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TNO report

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Bayesian estimation of characteristic S-N curves for reinforcement bars and proposal for the national annex of NEN-EN1992-1-1

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Contents

1 Introduction

The focus of this report is on the assessment of the characteristic (5% fractile) S-N curves for reinforcement bars, based on fatigue tests performed at constant stress amplitude [1] which have been performed in The Netherlands in the past years. Five datasets of experimental tests are considered in the investigation. The diameter of the bars varies between 12 and 25 mm and it includes straightened and welded bars.

The characteristic S-N curves have been derived by using a probabilistic fatigue model already available in literature [2] and a statistical technique for the characterization of the model parameters from the available experimental data.

The probabilistic model describes the fatigue life (in terms of the logarithm of the number of cycles at failure) of reinforcement bars as a bilinear function of the logarithm of the applied stress range. The chosen formulation of the fatigue model is representative of the fatigue behaviour of bars subjected to constant stress amplitudes.

A Bayesian approach is applied for the statistical inference of the parameters of the probabilistic fatigue model, based on the test results.

The outcome of the investigation is a proposal for the parameters of the characteristic S-N curves to be included in the Dutch national annex of the design code NEN-EN1992-1-1.

The report is organized as follows. Section 2 outlines the probabilistic fatigue model used for the assessment of the characteristic S-N curves. The Bayesian approach for the estimation of the parameters of the fatigue model is presented in Section 3. Moreover, the procedure for the assessment of the characteristic S-N curves is explained at the end of the section. Section 4 describes the application of the Bayesian procedure to datasets of fatigue tests performed on bars with diameter between 12 and 25 mm. On the basis of the estimated characteristic S-N curves, a proposal to be included in the national annex of the design code NEN-EN1992-1-1 is presented in Section 5.

The report has been prepared by TNO, for which a task group was formed consisting of:

- D.L. Allaix (TNO)
- A. de Boer (RWS)
- T. Breedijk
- F.B.J. Gijsbers (TNO)
- C. van der Veen (TU Delft)

The work has been financially sponsored by RWS.

2 Probabilistic fatigue model

The probabilistic fatigue model presented in [2] is used in the present investigation. Let N be the number of cycles that leads to fatigue failure (also called the fatigue life) of a specimen under a prescribed stress range s. The following probabilistic model is used for the fatigue life of reinforcement bars:

$$Y = \frac{\beta_0 + \beta_1 x}{H(V - x)} + \varepsilon(0, \sigma) \tag{1}$$

where Y=ln(N), x=ln(s), V=ln(s_{lim}), s_{lim} is the fatigue limit, H(·) is the step function and ε is the error term, which accounts for the deviations from the model. The fatigue limit is the value of the stress range below which fatigue failure of the bar does not occur. The model Eq.(1) is representative of the fatigue behaviour of bars subjected to constant stress amplitude.
The normal distribution is assumed for Y, V and ε [2], while x is deterministic because the stress range is under control during the experimental test. The probabilistic fatigue model is completely defined by the probabilistic characterization of the following vector of parameters:

$$\boldsymbol{\theta} = (\beta_0, \beta_1, \sigma, \mu_V, \sigma_V) \tag{2}$$

Where μ_V and σ_V are the mean value and the standard deviation of the random variable V.
A schematic representation of the model is plotted in Figure 1. The step function H(·) leads to a bilinear representation of the relationship between the natural logarithms of the applied stress range and the number of cycles at failure.

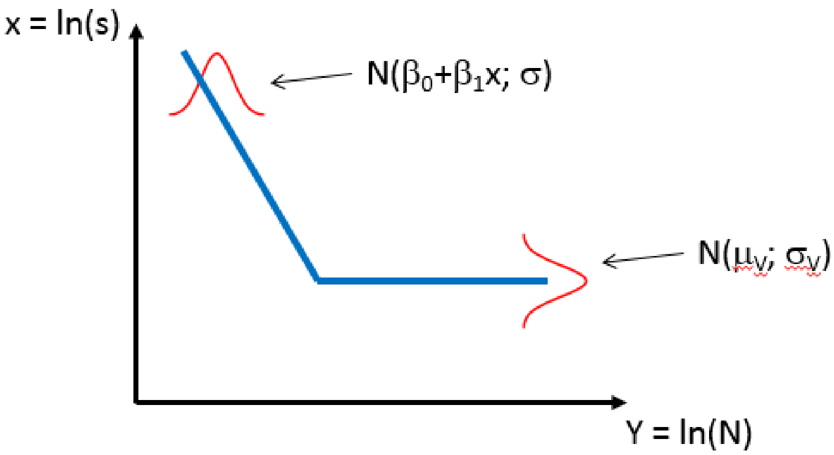

Figure 1. Schematic representation of the probabilistic fatigue model.

The probability density function of V can be written as:

$$f_V(v|\mu_V, \sigma_V) = \frac{1}{\sigma_V} \varphi\left(\frac{v - \mu_V}{\sigma_V}\right) \tag{3}$$

where $\varphi(\cdot)$ is the probability density function of the standard normal distribution. It is assumed that, conditioned on a fixed value of V < x, the conditional distribution of Y|V is also a normal distribution:

$$f_{Y|V}\left(y|\beta_0,\beta_1,\sigma,x\right)=\frac{1}{\sigma}\varphi\left(\frac{y-\left[\beta_0+\beta_1 x\right]}{\sigma}\right) \tag{4}$$

$$F_{Y|V}\left(y|\beta_0,\beta_1,\sigma,x\right)=\Phi\left(\frac{y-\left[\beta_0+\beta_1 x\right]}{\sigma}\right) \tag{5}$$

with mean value $\beta_0+\beta_1 x$ and standard deviation σ. The cumulative distribution function of the standard normal distribution is denoted by $\Phi(\cdot)$.
The marginal probability density function of Y is given by:

$$f_Y\left(y|x,\boldsymbol{\theta}\right)=\int_{-\infty}^{x} f_{Y|V}\left(y|\beta_0,\beta_1,\sigma,x\right) f_V\left(v|\mu_V,\sigma_V\right)dv \tag{6}$$

