

division of technology for society

netherlands organization for
applied scientific research

WINDTURBINE WAKE EFFECTS

by

P.J.H. Builtjes

organization for
industrial research



p.o. box 342
7300 AH apeldoorn

address
laan van westenenk 501

telex 49095 tnoap
phone 055 - 77 33 44

Ref. nr. : 79-08375

File : 8710-8119

Date : August 1979

Archief TNO Apeldoorn

- 5 OKT. 1979

Ref.:

Bijl.

Dossier:

8710-8119

Beantw.:

Archief: - 8 OKT. 1979

„Client is referred to the General
Conditions for the Acceptance of
Assignments of TNO with regard
to his rights and duties concerning
the contents of this report”.

No part of this report may be re-
produced in any form, by print,
photoprint, microfilm or any other
means without prior written per-
mission from TNO.

TNO accepts no liability whatso-
ever with regard to the contents
and/or the form of this report.



CONTENTS

page

SUMMARY

3

NOTATIONS

4

1. INTRODUCTION

5

2. CONCLUSIONS

19

3. LITERATURE

20

TABLES: 1-2



SUMMARY

The aim of this investigation is to evaluate turbine wake effects on the power output of wind-turbine arrays in order to be able to determine the required array pattern and minimum distances between the turbines for the most efficient wind-energy powerstation on a given ground area. For this purpose firstly the wake of a turning model of a Darrieus rotor has been investigated in a windtunnel. Thereupon a stationary object has been developed which simulates the found wake behaviour. Finally with a number of these stationary objects wake-interaction effects in arrays with different patterns and mutual distances have been investigated. The last experiments have been carried out in a windtunnel with a simulated atmospheric turbulent boundary layer. The power output of the stationary objects has been determined by measuring the wind force on them.

It is shown that for a limited array an economically acceptable power output per unit ground area of about 9 W/m^2 could be reached at a mutual distance of about 6 wind turbine diameters.

NOTATIONS

A	windturbine disc area [m^2]
b	width of the wake between the points at which the velocity difference is half the maximum velocity difference [m]
B	distance between turbines [m]
C_p	power coefficient
C_T	axial drag coefficient
d	diameter of the wind turbine [m]
F	force on the wind turbine [N]
K	dimensionless distance between turbines, B/d
N	number of turbines
P	power output of a turbine [W]
P_o	ditto, in an undisturbed velocity profile [W]
P_t	averaged power output of a turbine in an array [W]
$\bar{P}_t$	ditto, averaged over all wind directions [W]
P_∞	ditto, for an infinite array [W]
$P_{surf.}$	total power output of an array divided by the ground area [W/m^2]
R	radius of the wind turbine [m]
r	radial coordinate
U	mean velocity [m/s]
U_o	mean undisturbed velocity [m/s]
U_c	mean velocity at the centre of the wake [m/s]
U_{max}	maximum mean velocity [m/s]
w	ratio between disc area and needed ground area
x,y,z	coordinate in the axial, transverse and vertical direction
λ	tipspeed ratio
ρ	density of air [kg/m^3]
σ_{uo}	standard deviation of longitudinal velocity fluctuations in the undisturbed velocity field relative to U_o
σ_u	ditto in the centre of the wake relative to U_c
$\Delta\sigma_u$	$\sqrt{\sigma_u^2 - \sigma_{uo}^2}$



1. INTRODUCTION

This report gives a survey of the research on windturbine wakes performed by TNO, Netherlands organization for applied scientific research, on behalf of Sub-Task A3 of IEA R & D WECS Annex I.

As this Sub-Task has been concluded on May 1, 1978 this report only deals with the work carried out at TNO up to that date.

One of the problems in applying windturbines for power generation is the ground area needed for say a 1000 MW power station. This is especially important in densely populated countries, where only very little area is available for placing large windturbine arrays. But it is also of importance for offshore erection of windturbines, because smaller mutual distances between turbines will reduce the costs of connecting cables and construction and will diminish the part of the sea withdrawn from shipping. However, putting windturbines close together will cause a power output reduction resulting from a decreasing wind velocity in the array due to wake effects. Consequently the question arises: how close can windturbines be placed together with a still acceptable power reduction.

The first who paid attention to this problem was Templin (1). He tried to estimate the effect of the windturbine density w (the ratio of total windturbine disc area divided by the total ground area) on the power output of an unlimited array of windturbines, i.e. the power levels generated when equilibrium is reached between the energy extraction by the turbines and the re-entrainment of flow energy into the atmospheric boundary layer due to increased shear and turbulent mixing. The basis of his calculations is the idea that individual wakes will smooth out rapidly. Then the erected windturbines can be regarded as an extra roughness added to the existing ground surface roughness, creating a new logarithmic velocity profile different from the profile above the original surface. For that new velocity profile the power output per windturbine can be calculated by cubing the velocity at hubheight.

In Fig. 1 the results of Templin's calculations are given for a flat-open country. The average power output P_{∞} of a turbine in an unlimited array



divided by the power output P_0 of a free-standing turbine is given as a function of the windturbine density w (the spacing K between the windturbines given by $K = B/d$ with B the distance between turbines with a diameter d is related to w by the expression $w = \pi/4K^2$). It is of importance to notice that as a result of the assumption of a rapid smoothing out of individual wakes, these results of Templin are independent of the wind direction, this theory is essentially one-dimensional. Newman (2) follows the same reasoning as Templin, but with slightly different assumptions. His results are also given in Fig. 1. Based on the same reasoning Crafoord (3) and Schmidt (4) also carry out such calculations. But additionally they incorporate some calculations for a limited array of turbines using an energy balance. Their main conclusion is that the power output of a turbine in a very large array reaches an equilibrium value after about the 40th row. Ljungström (5) also uses the energy-balance method of Crafoord. He suggests that an average power output per turbine of $0.95 P_0$ is possible in an array with a spacing of $10d$. Starting from the fact that in coastal areas a windturbine of 50 m diameter will give a power output of 1 MW (mean velocity at hubheight 11 m/s), Ljungström estimates that the maximum energy output per unit ground area will be about 6 W/m^2 .

For the unlimited array of Templin this value is only about 0.25 W/m^2 ; Raily (6) even calculates a value of 0.1 W/m^2 .

This large difference between the results of Ljungström and Templin is caused by the fact that Templin considers an unlimited, wide-spread array with a minimum of power reduction, whereas Ljungström calculates the maximum of power production. Also whether or not taking into account winddirectional effects influence the outcomes.

As a compliment to these theoretical calculations two experiments have been carried out to investigate the power reduction caused by wake effects using small turning models in a windtunnel. Ljungström (7) as well as Phillips and Robertson (8) have determined the distance between turbines at which no wake effect is present anymore. Both experiments confirm that if a turbine is to be located directly downstream from another, a spacing of 40 diameters is required if the downstream turbine is to deliver full power.



The main imperfection of the above-mentioned theoretical investigations is the assumption that the wakes of turbines smooth out rapidly and that consequently there is no direct wake effect between turbines. Lissaman (9) has tried to overcome this shortcoming. He assumes that the wake of a turbine has a jet-like character and will be influenced by the performance of the turbine, the hubheight above the ground, the mechanically generated turbulence of the turbine, and, especially further downstream, by the ambient turbulence. With a computer program Lissaman calculates the wakes of all turbines in an array. Assuming that the windvelocity profile in front of a turbine is the result of the velocity deficits in the different wakes reaching the turbine, its power output is found by integrating the cube of this velocity. However, although the computer program takes the mentioned factors of influence into account, there is no complete experimental indication of the magnitude of their influence.

In view of the above-mentioned contradictions in the results of the theoretical investigations, and especially of the lack of experimental results concerning interacting windturbine wakes, it has been decided upon to carry out an experimental investigation, consisting of the following three parts, in the windtunnels of TNO in Apeldoorn. Firstly the wake of a turning model of a Darrieus-rotor has been measured. Next a stationary object, which simulates the wake of the turning model, has been developed. Finally with a number of these objects the energy output of different arrays of interacting turbines has been investigated.

In the following these experiments are discussed in detail.

The wake of a turning model of a Darrieus rotor

At the start of the Dutch program on wind energy, in 1976, it was decided upon to focus primarily the attention to vertical-axis turbines, because about these less was known than about horizontal-axis turbines. So a Darrieus rotor was studied first. As at that moment no full scale Darrieus rotor was available in the Netherlands, a small scale turning model has been constructed for wake measurements in a windtunnel. One of the problems of such a small scale model is the fact that with the tests in the windtunnel the Re-number for the blade profile is very low, in each case much lower than for a full



scale rotor in the real atmospheric boundary layer. Consequently scale effects could be expected. To overcome this difficulty for the windtunnel model a blade profile has been used, that at these low Re-number behaves the same as the blade profile of the real turbine at much higher Re-numbers in the natural wind. It is known that at a full-scale Darrieus rotor the boundary layer along the blades with profile NACA-0012 remains attached during the whole revolution. But this type of profile, however, will at the low Re-number of the small-scale model in the windtunnel stall at angles of attack of 5° (see (10), (11), (12)), while the maximum angle of attack at optimal working conditions is about twice as large. Consequently for the turning model the blade profile DVL 00009-1.150 has been chosen that probably would not stall at that maximum angle of attack and low Re-number. To be sure of the desired behaviour of this profile a two-dimensional model of it with a chord of 50 mm and a span of 500 mm has been investigated in a windtunnel with a measuring section of $2.65 \times 1.2 \text{ m}^2$ cross-sectional area and a length of 6 m. The blade has been investigated in a homogeneous flowfield at a mean velocity of 10 m/s and a turbulence intensity in the mean flow direction of 3.5% with an integral length scale of about 0.23 m. At a velocity of 10 m/s the Re-number based on the chordlength is about $3.1 \cdot 10^4$. The stalling of the blade has been investigated by measuring the wake close behind the blade at different angles of attack. A sudden increase of the wake width indicates that flow separation has occurred. The results have been checked by considering the behaviour of tufts on the blade surface. These tests showed that at this Re-number the boundary layer remains attached up to angles of attack of about 9° . It proved to be impossible to enlarge this angle by roughening the blade.

The turning model of the Darrieus rotor has two blades with profile DVL 0009 - 1.150 and a diameter of 0.2 m. The blade chord is 10 mm, and the vertical axis has a diameter of 10 mm. Nearly all the wake measurements have been performed in the above described homogeneous flowfield.

The model turns at a tipspeed ratio of $\lambda=5.5$, which should be close to the tipspeed ratio at the optimum power output C_p . This means that the tipspeed at the largest diameter is 55 m/s, i.e. the model turns at about 5200 cycles/minute. It should be noticed that no actual measurements of the $C_p - \lambda$ curve have been carried out, neither has the drag coefficient C_T been measured.



From these values of tip speed and undisturbed velocity it follows that the largest angle of attack of the blades is about 10.4° . Consequently it seems possible that for a small time-interval during each revolution the blades stall; but it is more likely that this is suppressed by hysteresis effects. Moreover it seems unlikely that small differences in blade behaviour of the model and the real turbine have a large effect on the wake structure. A check on these ideas has been carried out by changing the rotor speed by 10% at the same wind velocity. This showed to have no influence on the wake characteristics.

The turning model has been placed in the centre of the measuring section in the homogeneous flowfield (see Fig. 2). The measurements in the wake of the mean velocity and turbulence intensity have been carried out with a single hot wire anemometer, DISA type 55 D01, attached to a traversing system.

