 01



DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

Figure 4 6

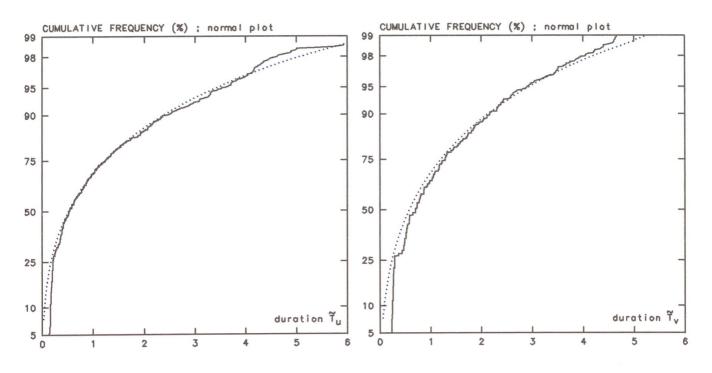
group 02



DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

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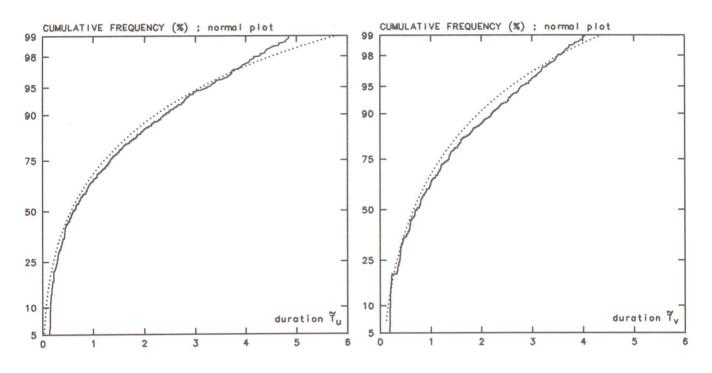
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DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

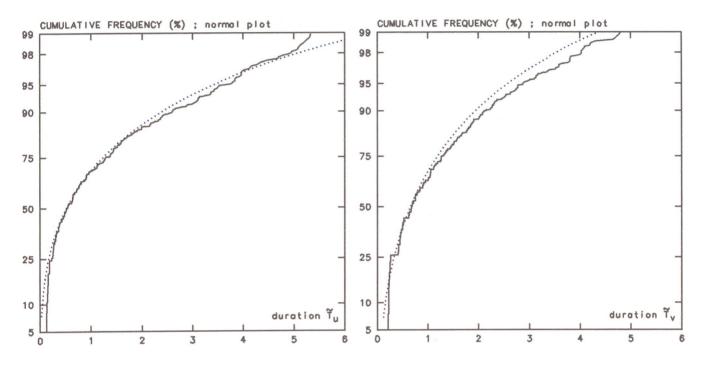
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group 04



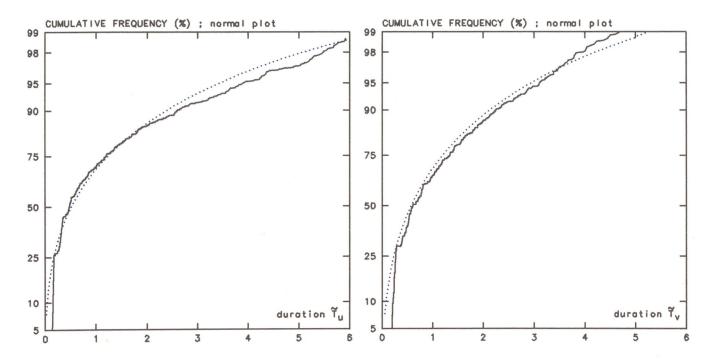
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 05



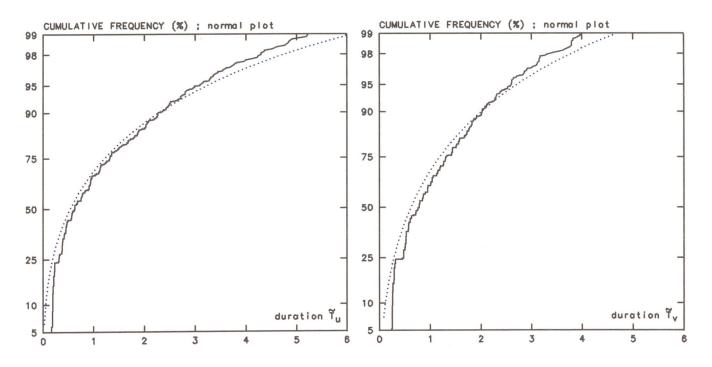
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 06



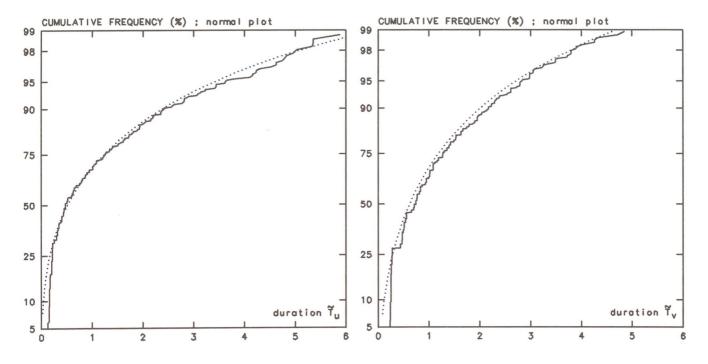
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 07



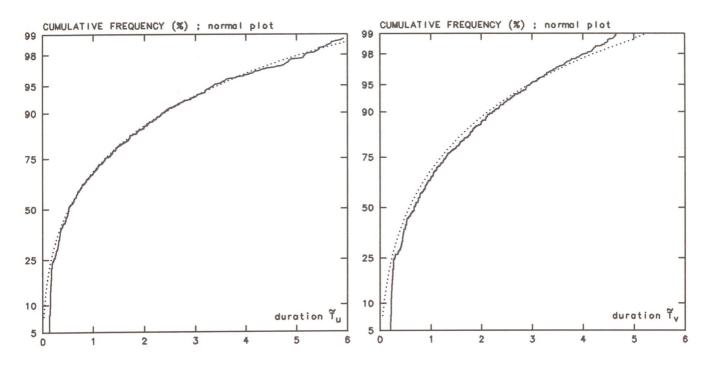
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 08