The marginal cumulative distribution function of Y is obtained as follows:

$$F_Y\left(y|x,\boldsymbol{\theta}\right)=\int_{-\infty}^{x} F_{Y|V}\left(y|\beta_0,\beta_1,\sigma,x\right) f_V\left(v|\mu_V,\sigma_V\right)dv \tag{7}$$

The density and the cumulative distribution functions $f_{Y|V}(\cdot)$ and $F_{Y|V}(\cdot)$ are not functions of the fatigue limit V, as it can be observed from Eqs.((4) and (5)). Therefore, the density and cumulative distribution functions of Y (Eqs.(6) and (7)) can be rewritten as follows:

$$f_Y\left(y|x,\boldsymbol{\theta}\right)=f_{Y|V}\left(y|\beta_0,\beta_1,\sigma,x\right)\Phi\left(\frac{x-\mu_V}{\sigma_V}\right) \tag{8}$$

$$F_Y\left(y|x,\boldsymbol{\theta}\right)=F_{Y|V}\left(y|\beta_0,\beta_1,\sigma,x\right)\Phi\left(\frac{x-\mu_V}{\sigma_V}\right) \tag{9}$$

3 Bayesian estimation of the parameters of the fatigue model and inference about the characteristic S-N curve

3.1 Introduction

Statistical techniques are resorted for the estimation of the parameters of the fatigue model, which is described in the previous section. The method of Maximum Likelihood Estimation is used in [2]. In the present investigation, the Bayesian approach is proposed in order to overcome undesirable sensitivities of the method of Maximum Likelihood Estimation to the starting point of the procedure.

The Bayesian approach to parameters estimation is a statistical methodology that allows to combine prior knowledge with experimental data. The Bayesian approach relies on three items:

- a sample **y** of experimental tests;
- a prior distribution $f_{\boldsymbol{\theta}}(\boldsymbol{\theta})$ of the unknown distribution parameters;
- the Bayes theorem for updating the prior distribution by using test results.

The prior distribution reflects the state of knowledge before performing the experimental tests. By applying the Bayes theorem, the posterior distribution $f_{\boldsymbol{\theta}}(\boldsymbol{\theta} \mid \mathbf{y})$ of the parameters can be determined as follows:

$$f_{\boldsymbol{\theta}}(\boldsymbol{\theta}|\mathbf{y}) = \frac{f_{\boldsymbol{\theta}}(\boldsymbol{\theta})L(\mathbf{y}|\boldsymbol{\theta})}{\int_{\boldsymbol{\theta}} f_{\boldsymbol{\theta}}(\boldsymbol{\theta})L(\mathbf{y}|\boldsymbol{\theta})\mathbf{d}\boldsymbol{\theta}} \tag{10}$$

where $L(\mathbf{y} \mid \boldsymbol{\theta})$ is the likelihood function. The posterior distribution represents the knowledge that has been gained by means of the experimental tests.

3.2 Prior distributions

Non-informative prior distributions are assumed for the parameters of the fatigue model $\boldsymbol{\theta} = (\beta_0, \beta_1, \sigma, \mu_V, \sigma_V)$, due to lack of knowledge before performing the experimental tests.
A convenient non-informative prior distribution is uniform on $(\beta_0, \beta_1, \ln(\sigma), \mu_V, \ln(\sigma_V))$ or, equivalently [3]:

$$f_{\boldsymbol{\theta}}(\boldsymbol{\theta}) \propto \sigma^{-1}\sigma_V^{-1} \tag{11}$$

3.3 Likelihood function

This function represents the probability of observing the sample **y**, given the vector of parameters **θ**. Given n independent sample data $y_1=\ln(N_1)$, …, $y_n=\ln(N_n)$ at the log stress range $x_1=\ln(s_1)$, …, $x_n=\ln(s_n)$, the likelihood function is defined as the product of the likelihood of each experimental data:

$$L(\mathbf{y}|\boldsymbol{\theta}) = \prod_{i=1}^{n} \left[f_Y(y_i|x_i,\boldsymbol{\theta}) \right]^{\delta_i} \left[1 - F_Y(y_i|x_i,\boldsymbol{\theta}) \right]^{1-\delta_i} \tag{12}$$

where:
- $\delta_i = 0$ if y_i is a failure point;
- $\delta_i = 1$ if y_i is a run-out point.

The assumption of independent sample data is considered applicable also in this investigation, at least because the samples came from different producers. In the likelihood function, the term related to the failure point *i* is the probability of observing $y_i=\ln(N_i)$ at $x_i=\ln(s_i)$. The term related to the run-out point is the probability that the Y is larger than y_i, which is in this case the logarithm of the maximum number of cycles performed during the test.
Only for computational reasons, it is preferred to work with the log-likelihood function:

$$\log\left[L(\mathbf{y}|\boldsymbol{\theta})\right] = \sum_{i=1}^{n} \delta_i \log\left[f_Y(y_i|x_i,\boldsymbol{\theta})\right] + (1-\delta_i)\left[1 - F_Y(y_i|x_i,\boldsymbol{\theta})\right] \tag{13}$$

3.4 Posterior distribution

As a consequence of the choice of the prior distributions and due to the expression of the likelihood function, a closed form expression of the posterior distribution $f_\theta(\boldsymbol{\theta} \mid \mathbf{y})$ (Eq. (10)) is not available. However, it is possible to sample directly from the posterior distribution $f_\theta(\boldsymbol{\theta} \mid \mathbf{y})$ by using a Markov chain Monte Carlo method (MCMC). The basic idea behind MCMC is to generate a large set of samples $\boldsymbol{\theta}$, whose empirical distribution approximates the posterior distribution $f_\theta(\boldsymbol{\theta} \mid \mathbf{y})$. Let suppose that n samples are already available. A new sample $\boldsymbol{\theta}^{(n+1)}$ close to $\boldsymbol{\theta}^{(n)}$ should be reasonably added if $f_\theta(\boldsymbol{\theta}^{(n+1)} \mid \mathbf{y})$ is larger than $f_\theta(\boldsymbol{\theta}^{(n)} \mid \mathbf{y})$ (or, in other words, the probability of observing $\boldsymbol{\theta}^{(n+1)}$ given the experimental data **y** is larger than the probability of $\boldsymbol{\theta}^{(n)}$).