In Fig. 3 the velocity deficit decay at the centre of the wake is given as a function of the distance behind the rotor. U_o is the undisturbed velocity of 10 m/s, U_c is the velocity at the centre of the wake. The figure shows the velocity deficit in the homogeneous flowfield with a turbulence intensity of 3.5% (the values are given in table 1), while also some points are given for a flowfield with a turbulence intensity of 6.5% (and an integral scale of 0.20 m). It shows that at higher turbulence levels the velocity deficit decreases caused by the broadening of the wake.

The constant velocity deficit in the near wake, up to about $x/d = 3$, is determined by the performance of the turbine, and is therefore a function of the tip speed ratio λ . After the near wake region the deficit decays as:

$$\frac{U_o - U_c}{U_o} \propto (x/d)^{-1.25} \quad (1)$$

In table 2 also some results are given of the width of the wake b , defined as the width between the points where the velocity deficit is half the maximum velocity deficit.

For a first estimation of the wake interactions in an array it is of interest which amount of wind energy is available in the velocity profile behind a turbine.



For this purpose a power ratio of a second turbine placed behind the first one can be defined by:

$$P/P_0 = \frac{\int_0^R 2 \pi r \frac{1}{2} \rho U^3(r) dr}{\pi R^2 \frac{1}{2} \rho U_0^3} \quad (2)$$

with R the radius of the turbine and P_0 and P the wind energy in the undisturbed velocity profile and in the wake. The results are shown in Fig. 4. It should be noticed that this does not mean that a second turbine at 10 diameters should give a power output of $0.55 P_0$. This depends on the performance of the second turbine. In case the outer part of the turbine is more effective than the inner part, this value could be higher. But as the velocity profile a second turbine placed at a distance of 10 diameters "feels" will be about the velocity profile at 9 diameters, the power output could also be smaller. The ratio P/P_0 gives only the energy available in the wind velocity profile for the disc area.

Finally Fig. 5 gives the turbulence intensity at the centre of the wake defined as:

$$\Delta\sigma_u = \sqrt{\sigma_u^2 - \sigma_{u0}^2} \quad (3)$$

where σ_u is the turbulence intensity relative to the centerline velocity U_c and σ_{u0} the undisturbed turbulence intensity relative to U_0 of 3.5%. From this figure it is clear that the additional turbulence intensity behind the Darrieus rotor can be quite large, from 30% at 3 diameters, up to still 10% at 10 diameters. This additional turbulence intensity will have an influence on the performance of a second turbine, and on the needed strength of the blades.

The development of a stationary wake-simulating object

The best way to investigate interactions between turbines in an array would be to place a number of turning models in the windtunnel. In that case, however, due to the limited dimensions of the testsection models with a diameter of only 50-100 mm could be used. But the construction of turning models



of that size which give a correct simulation of a full scale Darrieus rotor would be very difficult, if not impossible; not to mention the equipment problems for the turning at very high speeds of a lot of such models. In view of these facts it was decided upon as to try to develop a stationary object which gives the same wake as found behind the turning model.

In case of an optimal performance of a windturbine, at which $C_p = 0.59$ (Betz-coefficient), the total force F of the wind acting on the turbine with disc area A is given by:

$$F = 8/9 \cdot \frac{1}{2} \rho U_0^2 A \quad (4)$$

The factor $8/9 \approx 0.9$ is called the dragcoefficient of the turbine. This dragcoefficient determines the momentum deficit in the wake, and is consequently of major importance for the total wake behaviour. So, a Darrieus rotor simulating stationary object should have a dragcoefficient of the order of 0.9. It should, of course, be noticed that a stationary object can never simulate the instationary wake close behind a turning model, but the wake further downstream will be determined by the drag, and not by this instationary behaviour.

From the literature (for example (14) and (15)), it is known that a gauze with a porosity of 45-50% has a dragcoefficient of the order of 0.9. So at first several circular gauzes with a diameter of 50 mm and a porosity ranging from 43-52% have been tested. It showed, however, that, although the dragcoefficient for the gauzes was around 0.9, the centerline velocity deficit $(U_0 - U_c)/U_0$, measured in the above described homogeneous flowfield was much lower than behind the Darrieus rotor, ranging from 0.25 at $x/d = 5$ to 0.1 at $x/d = 10$. It turned out that the width of the wake behind the gauzes was larger than behind the Darrieus rotor, and consequently the centerline velocity was higher. The broad wake behind the gauze could be caused by the large vortices coming from the edges of the gauze. So it seemed necessary to restrict the width of the wake behind the gauzes. After some trial and error it was found that a gauze with a porosity of 47%, attached to a diffuser with a half angle of 25° and a length of 24 mm produces a wake



which is quite similar to the wake behind the Darrieus rotor. Fig. 6 gives a sketch of this tea-strainer like object. However, the attached diffuser creates a problem in defining the diameter of the object. As shown in Fig. 6 the object has now two diameters of 50 mm and 70 mm and the question rises which diameter is the wake-determining diameter. In Fig. 3 and Fig. 4 the results behind the gauze with diffuser are shown for diameters of 50 mm and 70 mm.

For the centerline velocity the values for a diameter of 70 mm are somewhat closer to the Darrieus rotor results than for the 50 mm. However, as shown in Fig. 4, for P/P_0 the results for the 50 mm diameter are clearly better. In this case the quite large difference between the results for the 50 mm and 70 mm diameters are caused by the fact that the integration according to formula (2) for the 70 mm also takes place over parts of the velocity profiles with higher velocities. Also in Fig. 4 some results are shown for P/P_0 of a turning model of a horizontal-axis turbine, which have been measured after May 1, 1978. There is a good agreement between these results and the values for the 70 mm diameter.

Finally, according to this problem a remark has to be made concerning the drag. The single gauze has a dragcoefficient of about 0.9. The gauze with diffuser has also a dragcoefficient of 0.9, based on a diameter of 70 mm. This means that the gauze with diffuser has a dragcoefficient of about 1.75 based on a diameter of 50 mm.

As has been mentioned earlier, neither the C_p - λ curve nor the C_T - λ curve of the Darrieus rotor has been measured. The fact that the wake of the Darrieus rotor is well simulated with an object with a high dragcoefficient means that probably the dragcoefficient of the Darrieus rotor C_T is high. This means that at the tipspeed ratio of $\lambda=5.5$ the value of C_p is less than the optimum value, the maximum value of C_p is reached at a lower value of λ . It should, however, be reminded that P/P_0 gives only information of a part of the velocity profile. This question needs further investigation and additional measurements behind the Darrieus rotor are needed, especially closer to the rotor, to confirm the above discussed results. It seems also premature to claim at this moment that there is a difference in wake behaviour between a Darrieus rotor and a horizontal-axis turbine.



In the following the discussions are restricted to the results for 50 mm diameter of the gauze models, unless otherwise stated.

As mentioned earlier, all the wake measurements up to this moment have been carried out in a homogeneous flowfield with a turbulence intensity of 3.5%. To determine the interaction between turbines in the real atmosphere, it is necessary to measure the interaction between the models in a simulated atmospheric boundary layer. In the same windtunnel as described above a simulated atmospheric boundary layer belonging to a neutral stable atmosphere above grassland has been created on a scale of 1:500 (for details see (16)). The wake of the gauze with diffusor has been measured in this boundary layer, with the centre of the gauze 75 mm (1.5d) above the floor. The found wake behaviour is only equivalent to the wake of a full scale Darrieus rotor in the atmosphere if the following assumptions are right: 1) the turning model represents a full scale machine; and 2) the gauze model simulating the wake of the Darrieus rotor model in a homogeneous flowfield, will also simulate the wake of a Darrieus rotor in a boundary layer. The simulated atmospheric boundary layer gives at the centre of the gauze a mean velocity of 6.2 m/s and a turbulence intensity of 12%.

In Fig. 7 horizontal wakes behind the gauze model are given relative to the undisturbed velocity of 6.2 m/s and in Fig. 8 the vertical profiles are given. Finally, the wind energy in the velocity profile across a second turbine at different distances behind the first turbine is given in Fig. 9. It turns out that these results are higher than in the case of a homogeneous flowfield. The higher turbulence intensity in the atmospheric boundary layer broadens the wake, which leads to higher values of P/P_0 .

With a number of the above described gauze models interaction measurements in different arrays have been carried out.

The interactions between turbines in an array

Three different arrays have been investigated arranged on a turntable of 2.3 m diameter in the floor of the windtunnel.

All gauze models have been placed with their centerlines 75 mm above the floor in the simulated atmospheric boundary layer.

The arrays of circular shape consisted of respectively 37, 49 and 97 gauze models with decreasing mutual distance (see table 2). The turntable enables



measurements under different wind directions. They have been carried out at direction angles between 0° and 45° with steps of 5° . As the arrays are symmetrical, from these measurements the energy output for every wind direction could be calculated. The energy output has been deduced from measurements of the forces on a certain number of gauze models by means of strain-gauge balances. For that purpose the gauze models were mounted on poles, attached to the drag balances mounted below the turntable surface.

After calibration of the strain-gauge balances the force measurements have been performed using an on-line computer. Now the drag force of a gauze model is proportional to the square of the wind velocity while the power is proportional to the cube of that velocity. So by measuring the drag, due to the velocity profile approaching the gauze model, the power which a wind turbine can deliver in that flowfield can be determined. Only the assumption must then be made that a wind turbine reacts to the velocity profile in the same way as the gauze model.

In every array 20 gauze models were instrumented with strain gauges. These models were chosen in such a way that always several of them were facing the undisturbed velocity, and so measuring P_0 . The other instrumented models were representative for a part of the array. Each part of the array with about the same power output has been covered with at least two instrumented gauze models. The representativeness of the models has been determined by using the computer program of Lissaman, described earlier. For this calculation the computer model is considered somehow as a black box. The input parameters were chosen in such a way that the calculated power output behind a turbine was equal to the measured values as shown in Fig. 9. With this adapted program calculations for the different parts of the arrays have been carried out to determine the 20 representative gauze models.

The averaged standard deviation in measured power between the different models which, according to their situation, should give the same results, was about 8%.



Most of the measurements have been carried out at a free-stream velocity $U_{\max}$ of 15 m/s. The power results at a free-stream velocity of 25 m/s showed to be about 5-10% higher.

For the calculation of the power according to the measured force there is the following additional difficulty. As a representative diameter 50 mm has been chosen. However, the velocity profile causes a force on the combination of gauze and diffusor with its maximum diameter of 70 mm. Therefore a correction has to be made for the additional force acting on the diffusor. This correction can only be made in a direct way in case two gauze models are placed on a straight line. For that situation the output of the second gauze model can be corrected because the approaching velocity profiles are known, namely the energy in the velocity profile can be calculated over an area with a diameter of 50 mm and of 70 mm. In that way the correction is a function of the power output; the correction is large at lower power output and smaller for larger power output. To apply this correction for every situation in the array it is assumed that the above described correction can be used for every gauze model as a function of the power output. It should be noticed that such a, rather arbitrary, correction is not necessary in case a 70 mm diameter is used.

For the three arrays the force measurements have been carried out at the afore-mentioned wind directions. One way of presenting the results is to determine for every wind direction the array efficiency P_t/P_o , defined as the average power output of a turbine in that array at that angle, divided by P_o . The total output, at a certain angle, of the array of N turbine, is then equal to NP_t .