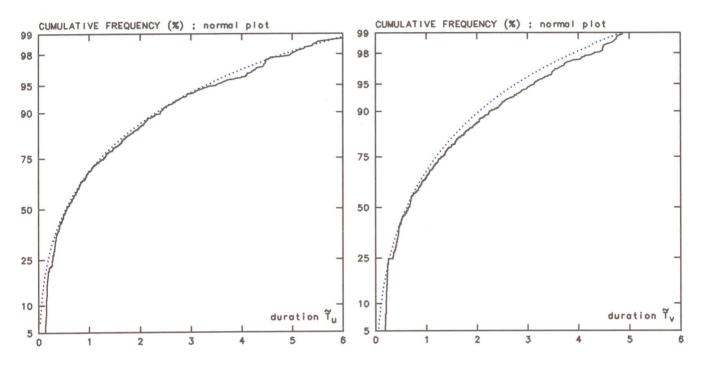
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group 09



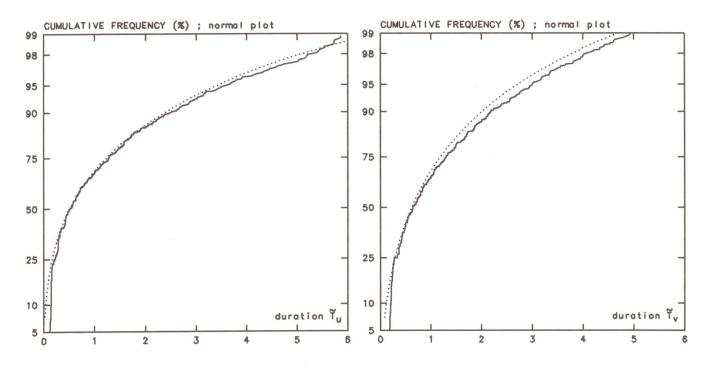
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 10



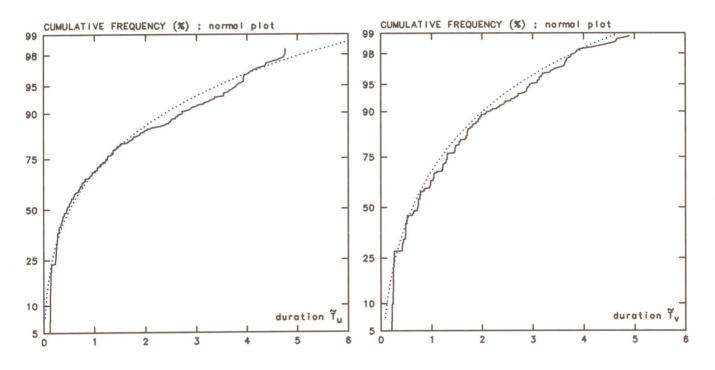
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 11



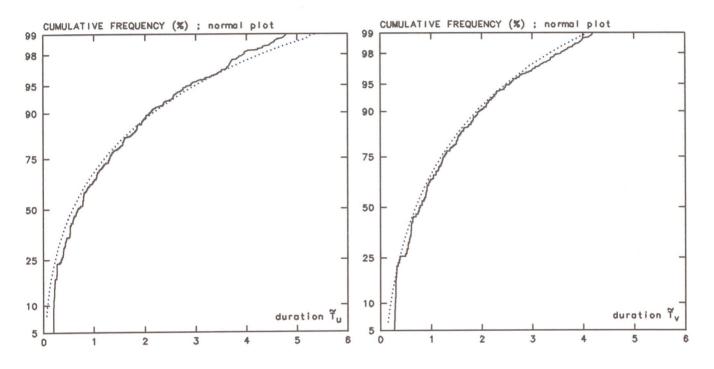
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

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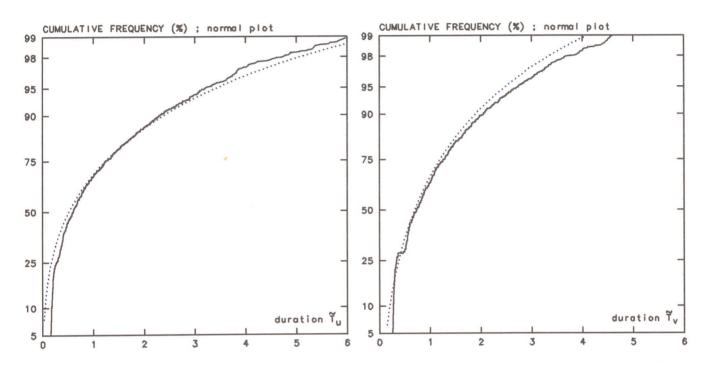
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 21



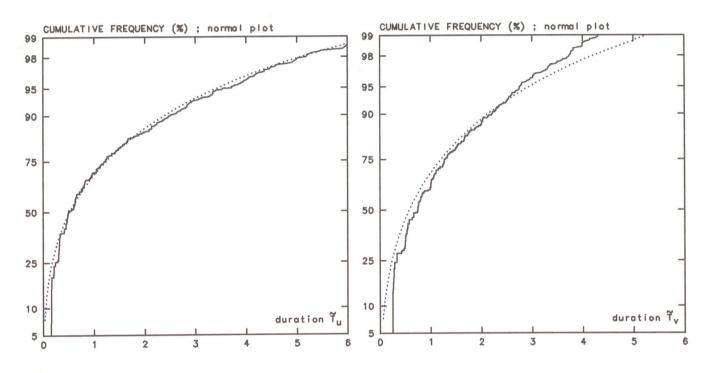
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 22