The Metropolis algorithm is used for the purpose. The algorithm works as follows [4]:
- given the sample $\boldsymbol{\theta}^{(n)}$, a new sample $\boldsymbol{\theta}^{(*)}$ is generated form a symmetric proposal distribution $J(\boldsymbol{\theta}^{(*)}|\boldsymbol{\theta}^{(n)})$
- the acceptance ratio r is computed as:

$$r = \frac{f_{\boldsymbol{\theta}}\left(\boldsymbol{\theta}^{(*)}|\mathbf{y}\right)}{f_{\boldsymbol{\theta}}\left(\boldsymbol{\theta}^{(n)}|\mathbf{y}\right)} = \frac{f_{\boldsymbol{\theta}}\left(\boldsymbol{\theta}^{(*)}\right)L\left(\mathbf{y}|\boldsymbol{\theta}^{(*)}\right)}{f_{\boldsymbol{\theta}}\left(\boldsymbol{\theta}^{(n)}\right)L\left(\mathbf{y}\boldsymbol{\theta}^{(n)}\right)} \tag{14}$$

- The new sample is chosen as:

$$\boldsymbol{\theta}^{(n+1)} = \begin{cases} \boldsymbol{\theta}^{(*)} & \text{with probability } \min(r,1) \\ \boldsymbol{\theta}^{(n)} & \text{with probability } 1\text{-}\min(r,1) \end{cases} \tag{15}$$

The choice of the proposal distribution $J(\boldsymbol{\theta}^{(*)}|\boldsymbol{\theta}^{(n)})$ is quite arbitrary. A normal distribution $N(\boldsymbol{\theta}^{(n)},\delta)$ is used in the present investigation . The parameter δ governs the convergence of the MCMC simulation. A low value determines a slow

convergence because the new sample will be very close to the previous one. On the opposite, a high value will determine a faster convergence, but the probability of acceptance of new samples will reduce due to the high probability of generating $\boldsymbol{\theta}^{(*)}$ far away from $\boldsymbol{\theta}^{(n)}$.
The term converge in the context of MCMC represent the attainment of a stationary behaviour of the Markov Chain, e.g. the same pattern can be recognized by plotting the Markov chain. In this investigation, 10^6 samples have been generated using MCMC and the parameter δ has been chosen in such a way the acceptance rate of the Markov Chain is between 0.20 and 0.50 [3].

3.5 Bayesian inference of the fractiles of the distribution of the fatigue life

The predictive distribution of the random variable Y is defined as:

$$F_Y(y|x) = \int_{\boldsymbol{\theta}} F_Y(y|x, \boldsymbol{\theta}) f_{\boldsymbol{\theta}}(\boldsymbol{\theta}|\mathbf{y}) \mathbf{d}\boldsymbol{\theta} \tag{16}$$

The samples obtained from the MCMC may be regarded as observations from the distribution $f_{\boldsymbol{\theta}}(\boldsymbol{\theta} \mid \mathbf{y})$ [5]. Therefore the predictive distribution $F_Y(y|x)$ can be approximated using the samples generated by the MCMC method:

$$F_Y(y|x) \approx \frac{1}{s} \sum_{i=1}^{s} \Pr(Y \le y|x) \tag{17}$$

Any fractile of the distribution of the fatigue life, given the stress range s can be obtained by means of Eq.(17).

4 Application to datasets of fatigue tests – estimation of the S-N curves

4.1 Introduction

The Bayesian approach described in Section 4 has been applied to the following datasets of fatigue tests of reinforcement bars:
- straightened bars, diameter 12 mm;
- straightened bars, diameter 16 mm;
- welded bars, diameter 12 mm;
- welded bars, diameter 16 mm;
- welded bars, diameter 25 mm.

The datasets are listed in Annex A. The bars were manufactured using reinforcement steel B500 (normally f_{yk} = 500 MPa for this type of steel) and the tests were performed at 0.6Re (300 MPa).

The dataset of straightened bars with diameter 12 mm consists of samples of steel type A and steel type B. Distinction is made between these two steel types. The S-N curve is not reported for the steel type A, because the probabilistic fatigue model leads to unrealistic results.

The datasets with bar diameter larger than 12mm consist of samples of steel type B.

The dataset of welded bars with diameter 25 mm contains results with failure within and outside the welded zone. The S-N curve is not reported for the case of the failure outside the welded zone, due to the insufficient sample size.
The characteristic (5% fractile) S-N curves obtained with the Bayesian approach are shown for the above-mentioned datasets in the Figures 2-6, as well as the experimental results (x = failure, o = run-out).

4.2 Dataset straightened bars, diameter 12 mm (steel type B)

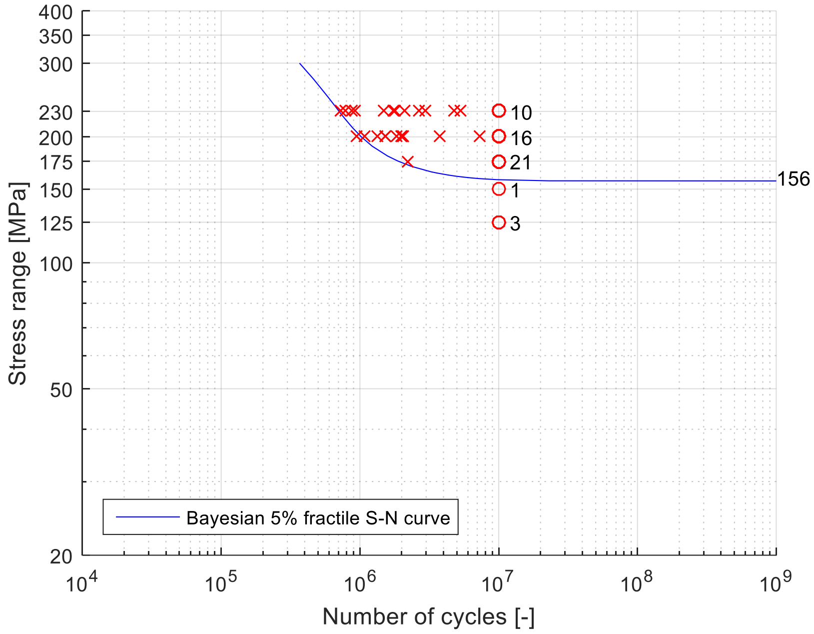

Figure 2. Characteristic S-N curve - straightened bars, diameter 12 mm.