Fig. 10 shows the array of 97 gauze models placed on the turntable in the windtunnel and Fig. 11 gives the results for P_t/P_o as a function of the wind direction for the three different arrays.

As can be seen, of course the array with the largest mutual distance gives the highest power output per turbine. It is also clear that in the case that the gauze models are placed on straight lines, the power output shows a minimum for the wind directions of 0° and 45° . The averaged power output of an array can be found by averaging P_t/P_o over the various wind directions. By doing so it is found that $\bar{P}_t/P_o = 0.80$ for 37, 0.70 for 49 and 0.48 for 97 gauze models. So the last array shows a power output decrease caused by



wake interference of about 50%! However, the total output of these arrays $\bar{NP}_t$, all placed on the same amount of ground area, increases from $30 P_o$ for 37 models to $35 P_o$ for 49 and $50 P_o$ for 97 models. Consequently, the increase of the number of gauze models in different arrays is larger than the decrease of the averaged power output, and so no maximum power output has been found. On the other hand, an output of $\bar{P}_t/P_o = 0.48$ does not seem economically sound. Or, said in a different way, the additional 48 gauze models of the array with 97 models in comparison to the array with 49 models give an additional power output of $15 P_o$, which means only $0.3 P_o$ per model. This means that for the determination of an optimal distance between turbines, also economical considerations have to be taken into account.

All these results are concerned with finite and even small arrays. The question arises what the power output would be of very large or even infinite, arrays, with the same mutual distances. About this the following can be said:

At wind directions of 0° and 45° the gauze models in the investigated arrays are placed on straight lines. In Fig. 12 measured results are shown for gauze models placed on these lines. The results indicate that an equilibrium situation is reached very quickly in such a line pattern. This is in contradiction to the theoretical results mentioned earlier. The results shown in Fig. 12, however, make a comparison between the theoretical results for infinite arrays possible. The only problem is that the wind direction has no influence on the theoretical results. The theoretical results are essentially one dimensional; the distance between turbines does not change with the wind direction, as is the case for the measurements. This raises the question whether a comparison between theoretical and measured equilibrium results should be made with results as given in Fig. 12, or from the results of measurements with large arrays averaged over all wind directions. At this moment it is thought that the best comparison can be made by means of line measurements as shown in Fig. 12; the comparison is shown in Fig. 1. As can be seen, the measured results compare very well with the theoretical results, although as indicated above, there are remaining questions.

The fact that equilibrium of gauze models on a line is reached very quickly leads to the idea that the central gauze models in the arrays may have a power output close to the power output for an infinite array. In this connec-



tion in Fig. 13, the power output of sorted gauze models averaged over all wind directions is given. It is thought that the figure for the central models is close to the results an infinite array would give. It should be noticed that these infinite results are much higher than the results in Fig. 1. This is caused by the fact that Fig. 13 shows the results for all wind directions. For an infinite array, all the three lines shown in Fig. 13 should be the same. However, at the largest array of 97 models the differences between these lines are larger than at the smaller arrays. This is an indication that the way in which an equilibrium situation is reached is a function of w (the results shown in Fig. 13 are the same results shown in (17), where they are given in a different way).

As has already been mentioned, the array of 97 gauze models is economically not acceptable. A yet economically acceptable, and from an aerodynamic point of view favourable array seems to lie in the range between the arrays of 37 and 49 models. This standpoint can be made more qualitative with the following reasoning. Starting from the assumption that a turbine with a diameter of 50 m gives an output of 1 MW (mean velocity of about 11 m/s), a power output per unit ground surface area can be found by calculating:

$$P_{\text{surf.}} = \frac{1\text{MW}}{\pi/4 \cdot d^2/w} \cdot \bar{P}_t = \frac{1\text{MW} \cdot \bar{P}_t}{B^2} \quad (5)$$

It is clear that this value at the same w is independent of the size of the rotor, because the poweroutput and B^2 are proportional to d^2 . The results of this calculation are shown in Fig. 14. The graph for the finite arrays is found by using the average result for the measured arrays; the infinite results are found by using the central gauze models only.

From Fig. 14 it is clear that for large arrays a maximum value of about 8 W/m^2 can be found at $w \approx 0.03$, which means an optimal mutual distance of about 5 diameters. It should be noticed that the average output of an individual turbine in that situation is only 50% of the output of a free-standing turbine.

The output of an array with an economical requirement that the turbines should deliver an average output of at least 70% of the power of a free-standing



windturbine can be calculated in the following way. From Fig. 13 it can be found that the 70% level is reached for a finite array at $w \approx 25 \cdot 10^{-3}$ (distance of 5.6 diameters), for an infinite array at $w \approx 15 \cdot 10^{-3}$ (distance of 7.2 diameters). With these values of w Fig. 14 gives as the output P_{surf} of a finite array about 9 W/m^2 , for an infinite array about 6 W/m^2 .

Now a remark has to be made concerning the turbulence levels in the arrays. Although the gauze-models are only stationary objects, the turbulence level in the centre of the array of 37 models is about 22%, increases to about 30% at the array of 49 and to approximately 50% at the array of 96 models. As has been said earlier, this is an indication that turbulence can be a severe problem when windturbines are placed close together.

Finally, a comparison has been made between the measurements of an array, and the results calculated with the model of Lissaman. As has been mentioned the parameters in Lissaman's model have been chosen in such a way that the calculated results behind a turbine are close to the measured results shown in Fig. 9 for the boundary layer. The results are shown in Fig. 15 for the array of 49 models. There is a rather large difference between the calculated and the measured results, although the behaviour of the graphs is the same. The average calculated value of $\bar{P}_t/P_o$ is 0.79, the measured value is 0.70. In Fig. 15 also the results are shown when 70 mm is taken as the representative diameter of the model (see also Fig. 4) and the value of $\bar{P}_t/P_o$ becomes about 0.75. This result is about 6% higher than for the 50 mm diameter. The mutual distance, however, would decrease by a factor 0.71 (50/70) for 70 mm diameter and w would increase by a factor of 2 $((70/50)^2)$. It is obvious that this point needs further investigation.



2. CONCLUSIONS

From this investigation the following conclusions can be drawn:

- the centerline velocity deficit behind a Darrieus rotor decays with the distance, after an initial stage, according to a -1.25 powerlaw.
- The wake of a Darrieus rotor can be simulated by a stationary object consisting of a gauze with a diffusor.
- The available wind energy in an atmospheric boundary layer above grassland in a straight line at distances behind a turbine of 10 and 20 diameters is about 60%, respectively 85%, of the full power.
- The poweroutput of different turbines placed on a straight line reaches an equilibrium situation already at about the 4th turbine.
- An, also economically acceptable, optimum power output of a limited array could be reached at about 6 diameters mutual distance and could give about 9 W/m^2 poweroutput per ground area.
- The maximum poweroutput per ground area for an infinite array is about 8 W/m^2 at a mutual distance of 5 diameters.

Acknowledgement

The author is indebted to mr. D.J. Milborrow (CEGB-Leatherhead, England) and mr. P.E.J. Vermeulen for their valuable comments.



3. LITERATURE

- (1) Templin, R.J.
Wind Energy Workshop, Stockholm
August 29-30, 1974
- (2) Newman, B.G.
Energy Conversion 16, 169, 1977
- (3) Crafoord, C.
An estimate of the interaction of a limited array of windmills
Rep. DM-16, Univ. of Stockholm, November 1975
- (4) Schmidt, W.L.
Wind power studies, Thesis Univ. of Waterloo
September 1977
- (5) Ljungström, O.
Wind Engineering 1, 1, 36, 1977
- (6) Raily, J.W.
Wind Engineering 1, 1, 23, 1977
- (7) Ljungström, O.
Windkraft, Swedish State Power Board Report, Stockholm, 1974
- (8) Phillips, P. and G. Robertson
An experimental analysis to determine the interaction of
wind turbines in widespread arrays,
University of Western Ontario, March 1976
- (9) Lissaman, P.
Energy effectiveness of arrays of wind energy conversion systems
AV FR 7058, AeroVironment Inc., Pasadena, U.S.A., May 1977



- (10) Riegels, W.
"Aerofoil Sections", Butterworths, London 1961

- (11) Kraemer, K.
"Flügelprofile im Kritischen Reynolds Zahl-Bereich"
For. a.d. Gebiet des Ingw. Band 27 No. P. 1961

- (12) Schmitz, F.W.
"Zur Aerodynamik der kleinen Re-Zahlen"
Jahrbuch Wiss. Gesells. f. Luftfahrt, Göttingen 1953

- (13) Boschloo, G.
Wake structure of a Darrieus rotor
TNO-report 77-07244, in Dutch, 1977

- (14) Graham, J.M.R.
J. Fluid Mech., 73, 3, 565, 1976

- (15) Castro, I.P.
J. Fluid Mech., 46, 3, 1971

- (16) Builtjes, P.J.H.
TNO-report 78-09681, 1978

- (17) Builtjes, P.J.H.
The interaction of windmill wakes,
Second Int. Symposium on Wind Energy Systems, October 1978
paper B5



TABLE 1

WAKE RESULTS OF THE DARRIEUS ROTOR

x/d	$(U_o - U_c)/U_o$	b/R	P/P_o
1	0.813		
2	0.843		
3	0.760		
4	0.567		
5	0.423		
6.25	0.322		
7.5	0.258		
8.75	0.196		
10	0.178	1.5	.55
12.5	0.145		
15	0.124	1.7	.70
20	0.088		
25	0.076		.82
29.5	0.056		