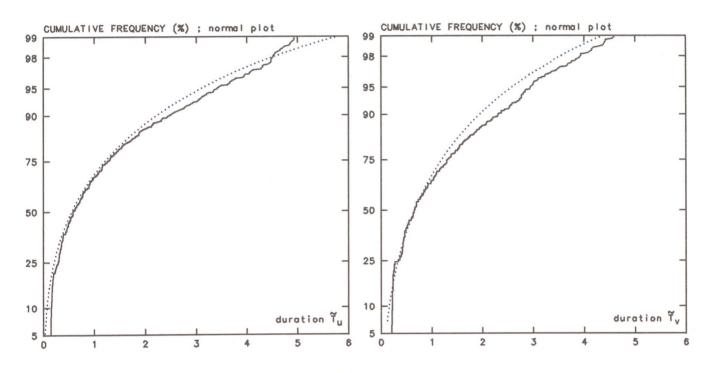
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 23



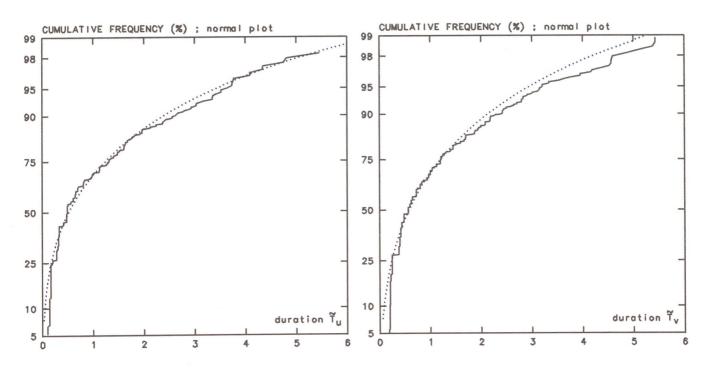
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 24



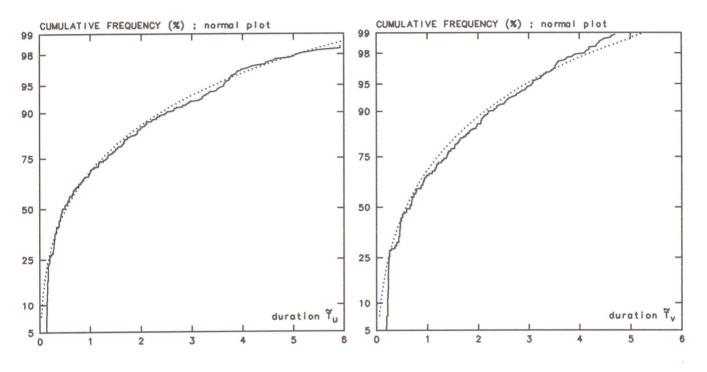
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 25



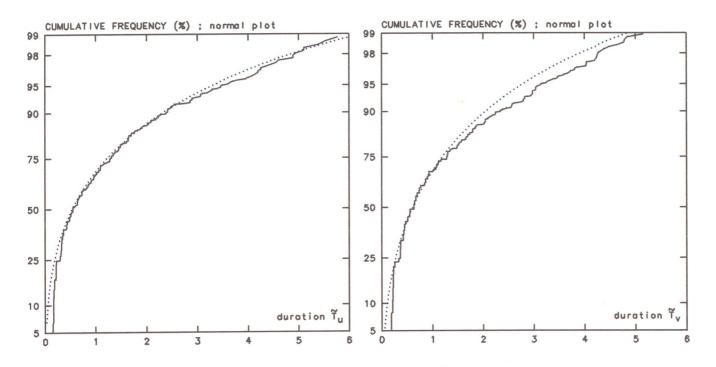
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 26



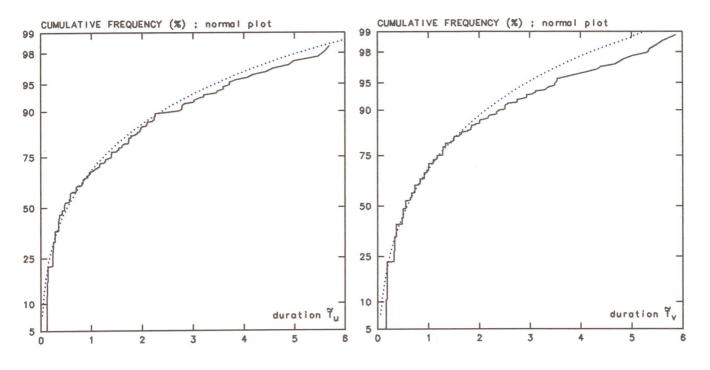
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 27



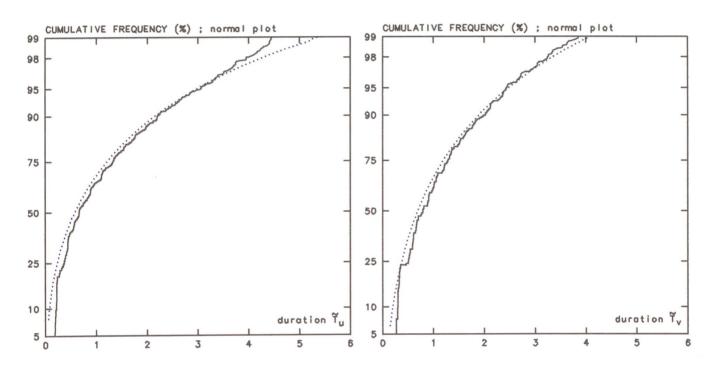
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 28



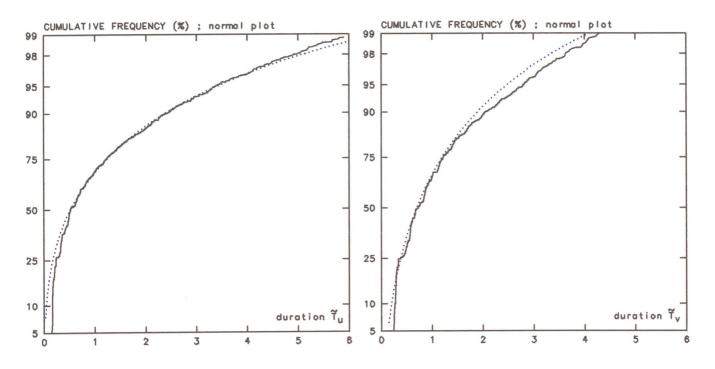
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 31