4.3 Dataset straightened bars, diameter 16 mm (steel type B)

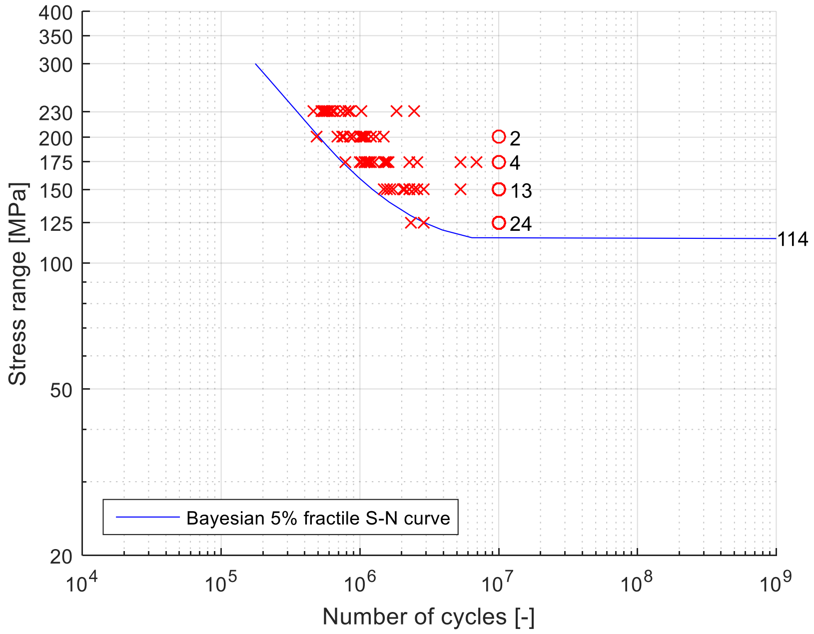

Figure 3. Characteristic S-N curve - straightened bars, diameter 16 mm.

4.4 Dataset welded bars, diameter 12 mm (steel type B)

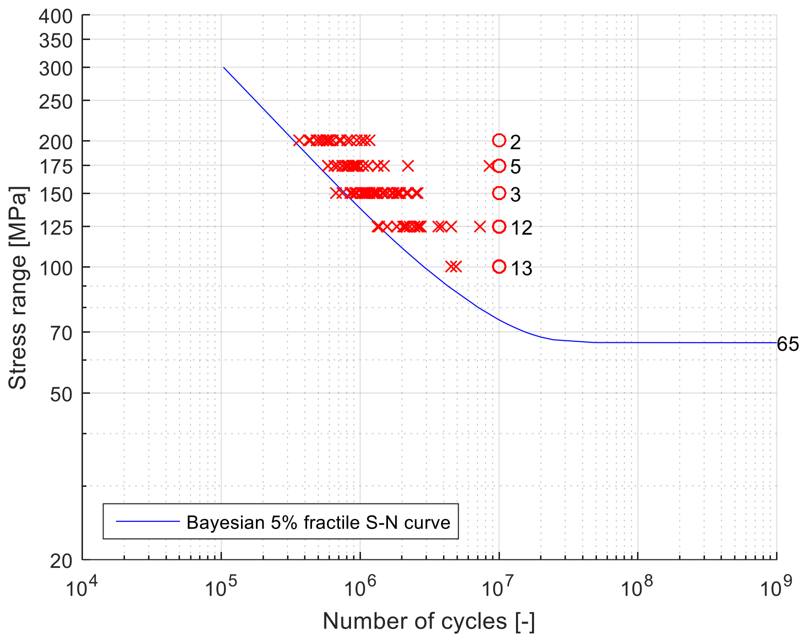

Figure 4. Characteristic S-N curve - welded bars, diameter 12 mm.

4.5 Dataset welded bars, diameter 16 mm (steel type B)

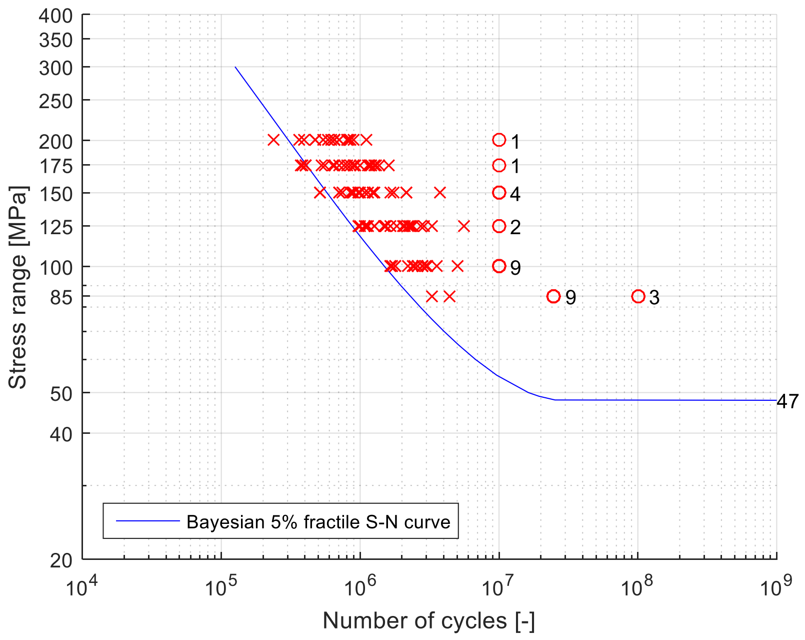

Figure 5. Characteristic S-N curve - welded bars, diameter 16 mm.