TABLE 2

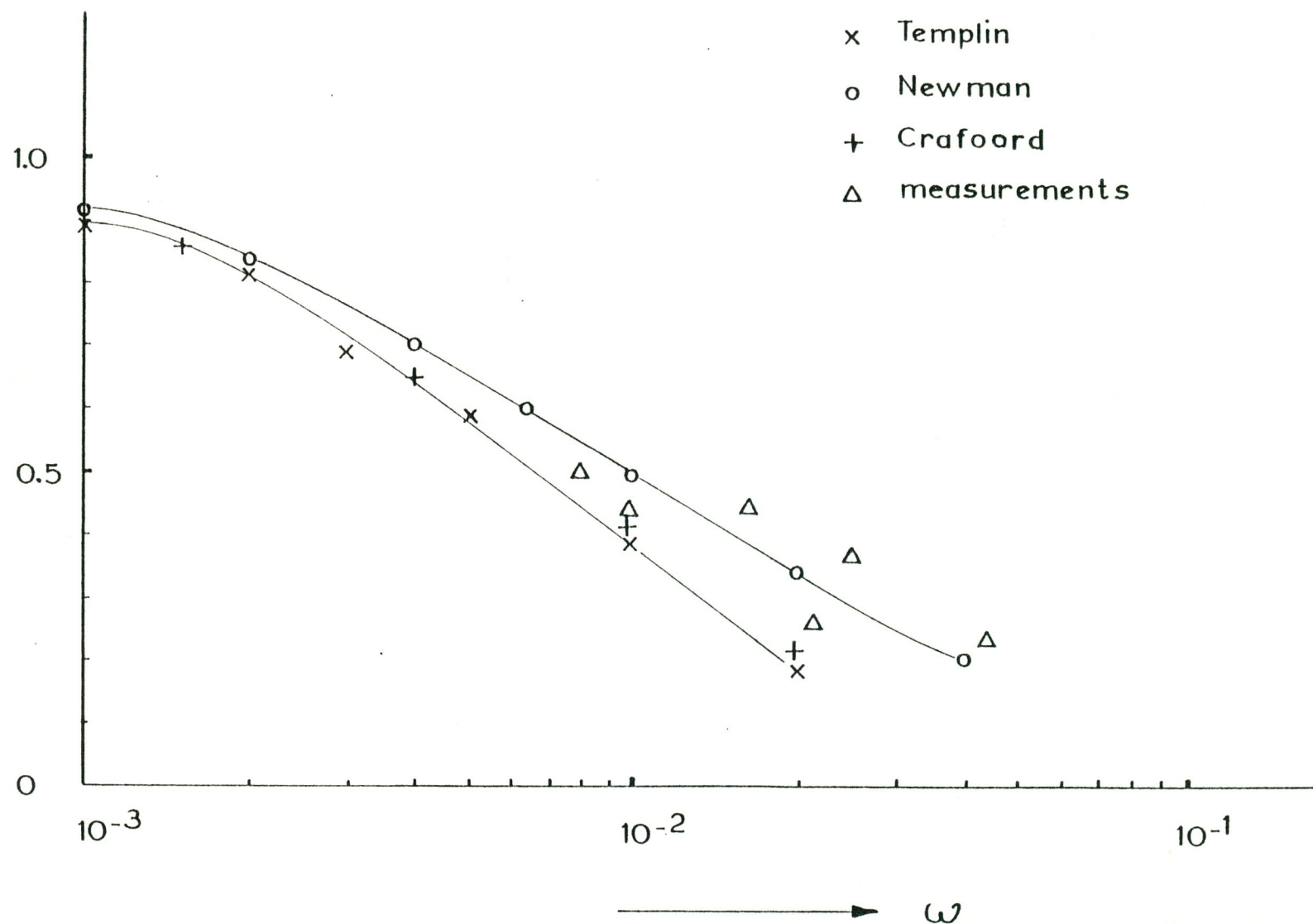
WINDTURBINE ARRAYS

number of gauzes	distance 0°	under 45°	windturbine density w
37	10d	7.1 d	.016
49	8d	5.7 d	.025
96	6d	4.25d	.044



Power output in an unlimited array.

$$\frac{P_s}{P_o}$$

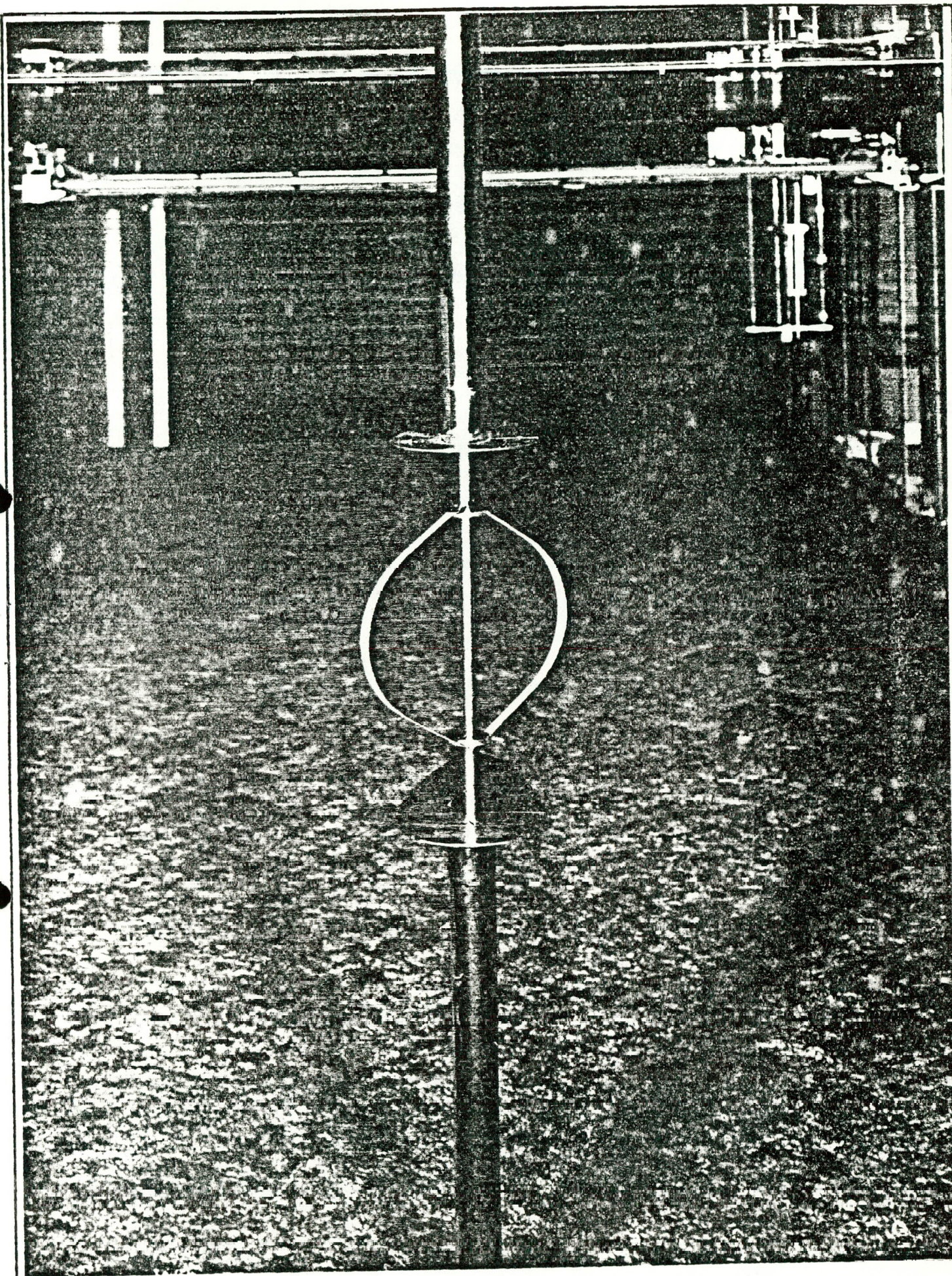


MT-TNO

8119

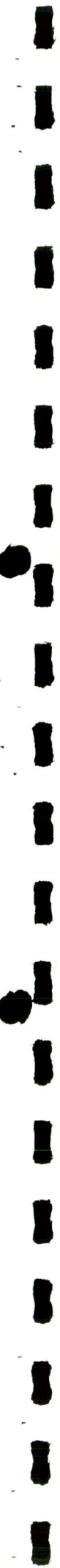
Fig. 1

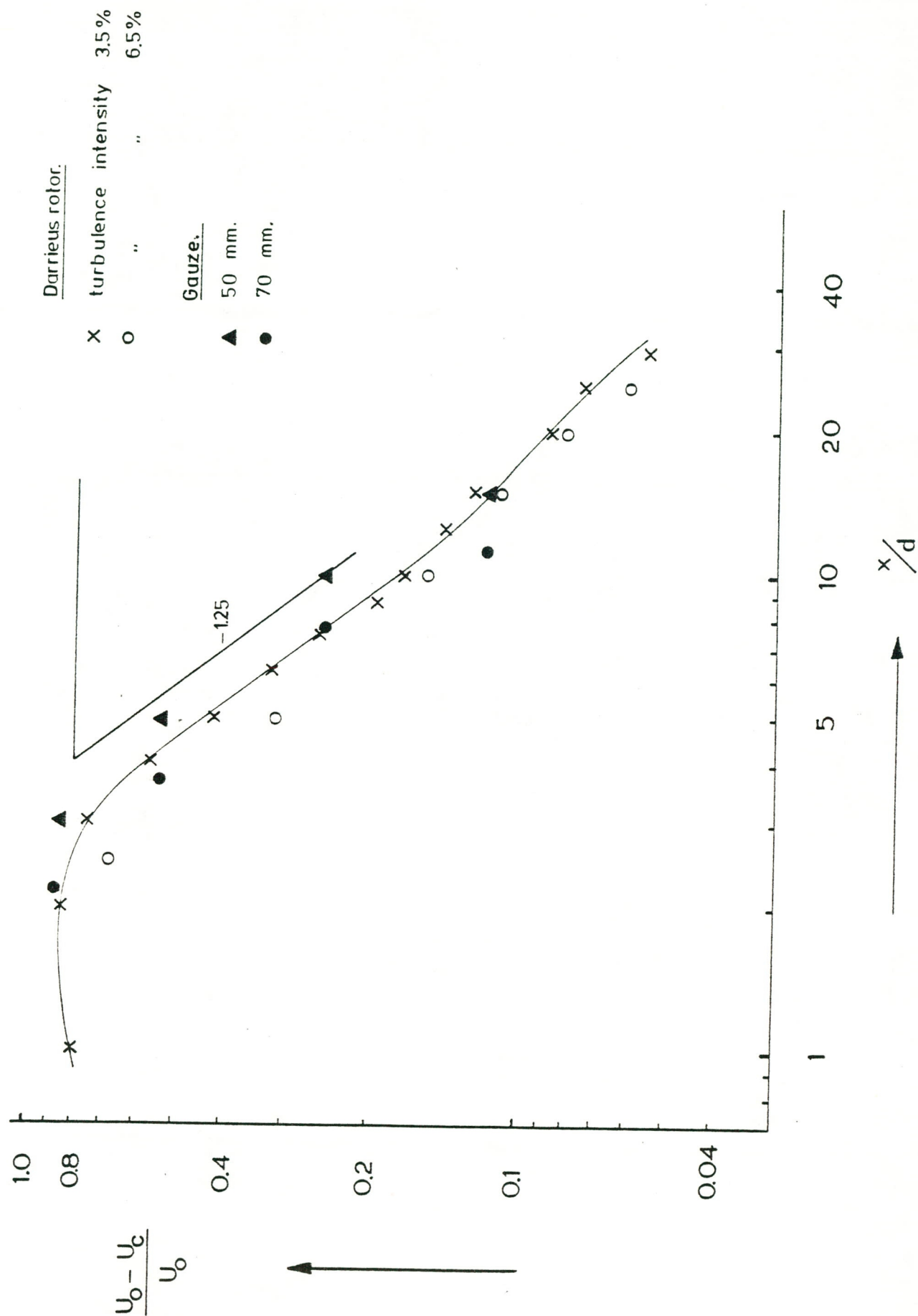




The model of the Darrieus rotor in the windtunnel.