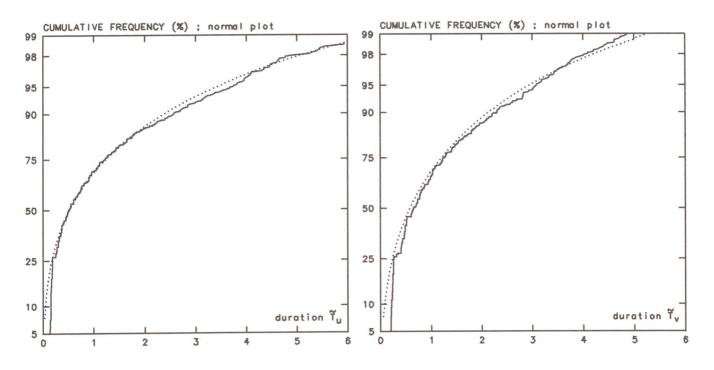
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 32



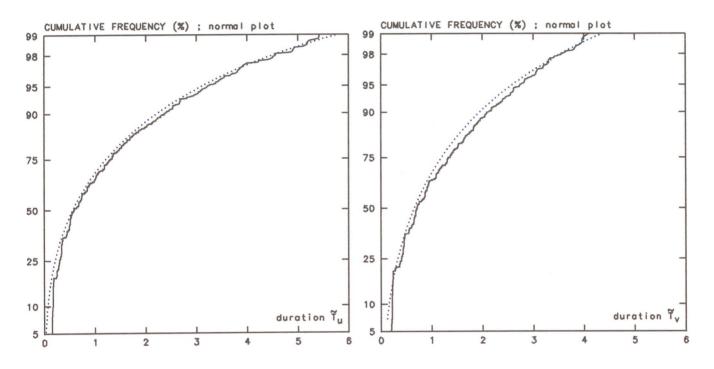
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 33



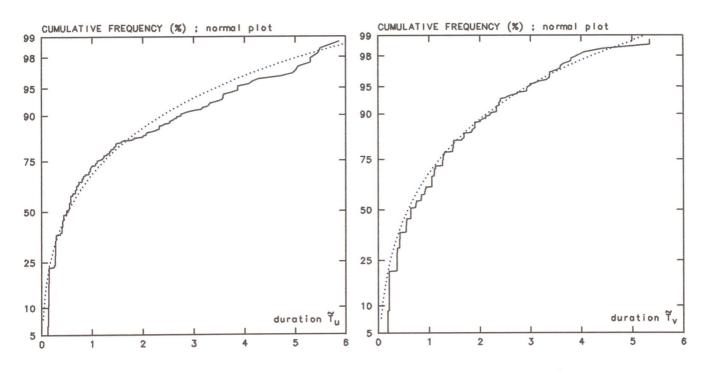
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 34



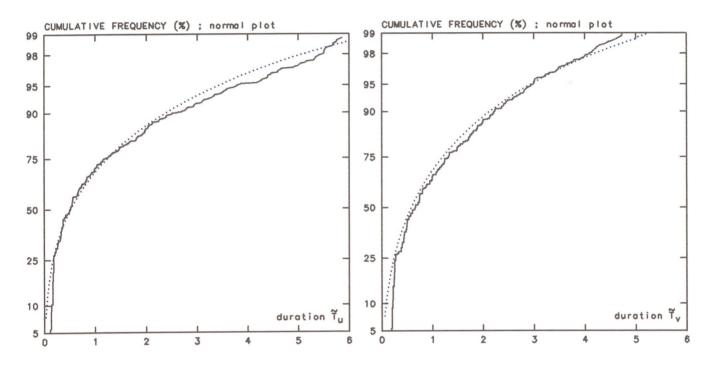
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 35



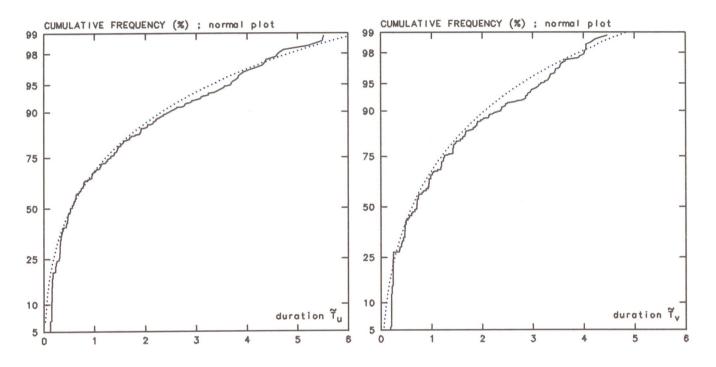
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 36



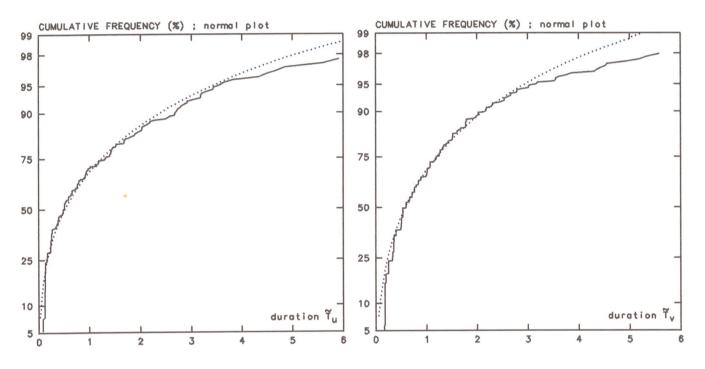
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 37



DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

group 38



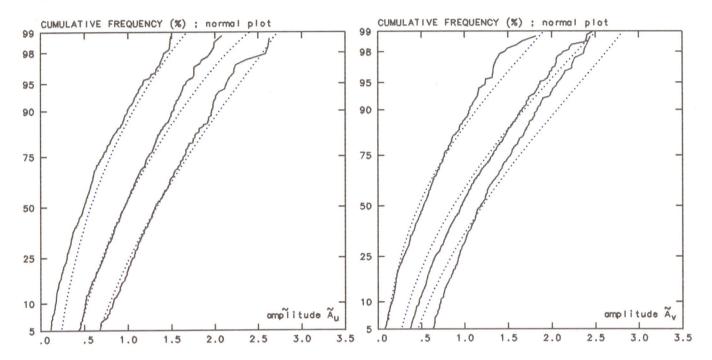
DISTRIBUTIONS OF NORMALIZED GUST DURATIONS