4.6 Dataset welded bars, diameter 25 mm (steel type B, failure at the weld)

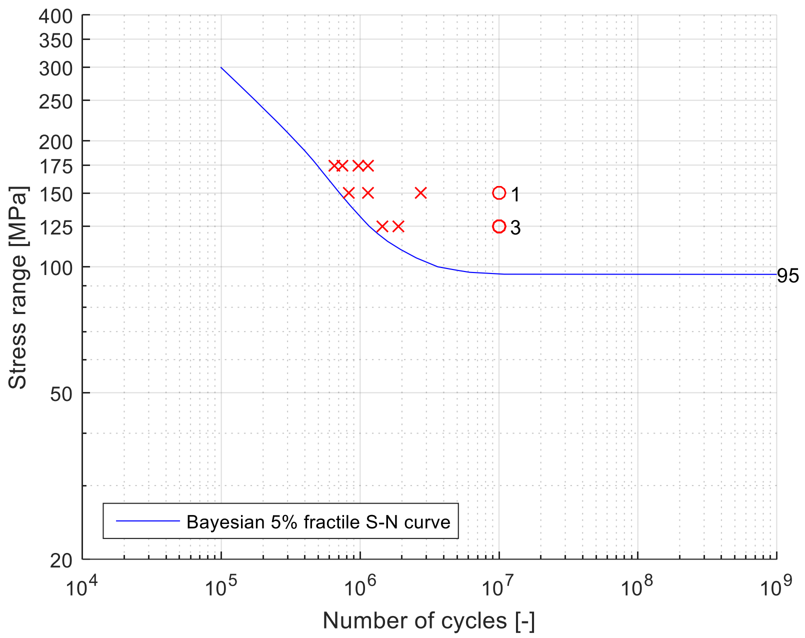

Figure 6. Characteristic S-N curve - welded bars, diameter 25 mm (experimental data with failure at the weld).

The type of weld used for the experimental tests is not representative for situations in practice and the S-N curve may be too unconservative. Further investigation is advised.

5 Proposal for the design code NEN-EN1992-1-1

5.1 Introduction

On the basis of the outcomes of the investigation about the datasets listed in Section 4, an update of the parameters N*, k1, k2, and $\Delta\sigma_{Rsk}$ at N* cycles is proposed, with reference to Figure 7.

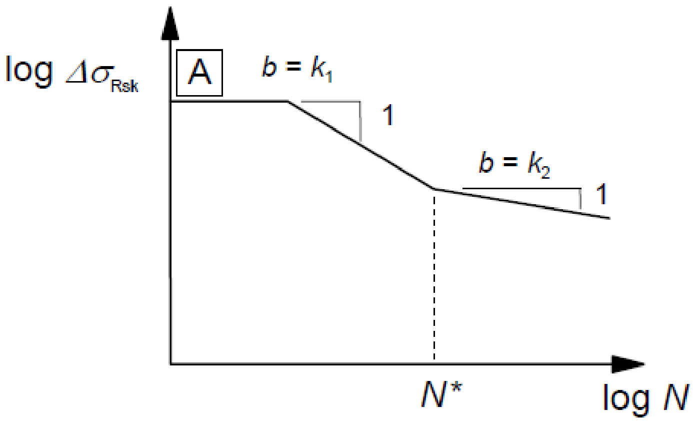

Figure 7. Shape of the characteristic S-N curve.

Three proposals are presented in the following:
- proposal “April 6, 2016”, which is the outcome of the meeting attended by Ane de Boer, Theo Breedijk, Cor van der Veen and Jan Gijsbers;
- proposal “Theo Breedijk April 8, 2016”;
- proposal “Hans Bongers April 11, 2016”;

The proposed values are listed in the Tables 1 and 2, for straight and straightened bars, and in the Tables 3, 4 and 5 for welded bars.

Table 1. Proposal “April 6, 2016”. Parameters N*, k1, k2 and $\Delta\sigma_{Rsk}$ (straight and straightened bars)

Diameter ϕ [mm]	**N***	**k_1**	**k_2**	**$\Delta\sigma_{Rsk}$ [MPa]**	
				Type A	**Type B and C**
$\phi \leq 12$	10^6	5	9	150	175
$12 < \phi < 20$	10^6	5	9	130	150
$\phi \geq 20$	10^6	5	9	NA	140

Table 2. Proposal “Theo Breedijk April 8, 2016”. Parameters N*, k1, k2 and $\Delta\sigma_{Rsk}$ (straight and straightened bars)

Diameter ϕ [mm]	**N***	**k_1**	**k_2**	**$\Delta\sigma_{Rsk}$ [MPa]**	
				Type A	**Type B and C**
$\phi \leq 12$	10^6	5	9	150	175
$12 < \phi \leq 25$	10^6	5	9	140	150
$\phi > 25$	10^6	5	9	NA	140

Table 3. Proposal “April 6, 2016”. Parameters N*, k1, k2 and $\Delta\sigma_{Rsk}$ (welded bars)

Diameter ϕ [mm]	**N***	**k_1**	**k_2**	**$\Delta\sigma_{Rsk}$ [MPa]**
				Type A, B and C
$\phi \leq 12$	10^7	3	5	58,5
$\phi > 12$	10^7	3	5	50

Table 4. Proposal “Theo Breedijk April 8, 2016”. Parameters N*, k1, k2 and $\Delta\sigma_{Rsk}$ (welded bars)

Diameter ϕ [mm]	**N***	**k_1**	**k_2**	**$\Delta\sigma_{Rsk}$ [MPa]**
				Type A, B and C
$\phi \leq 12$	$2 \cdot 10^6$	3	5	100
$\phi > 12$	$2 \cdot 10^6$	3	5	80

Table 5. Proposal “Hans Bongers April 11, 2016”. Parameters N*, k1, k2 and $\Delta\sigma_{Rsk}$ (welded bars)

Diameter ϕ [mm]	**N***	**k_1**	**k_2**	**$\Delta\sigma_{Rsk}$ [MPa]**
				Type A, B and C
$\phi \leq 12$	10^6	3	5	130
$\phi > 12$	10^6	3	5	110

Note that in all proposals a fixed relation is assumed between k_1 and k_2, namely $k_2 = 2k_1\text{-}1$, which is based on the work by Haibach [6]. The k_2 branch is relevant in connection with fatigue damage due to local effects with variable stress amplitude.

In the following paragraphs, the characteristic curves obtained by the Bayesian approach are compared with those of the proposals in the Figures 8-14.