MT-TNO
8119
Fig. 2

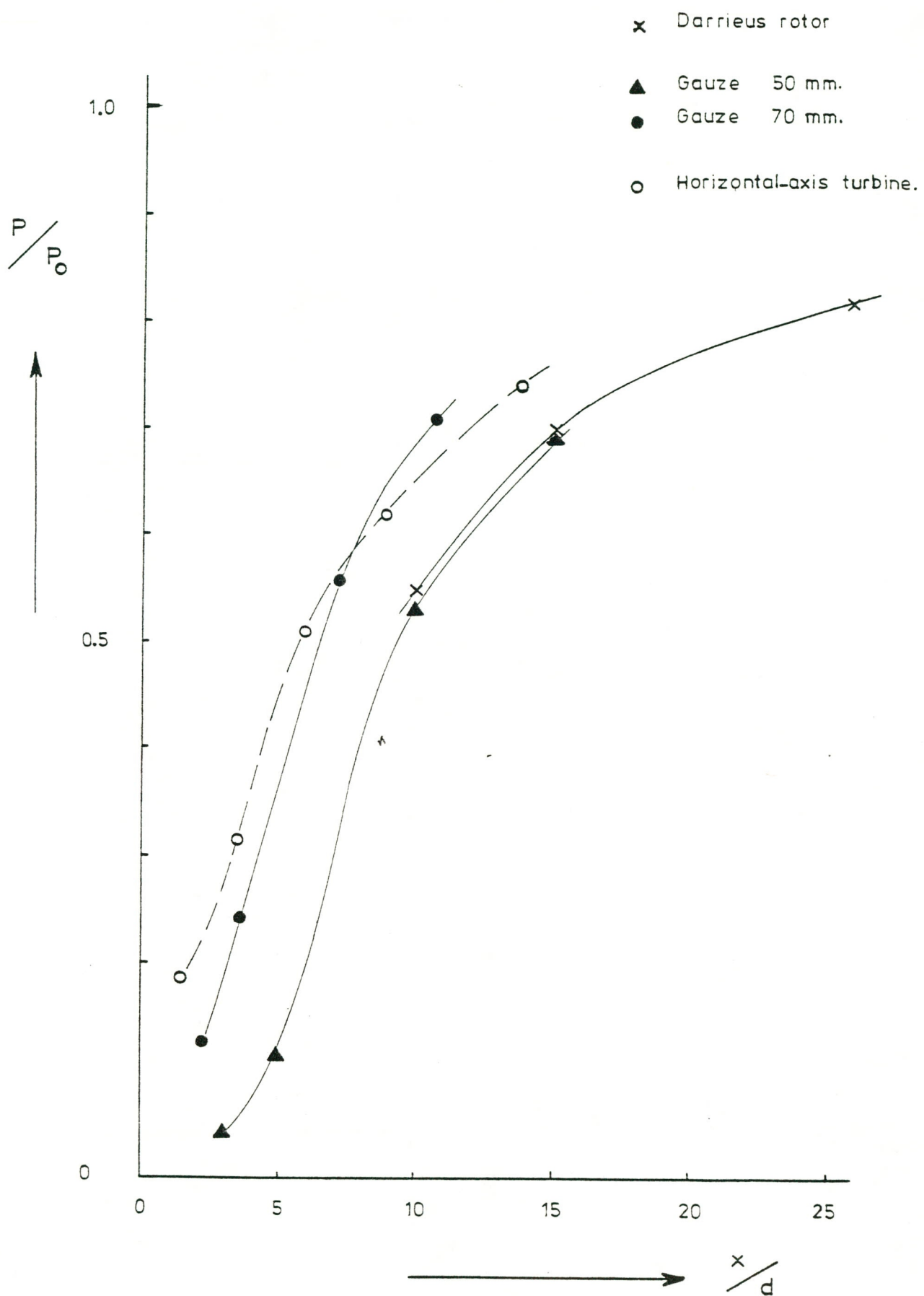




Wake centre-line velocity deficits.

MT-TNO
8119
Fig. 3

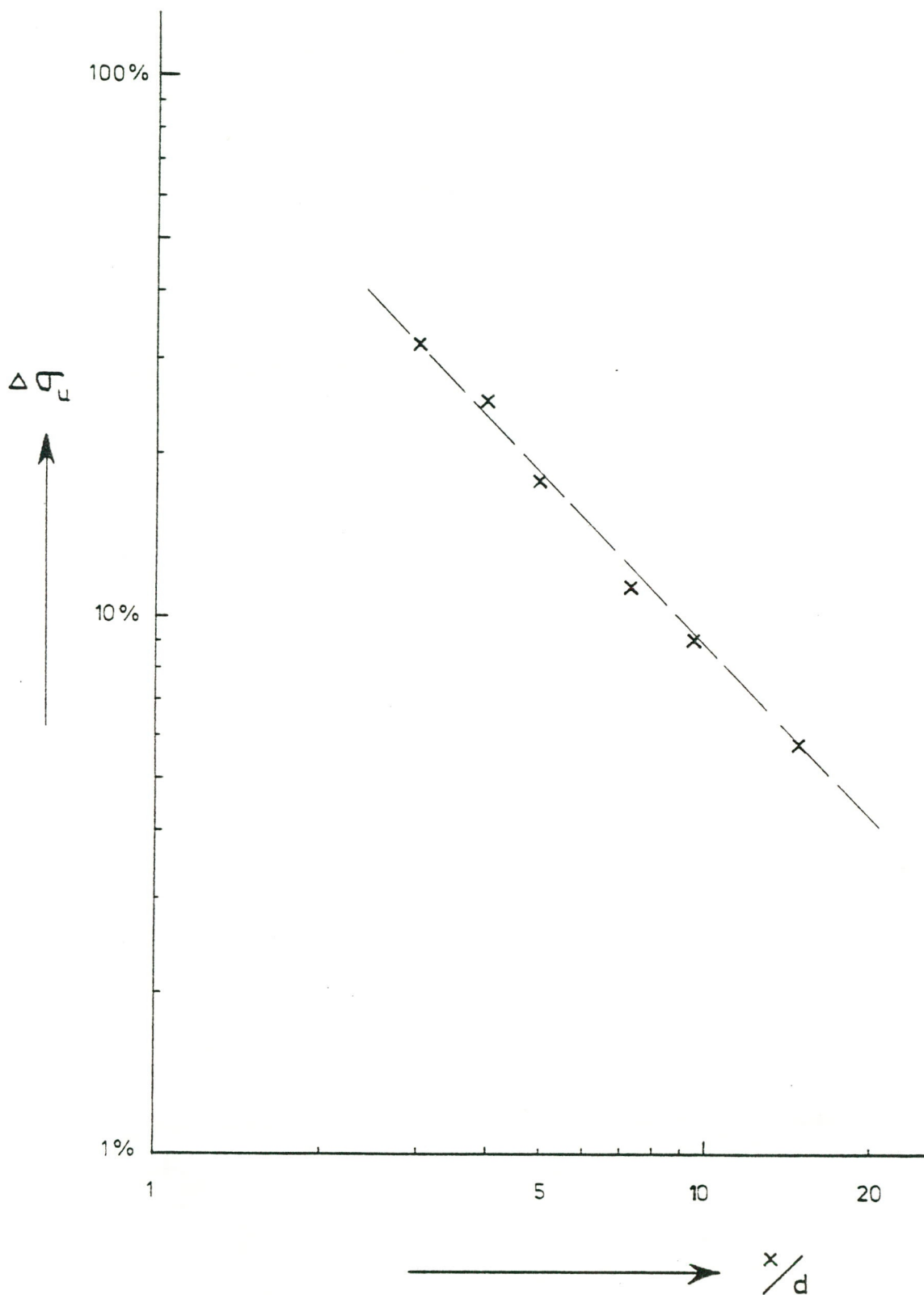




Available windpower behind a turbine in a homogeneous flowfield.

MT-TNO
8119
Fig. 4

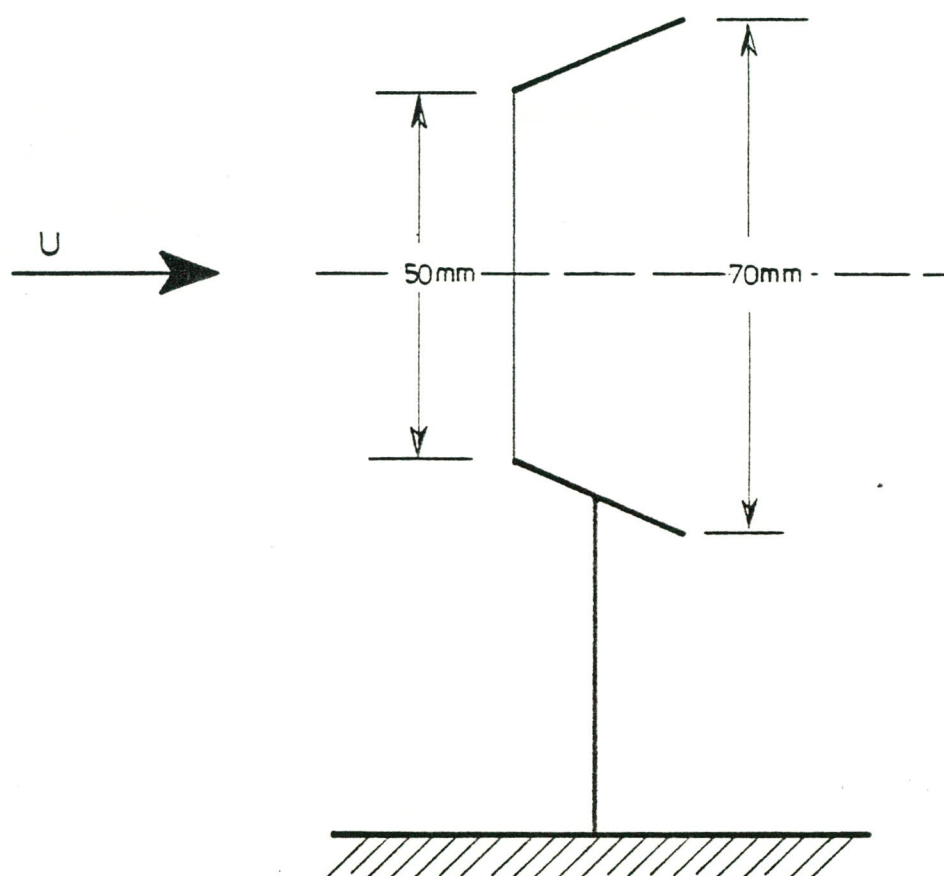




Turbulence intensity in the centre of the wake.