Wake and wind farm modelling Part B: Wake gust modelling

Figure 7

The cumulative frequency (in %) of the conditional probability density function of the normalized gust amplitudes for given normalized gust durations (wind component u on the left side of the figures, wind component v on the right side of the figures). The solid line represents the measured gusts, the dashed line represents a distribution function based on the functions mentioned at figures 4, 5 and 6. The relation has been drawn for all individual groups mentioned in table 3.

group 01

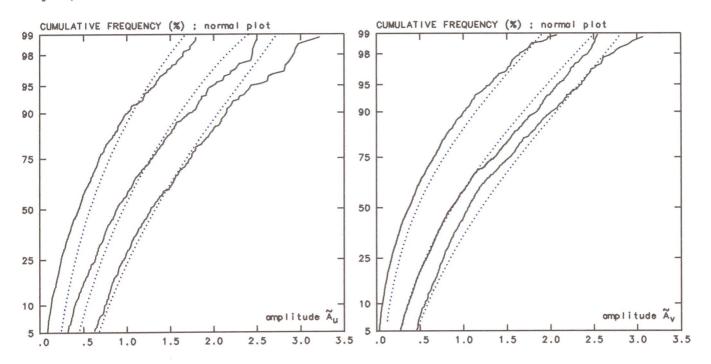


CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES

FOR GIVEN NORMALIZED DORALLOR

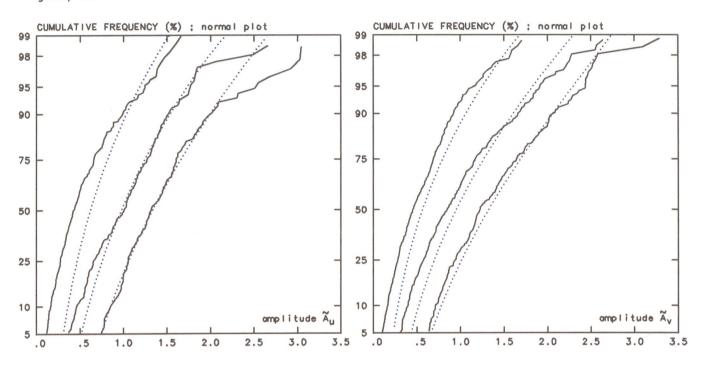
Figures \$ 7

group 02



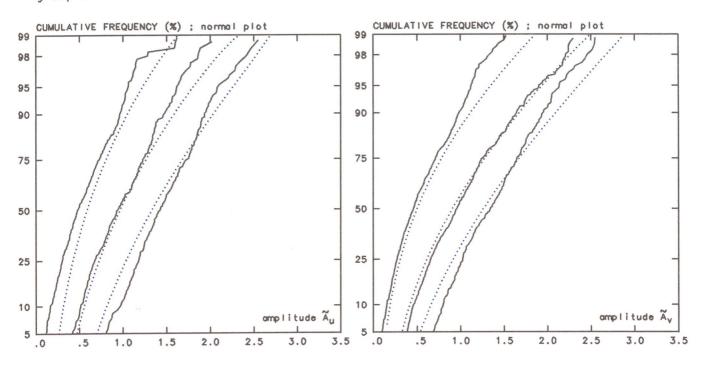
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 03



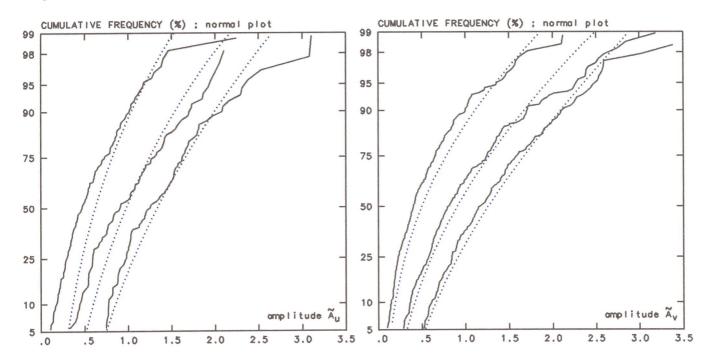
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 04



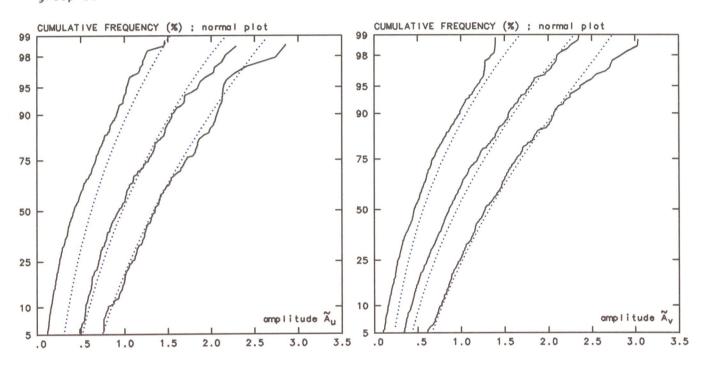
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 05



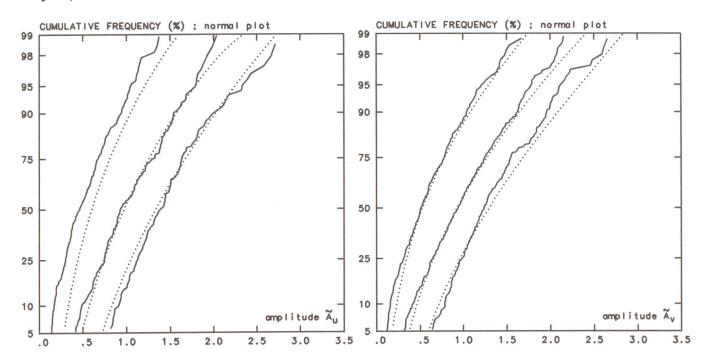
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 06