5.2 Dataset straightened bars, diameter 12 mm

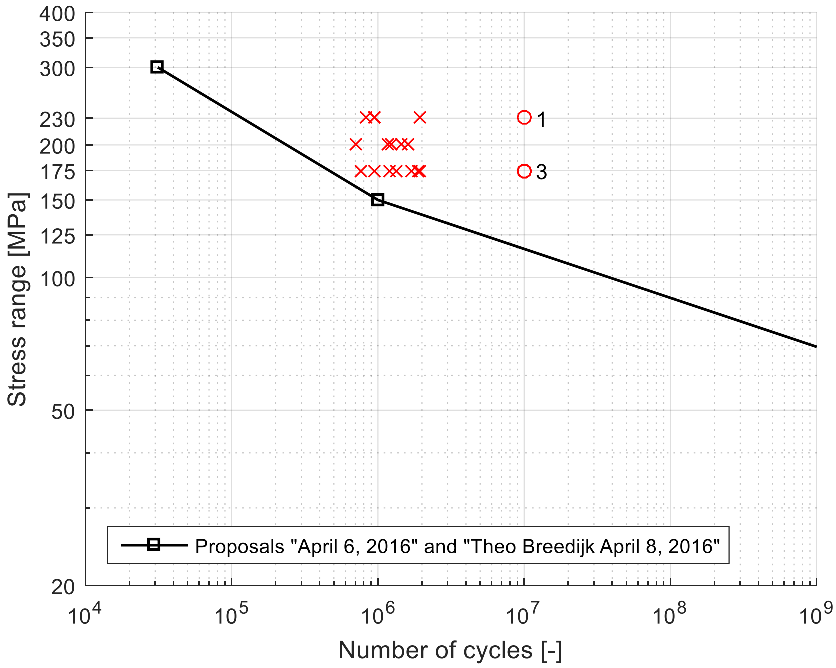

Figure 8. Proposed S-N curves - straightened bars, diameter 12 mm (steel type A).

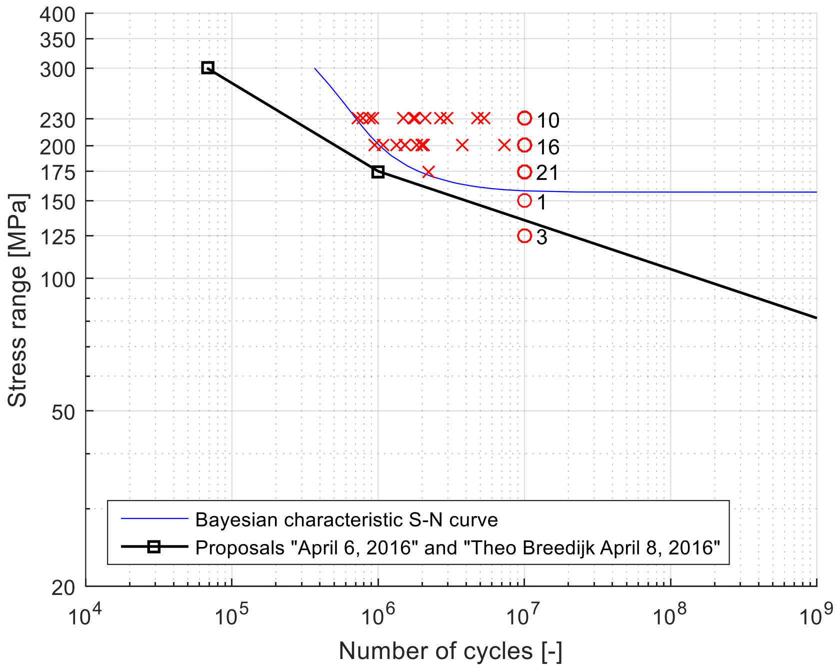

Figure 9. Proposed S-N curves - straightened bars, diameter 12 mm (steel type B).

For straightened bars, diameter 12 mm, there is agreement. The values listed in Table 6 are proposed for the national annex to NEN-EN1992-1-1.

Table 6. Proposal for straightened bars, diameter 12mm. Parameters N*, k1 and k2 and $\Delta\sigma_{Rsk}$.

Diameter ϕ [mm]	N*	k_1	k_2	$\Delta\sigma_{Rsk}$ [MPa]	
				Type A	Type B and C
12	10^6	5	9	150	175

5.3 Dataset straightened bars, diameter 16 mm

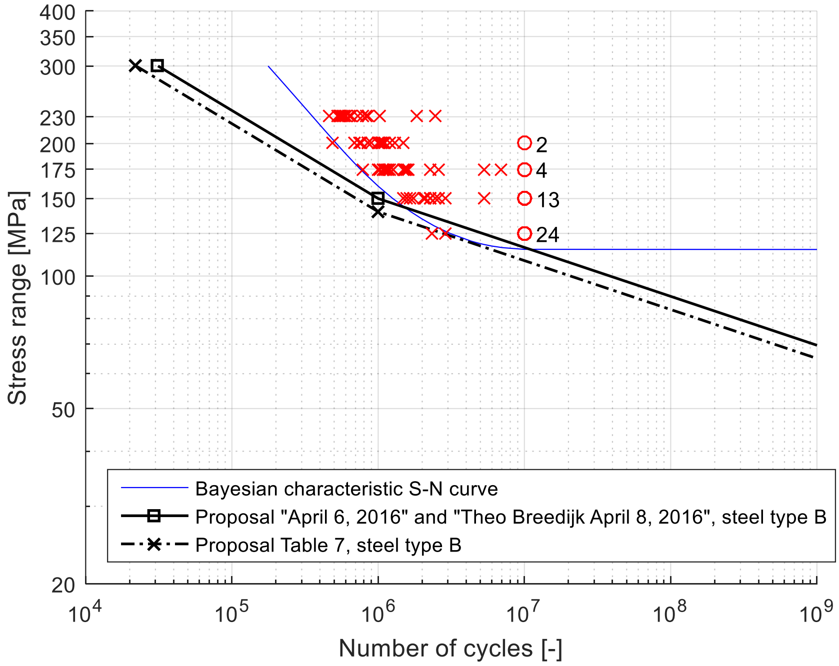

Figure 10. Proposed S-N curves - straightened bars, diameter 16 mm.