MT-TNO
8119
Fig. 5

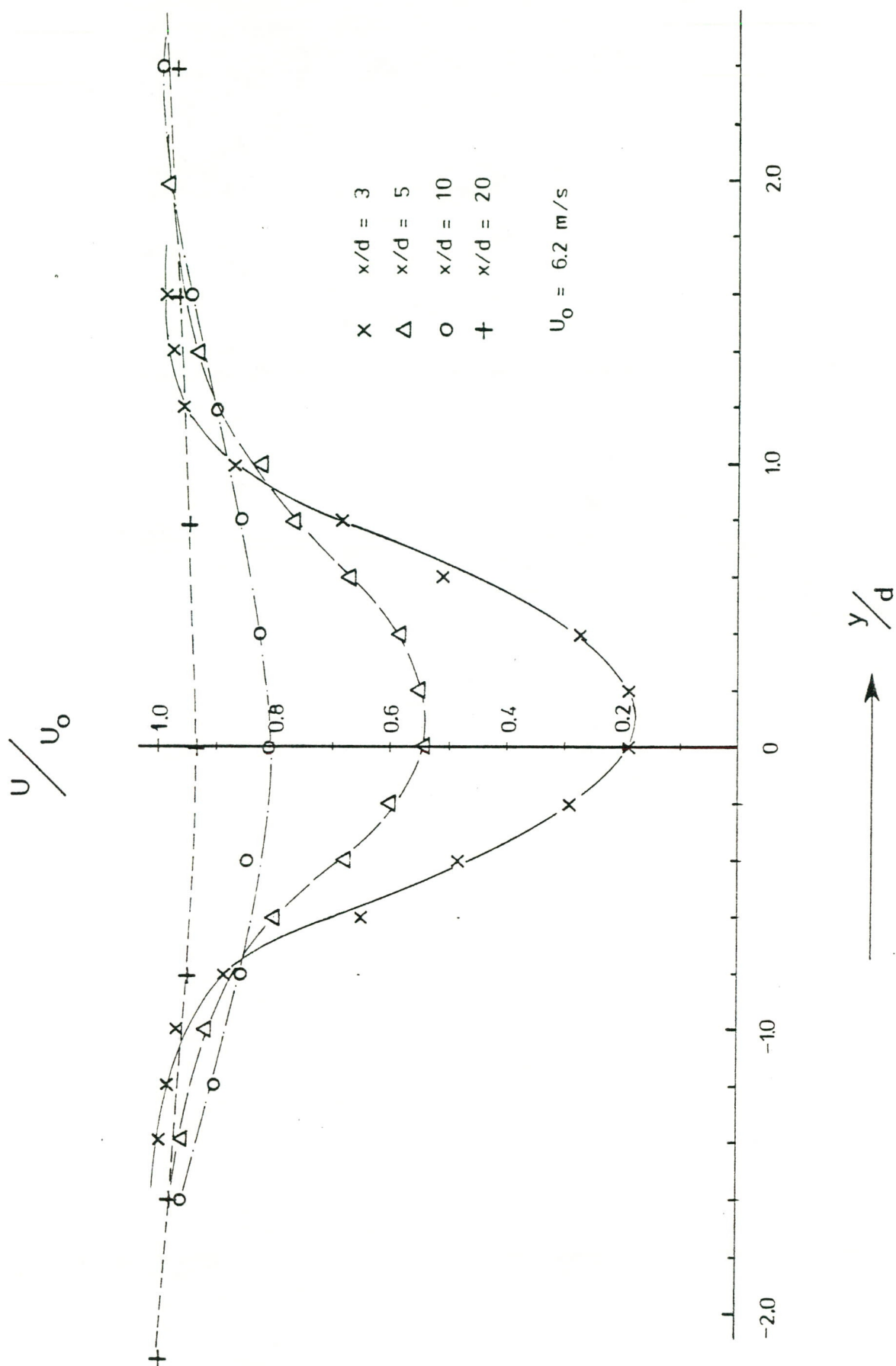




The wake simulating object

MT-TNO
8119
Fig. 6

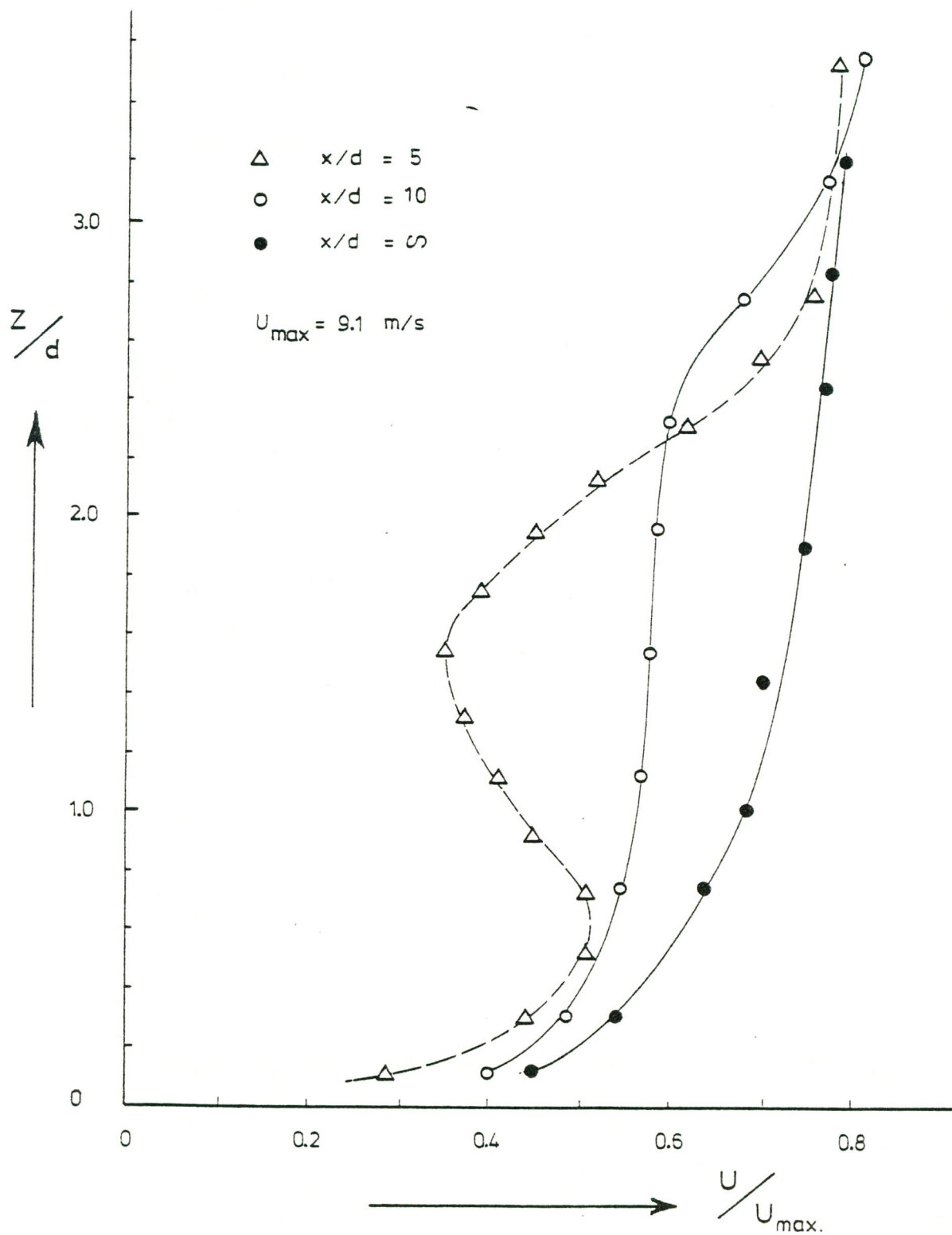




Mean velocity profiles in the horizontal direction.

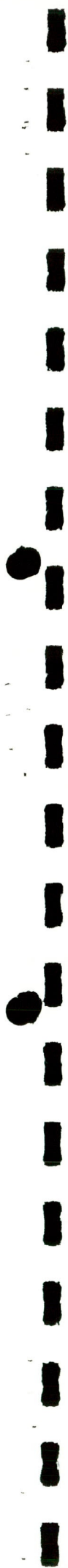
MT-TNO
8119
Fig. 7

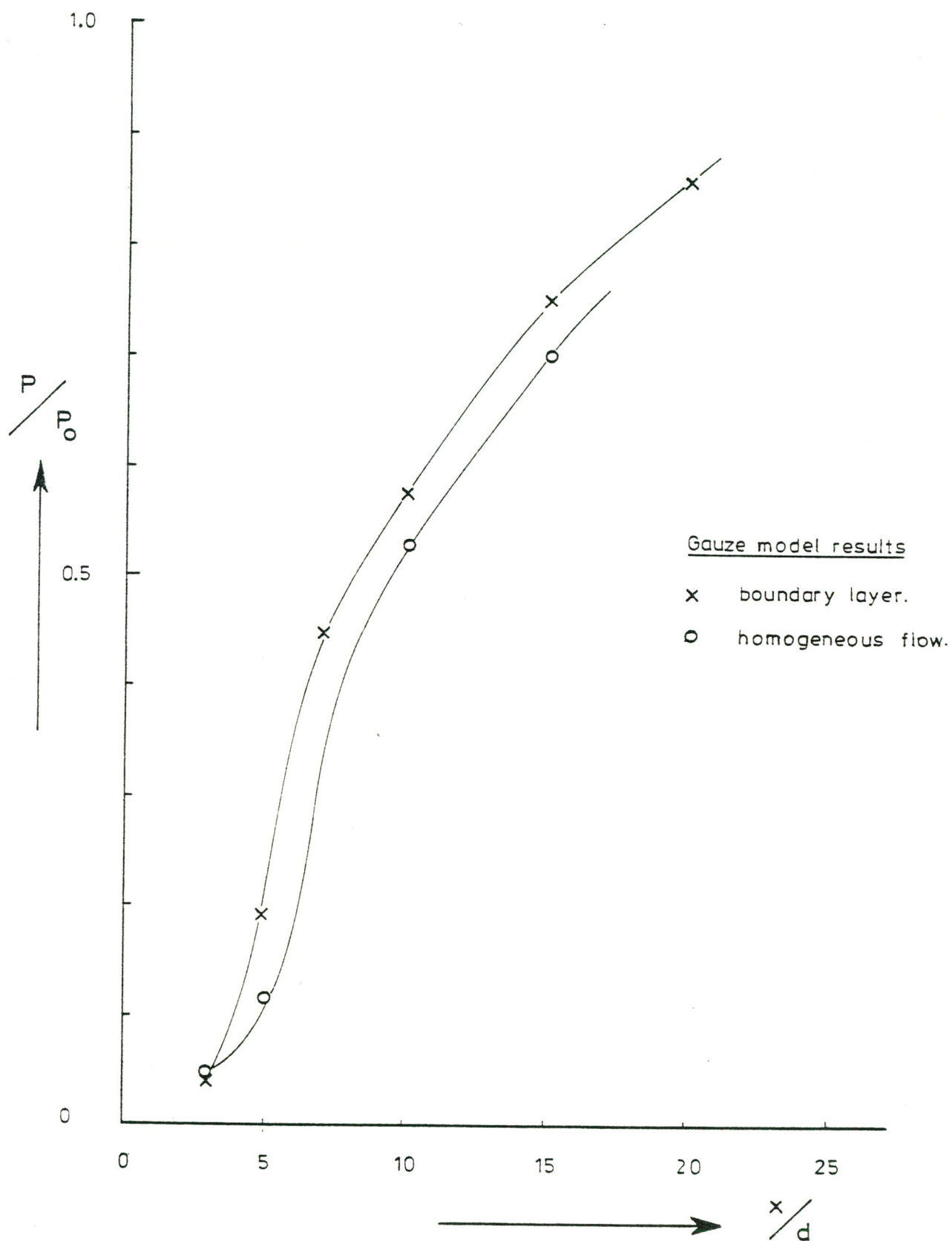




Mean velocity profiles in the vertical direction.