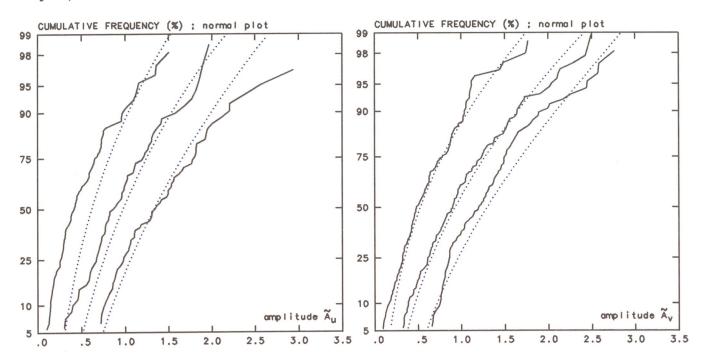
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 07



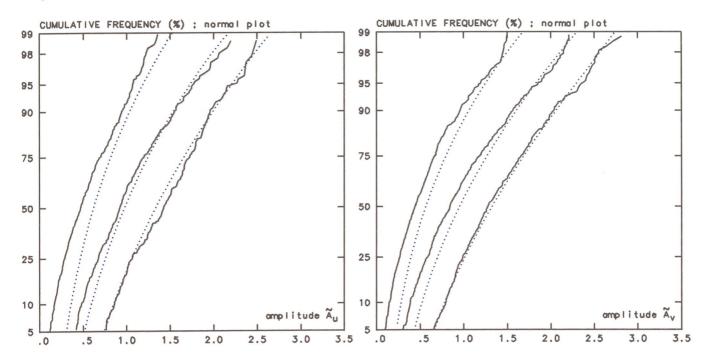
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 08



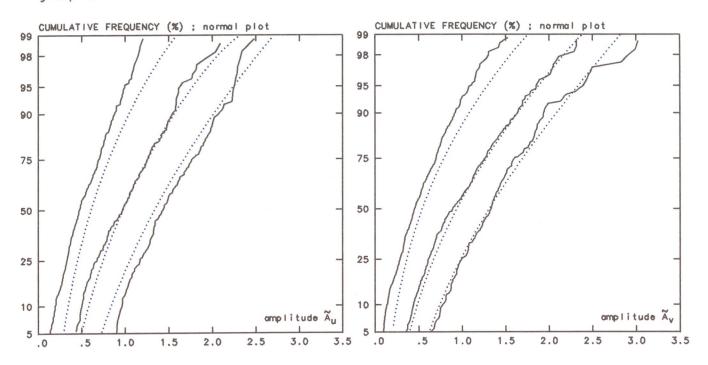
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 09



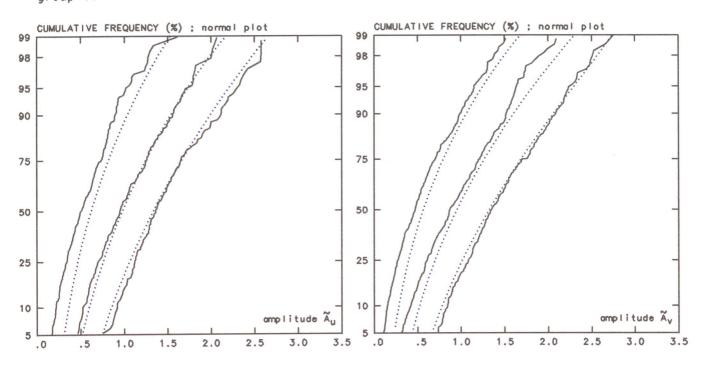
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 10



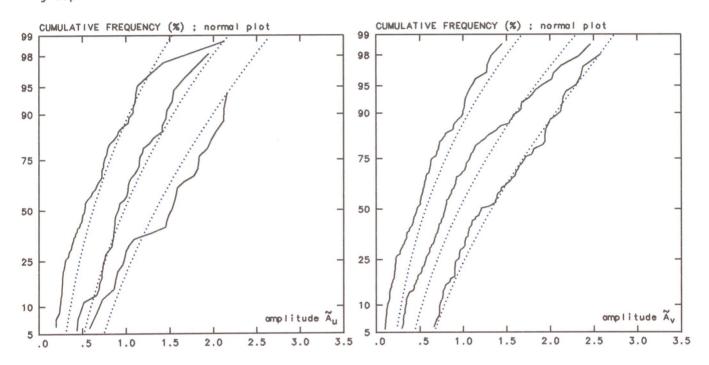
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 11



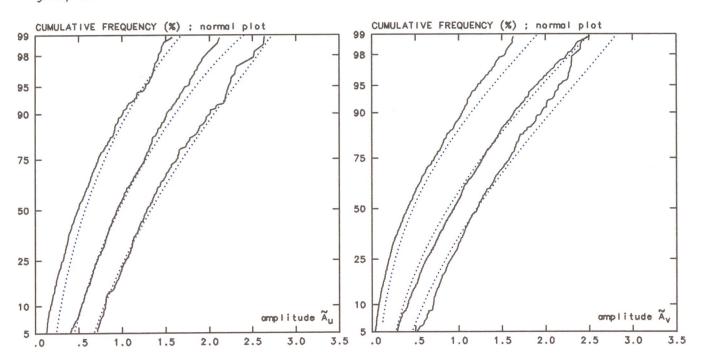
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 12



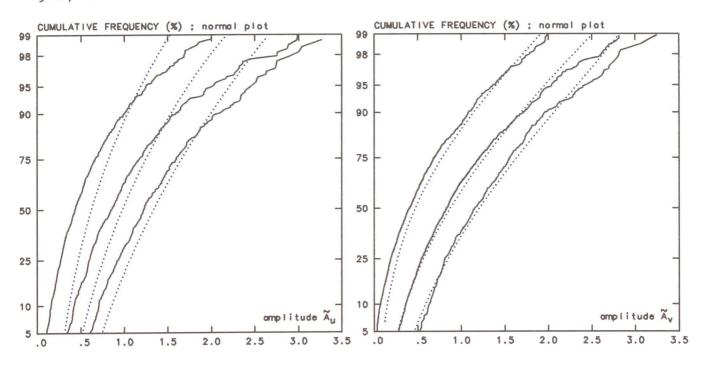
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 21