For straightened bars, diameter 16 mm, the proposals differ for steel type A. Note that the tests are covering steel type B only. Based on Figure 10, the values listed in Table 7 are proposed for the national annex to NEN-EN1992-1-1. The proposal for steel type B is also plotted in Figure 10.

Table 7. Proposal for straightened bars, diameter 16mm. Parameters N*, k1, k2 and $\Delta\sigma_{Rsk}$.

Diameter ϕ [mm]	N*	k_1	k_2	$\Delta\sigma_{Rsk}$ [MPa]	
				Type A	Type B and C
16	10^6	5	9	130	140 (*)

(*) There is no consensus about this value in the task group. Mr. T. Breedijk considers that a value of 150 MPa is more appropriate.

5.4 Dataset welded bars, diameter 25 mm (failure not at the weld)

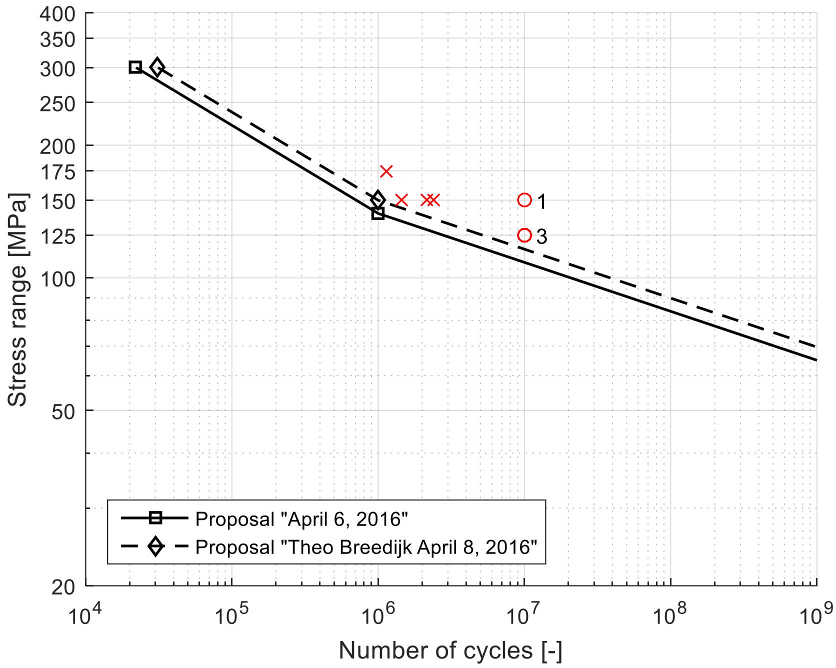

Figure 11. Proposed S-N curves - straight bars, diameter 25 mm (experimental data with failure not at the weld).

For straight bars, diameter 25 mm, the proposals differ. Based on Figure 11, the values listed in Table 8 are proposed for the national annex to NEN-EN1992-1-1.

Table 8. Proposal for straight bars, diameter 25mm. Parameters N*, k1, k2 and $\Delta\sigma_{Rsk}$.

Diameter ϕ [mm]	**N***	**k_1**	**k_2**	**$\Delta\sigma_{Rsk}$ [MPa]**	
				Type A	**Type B and C**
25	10^6	5	9	NA	140

5.5 Dataset welded bars, diameter 12 mm

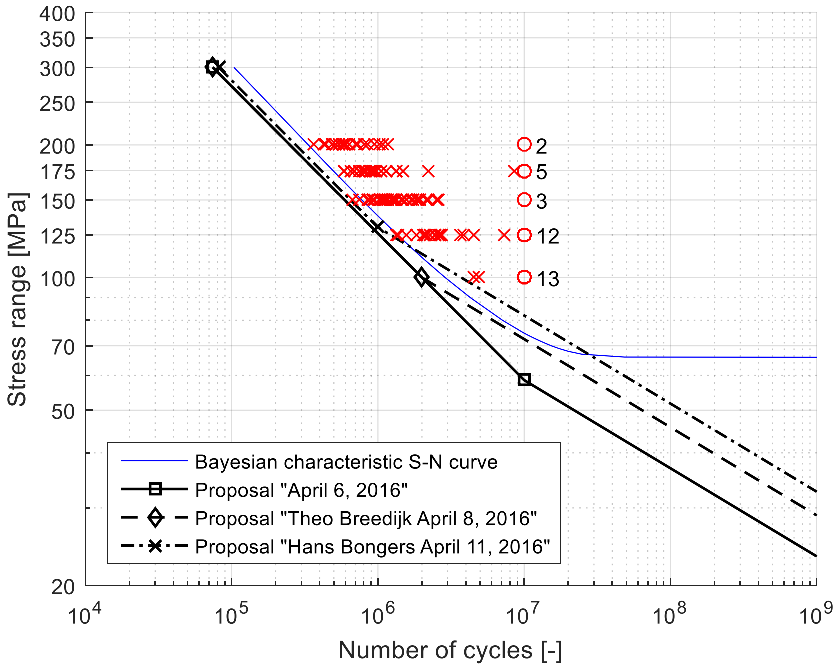

Figure 12. Proposed S-N curves - welded bars, diameter 12 mm.

For welded bars, diameter 12 mm, the proposals differ. Based on Figure 12, the values listed in Table 9 are proposed for the national annex to NEN-EN1992-1-1.

Table 9. Proposal for welded bars, diameter 12mm. Parameters N*, k1, k2 and $\Delta\sigma_{Rsk}$.