MT-TNO
8119
Fig. 8

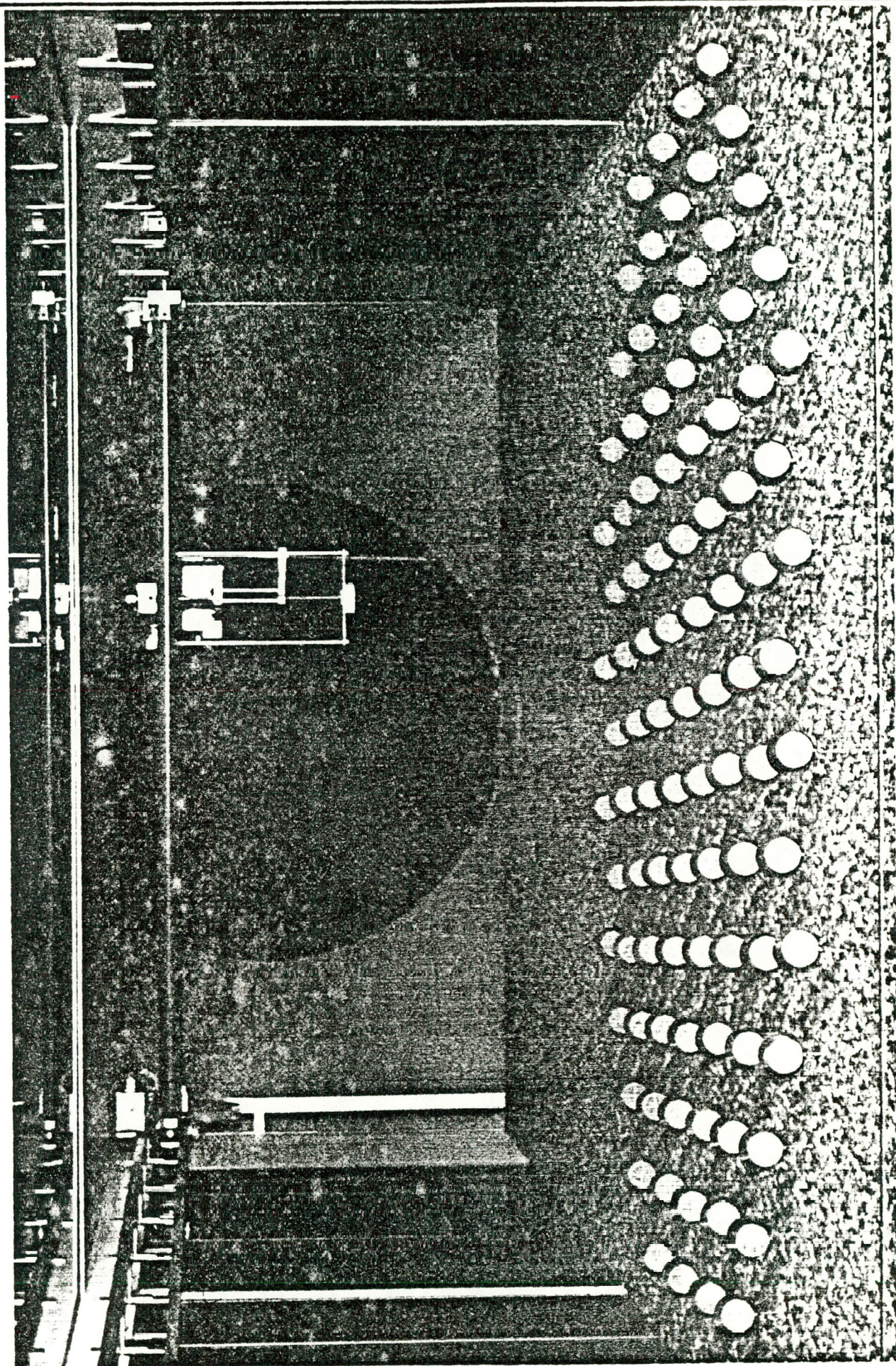




Available windpower behind a turbine
in a boundary layer.

MT-TNO
8119
Fig. 9





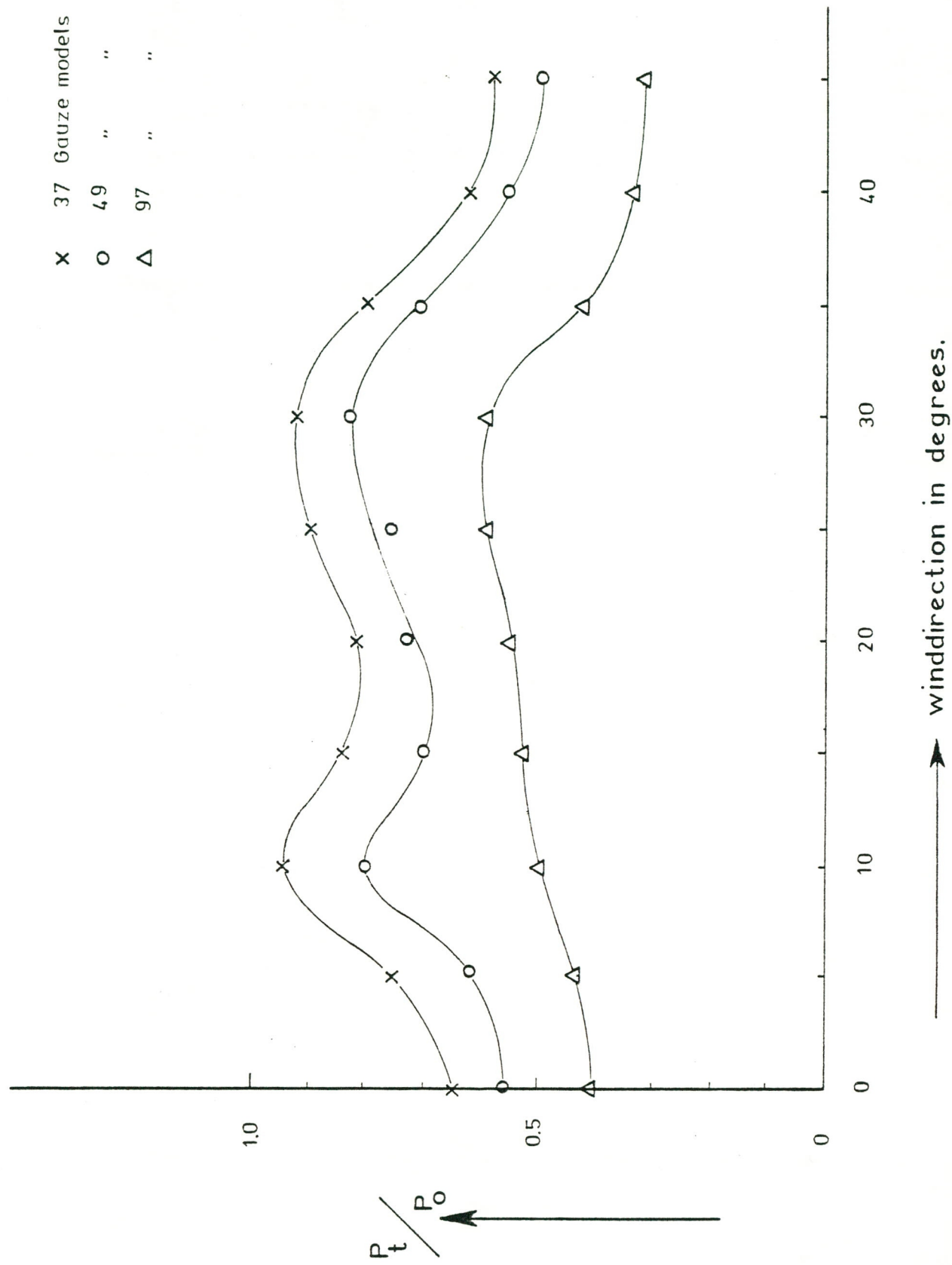
The array of 97 Gauze models placed
in the windtunnel.