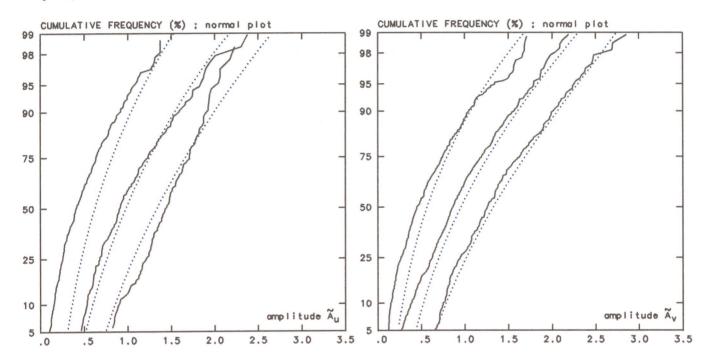


CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

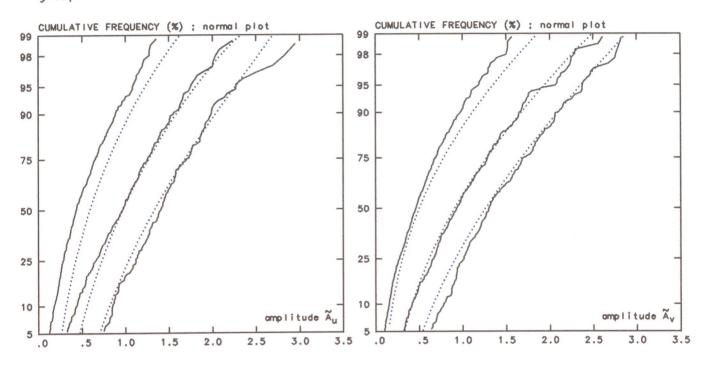
group 22



group 23

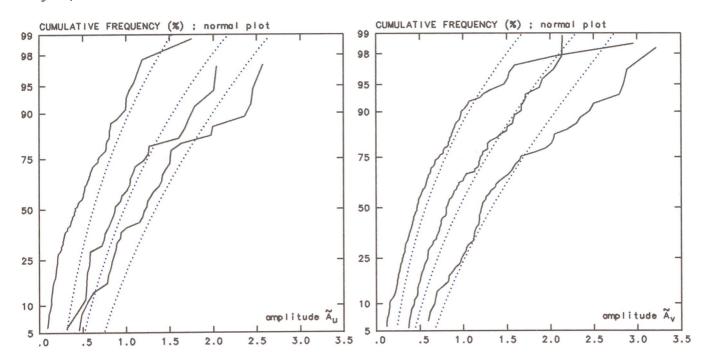


group 24

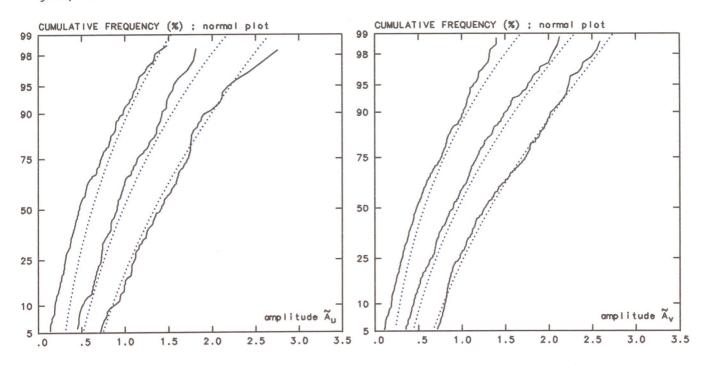


CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 25

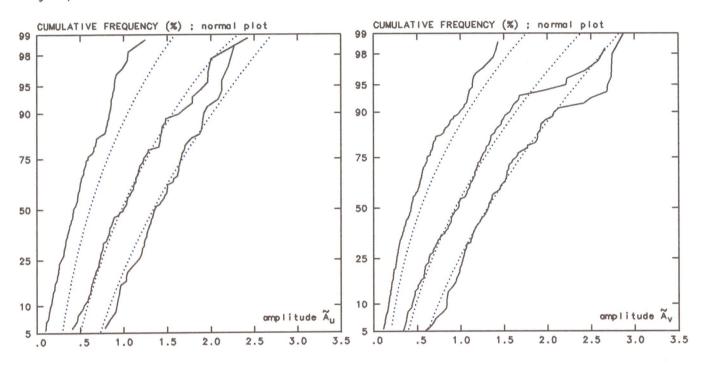


group 26



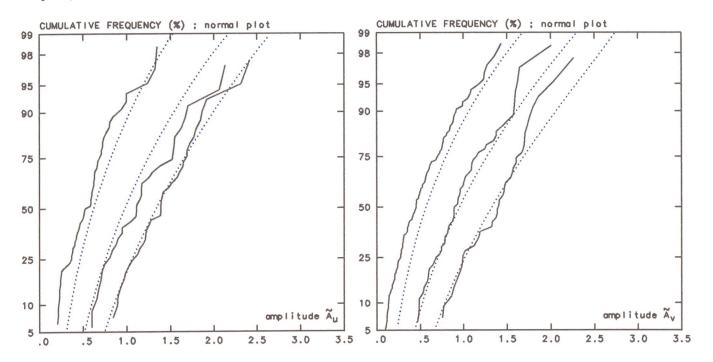
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 27