Diameter ϕ [mm]	**N***	**k_1**	**k_2**	**$\Delta\sigma_{Rsk}$ [MPa] Type B and C**
12	$2 \cdot 10^6$	3	5	100

5.6 Dataset welded bars, diameter 16 mm

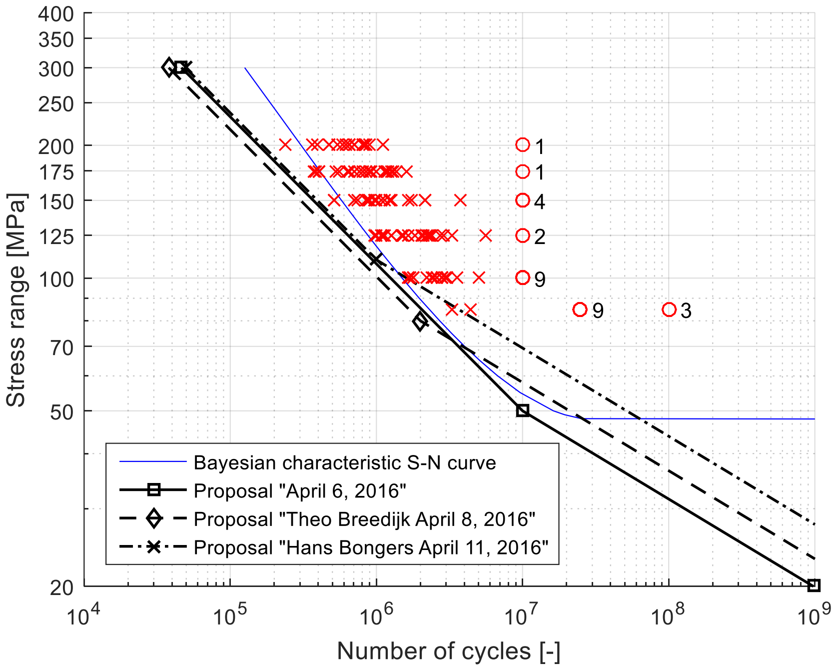

Figure 13. Proposed S-N curves - welded bars, diameter 16 mm.

For welded bars, diameter 16 mm, the proposals differ. Based on Figure 13, the values listed in Table 10 are proposed for the national annex to NEN-EN1992-1-1.

Table 10. Proposal for welded bars, diameter 16mm. Parameters N*, k1, k2 and $\Delta\sigma_{Rsk}$.

Diameter ϕ [mm]	**N***	**k_1**	**k_2**	**$\Delta\sigma_{Rsk}$ [MPa] Type B and C**
16	$2 \cdot 10^6$	3	5	80

5.7 Dataset welded bars, diameter 25 mm (failure at the weld)

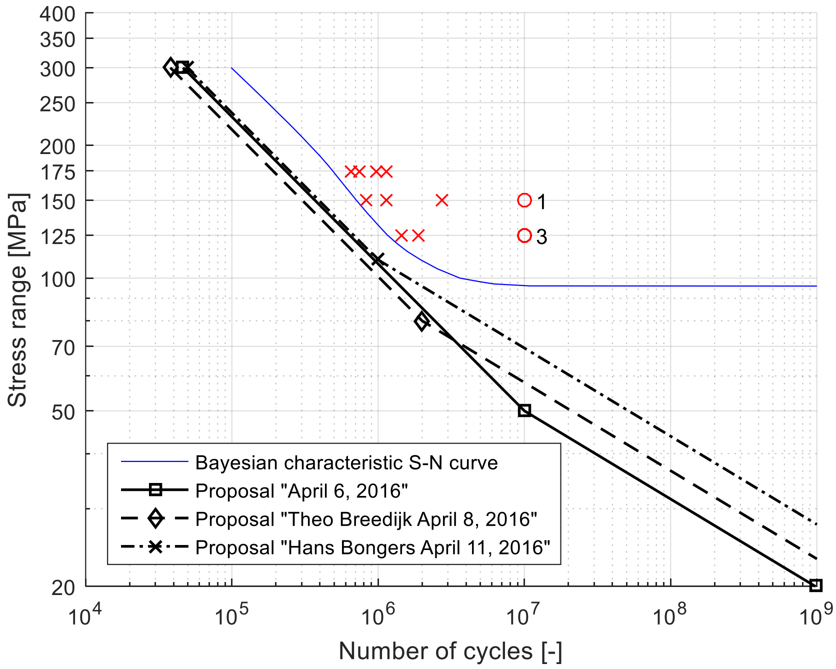

Figure 14. Proposed S-N curves - welded bars, diameter 25 mm (experimental data with failure at the weld).

For welded bars, diameter 25 mm, the proposals differ. Based on Figure 14, the values listed in Table 11 are proposed for the national annex to NEN-EN1992-1-1.

Table 11. Proposal for welded bars, diameter 25mm. Parameters N*, k1, k2 and $\Delta\sigma_{Rsk}$.

Diameter ϕ [mm]	N*	k_1	k_2	$\Delta\sigma_{Rsk}$ [MPa] Type B and C
25	$2 \cdot 10^6$	3	5	80

5.8 Proposal for the national annex to NEN-EN1992-1-1

Based on the previous paragraphs, the proposal for the national annex to NEN-EN1992-1-1 is summarized in the Tables 12 and 13.

Table 12. Proposed parameters N*, k1, k2 and $\Delta\sigma_{Rsk}$ (straight and straightened bars).

Diameter ϕ [mm]	**N***	**k_1**	**k_2**	**$\Delta\sigma_{Rsk}$ [MPa]**	
				Type A	**Type B and C**
$\phi \leq 12$	10^6	5	9	150	175
$12 < \phi \leq 16$	10^6	5	9	130	140
$16 < \phi < 25$	10^6	5	9	NA	140
$\phi \geq 25$	10^6	5	9	NA	140

Table 13. Proposed parameters N*, k1, k2 and $\Delta\sigma_{Rsk}$ (welded bars).

Diameter ϕ [mm]	**N***	**k_1**	**k_2**	**$\Delta\sigma_{Rsk}$ [MPa]**	
				Type A	**Type B and C**
$\phi \leq 12$	$2 \cdot 10^6$	3	5	100	100
$\phi > 12$	$2 \cdot 10^6$	3	5	NA	80

For comparison Table 14 shows the parameter values for S-N curves for reinforcing steel (straight bars and welded bars) in Table 6.3N of NEN-EN1992-1-1. These values are adopted in the present national annex to NEN-EN1992-1-1.

Table 14. Parameters for S-N curves for reinforcing steel in NEN-EN1992-1-1.

Type of reinforcement	**N***	**k_1**	**k_2**	**$\Delta\sigma_{Rsk}$ [MPa]**
Straight bars	10^6	5	9	162.5
Welded bars	10^7	3	5	58.5

6 References

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A Datasets

The datasets are restricted