MT-TNO

8119

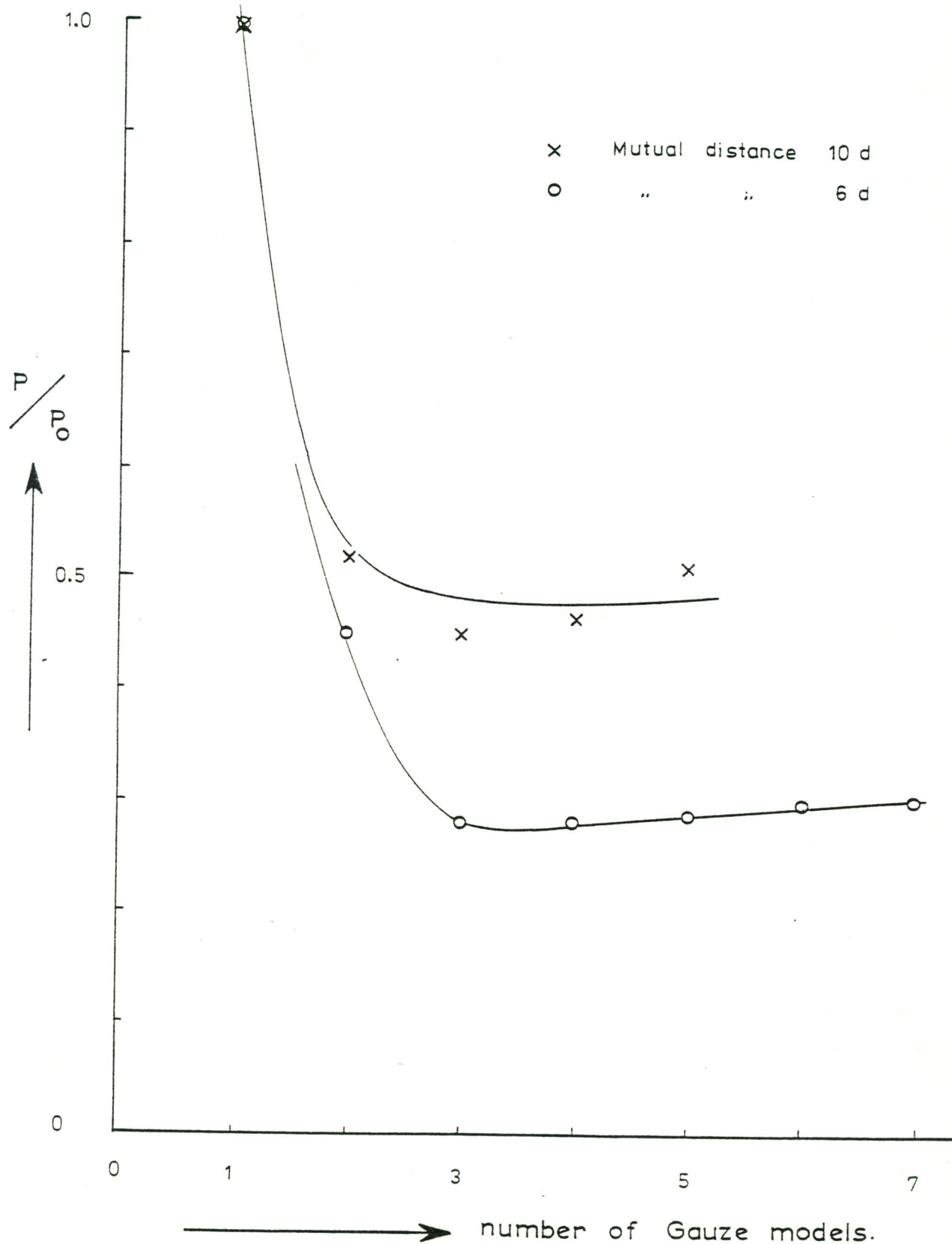
Fig. 10





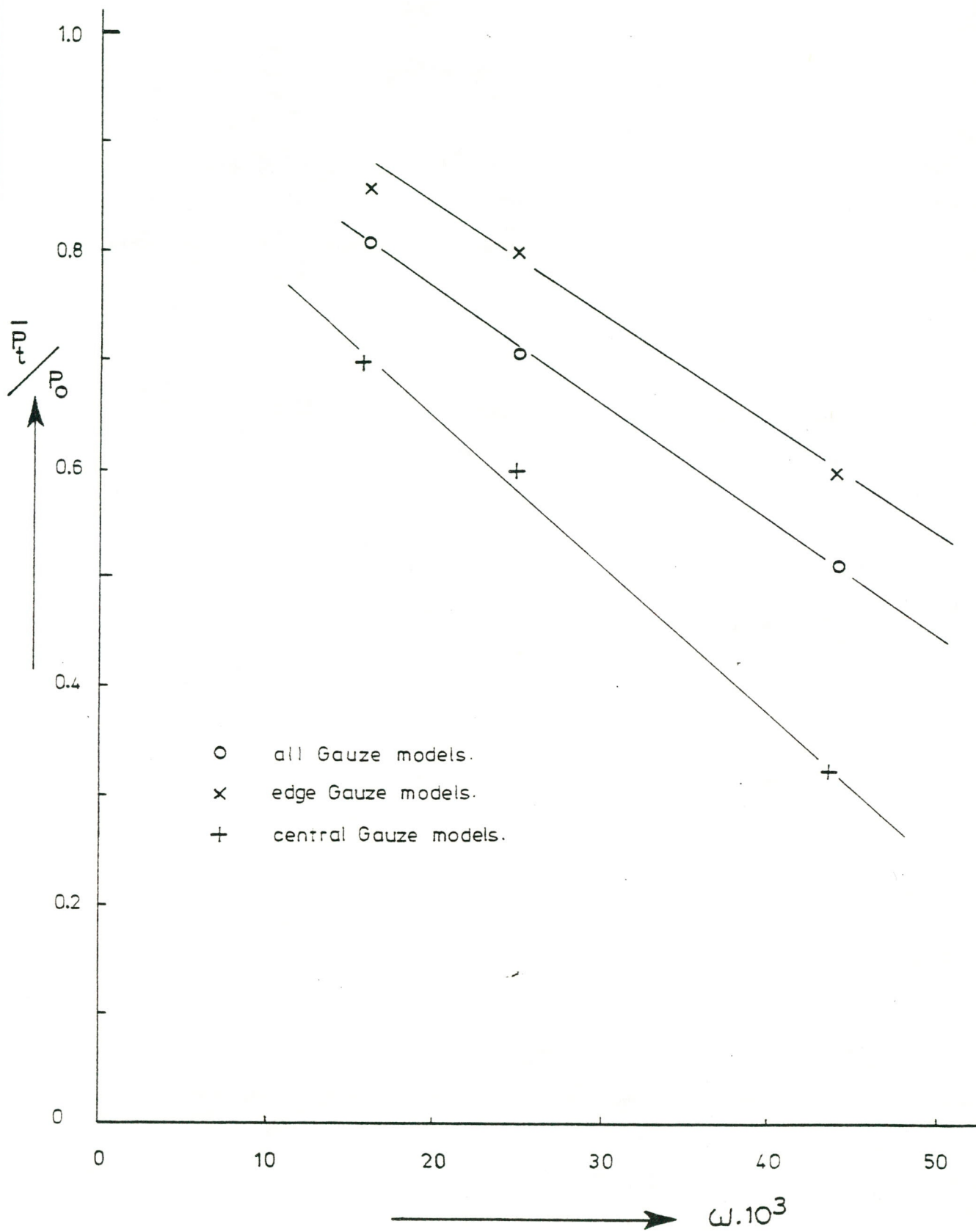
The power output averaged per Gauze model

MT-TNO
8119
Fig. 11



Measured power output in lines

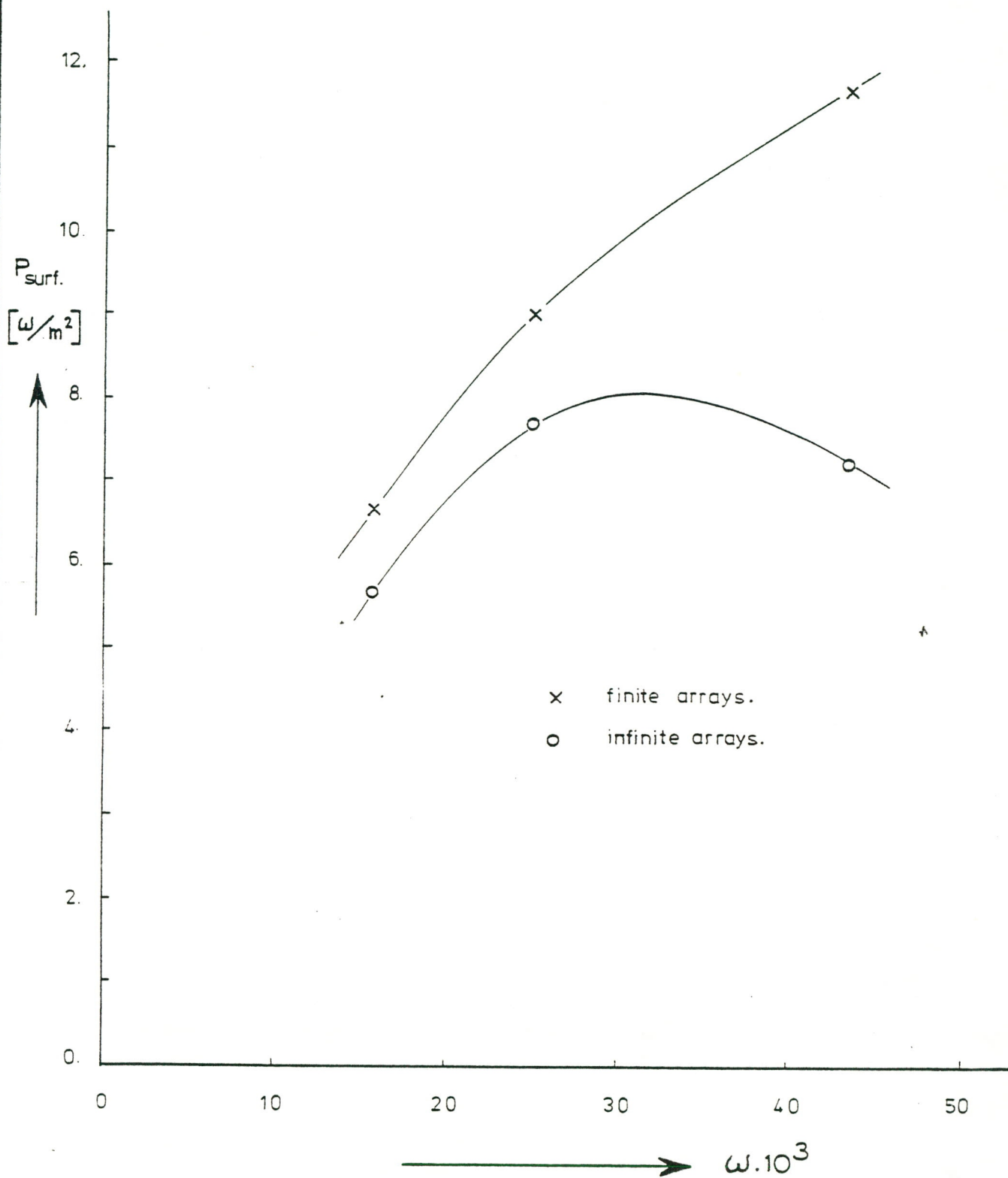
MT-TNO
8119
Fig. 12



Array results.

MT-TNO
8119
Fig. 13

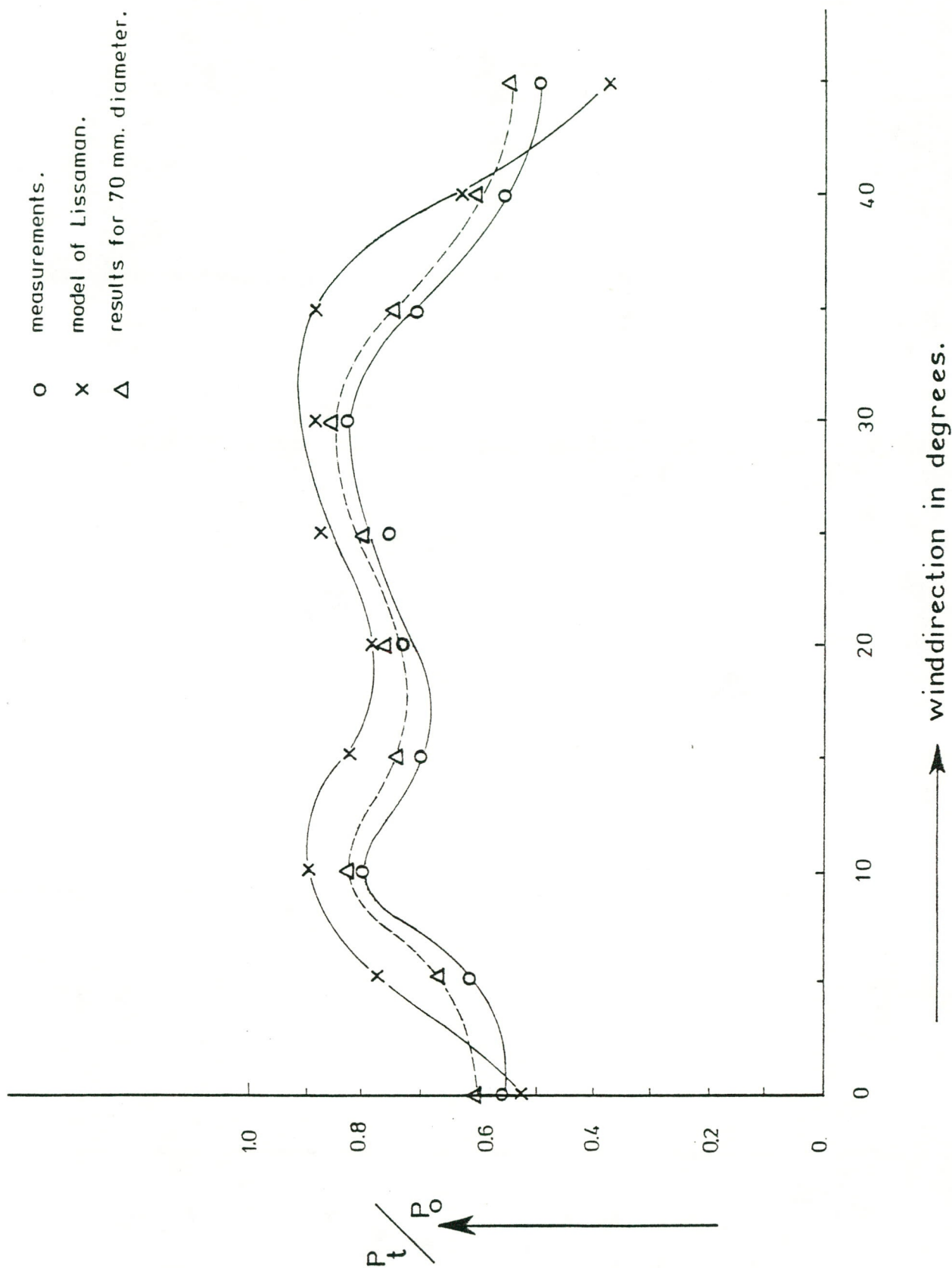




Power output per ground area

MT-TNO
8119
Fig 14





Comparison between results for the array of 49 Gauze models

MT-TNO
8119
Fig. 15