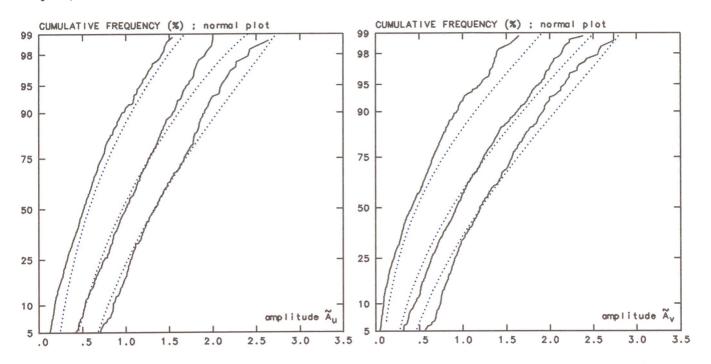
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 28



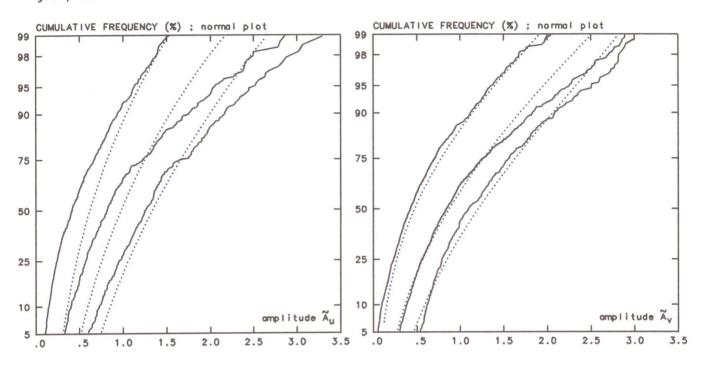
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 31



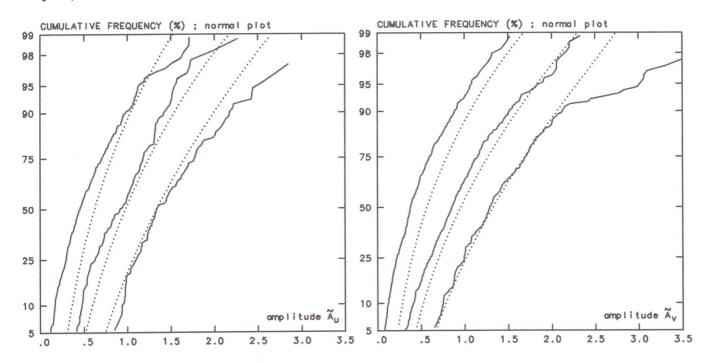
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 32

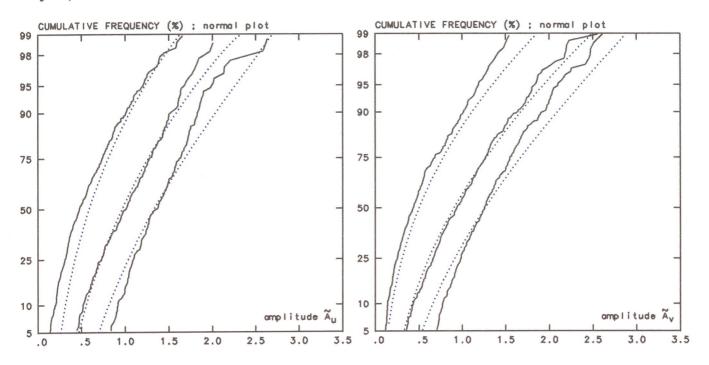


CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 33

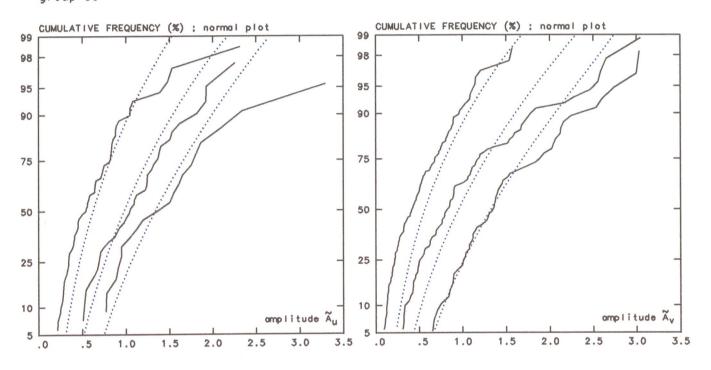


group 34



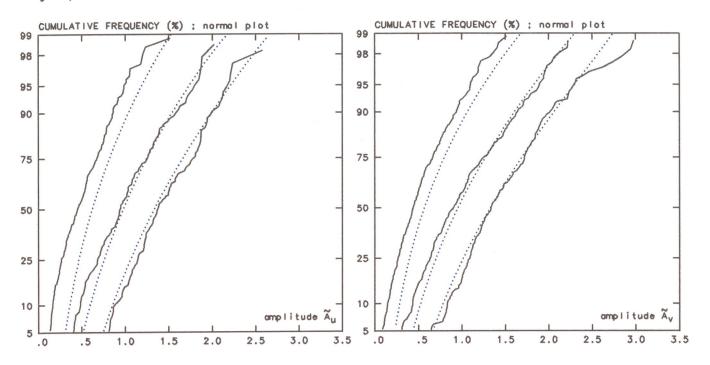
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 35



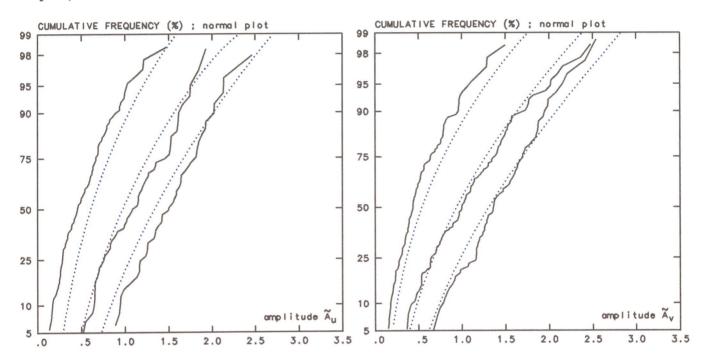
CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 36



CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 37



CONDITIONAL DISTRIBUTIONS OF NORMALIZED GUST AMPLITUDES FOR GIVEN NORMALIZED DURATION

group 38

