Biography, CMA work, CMA interest, and References for the Special Interest Group on Theory of Characteristic Modes (http://www.characteristicmodes.org/about-us/)

Written by Dave Bekers, Radar Technology, TNO, The Hague, The Netherlands

Written for Buon Kiong Lau [e-mail of 31-03-2022], chairman of the CMA SIG group and professor at Lund University, Department of Electrical and Information Technology

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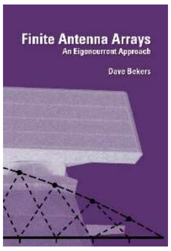
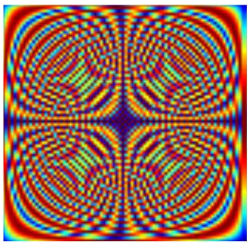




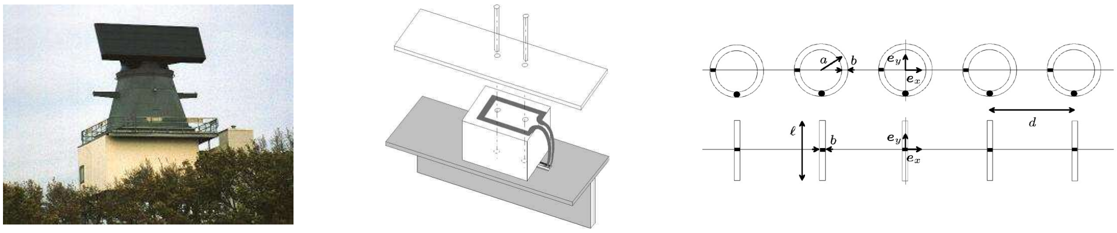

Summary of my work on CMA (Characteristic Mode Analysis and Eigencurrent Analysis)

My work story related to CMA started with modeling and analysis of phased array antennas for radar applications. Therefore, I will introduce my work from that perspective, as it is also summarized on the backside of my thesis.

On April 30, 1904, Christian Hülsmeyer patented his `Telemobiloskop', which became the first operational radar system for detecting ships through the transmission and reception of electromagnetic waves. Nowadays, radar systems are widely used, e.g., to control air traffic, to measure vehicle speeds, and to detect and track objects like ships and airplanes. Often, the transmit/receive unit of such a system is an array composed of separate antennas, their number varying from a dozen to thousands. An example is shown in Fig. 1. Since the development of antenna arrays is complex and costly, designs from simulations are made prior to the development.

Around August 2000, my final project for the post-master’s program Mathematics for Industry (MfI) at the Stan Ackermands Institute of the Eindhoven University of Technology (TU/e) started at the antenna and front-end group of Thales Nederland, Hengelo, The Netherlands. At that time, brute-force numerical approaches applied to a large array were still far too computationally expensive. Therefore, an array was, and still is, often considered as an infinite periodic structure, where symmetry is used to restrict the analysis to a single antenna of the array. This approach cannot completely describe the characteristic electromagnetic behavior of antenna arrays. In particular, it cannot predict the occurrence of standing-wave phenomena that may limit their bandwidth or that may be used for creating high/super gain/directivity with narrow bandwidth (high-Q). In my thesis, published in 2004 and online available, we propose an approach that describes the characteristic behavior of finite arrays accurately. Besides the prediction of standing-wave phenomena, the approach can indicate how these phenomena can be reduced for the entire scan range of the array. These aspects are detailed in [T1], Ch. 5 and 6, [J2], [J3], and [J4]. Ref. [T1] compares also with work of B.A. Munk et al. on low-Q resonances and with the modulated impedance oscillations observed by R.C. Hansen and C. Craeye. Ref. [J4] sheds more light on these modulated impedance oscillations, see also [J4], refs. [15] and [16]. Ref. [16] shows modulated impedance oscillations calculated by G. Fikioris, R.W.P. King, and T.T. Wu (1990) for a circular array of dipoles (no spectral decomposition, but trials to trace the resonances in forward simulations). Ref. [15] of Veremey (1995) links our work to a famous paper of C.J. Bouwkamp and N.G. De Bruijn demonstrating that there is no theoretical limit on the directivity obtainable by a continuous line source of a given length. [C.J. Bouwkamp and N.G. de Bruijn, The Problem of Optimum Antenna Current Distribution, Eindhoven, The Netherlands, June 1945].” This paper inspired many researches on high-gain and super-directive antennas.

In our approach, we describe the behaviour of an array by its `eigenvibrations' or eigencurrents. These eigencurrents are the eigenfunctions of the impedance operator that relates the currents on the separate antennas to their excitation fields. From a physical point of view, the eigencurrents are standing waves of the array. The concept of eigencurrent appears rather useful for the design, because eigencurrents are one-to-one related to properties of the array, like sum patterns, difference patterns, grating lobes, modulated impedance oscillations, and impedance variations attributed to surface waves, see Figs. 2 and 3 below. Besides a physical interpretation, the approach with eigencurrents leads to rapidly executable simulations; for, although the performance parameters of an array vary as a function of the geometry parameters, the eigencurrent expansions vary much less. Moreover, eigencurrents of (large) arrays are approximated as compositions of a small number of eigencurrents of the individual antennas in an array.



(a)

Fig. 1: (a) Smart-L around 2000 / 2001 on a test tower at Thales Hengelo, The Netherlands, see [T1], p. 1. (b) Sketch of a single antenna element as depicted in [T1], p. 2. (c) Linear arrays of microstrip loop antennas and microstrip dipoles with varying excitations (from [J4]). The loops and dipoles are excited by voltage gaps. For the (H-plane-oriented) dipoles, the positions of these gaps are indicated by black rectangles. For H-plane-oriented loops, the positions are similarly indicated, while for E-plane-oriented loops, they are indicated by black circles.

The eigencurrent approach was tested for the canonical examples of linear arrays of microstrip loops and dipoles. However, the approach is suitable for other types of elements and array geometries as well. As an example Fig. 4 shows the modal changes over an octave bandwidth as well as the relative differences (in 2-norm of the residual current) between the eigencurrent approach, the moment method, and the CBFM for different numbers of coupled groups of eigenvalues and different numbers of CBFs.

If one wonders what this story has to do with CMA, I can give a little historical and a technical account. The technical one is that on p. 115 on my PhD thesis [T1], the difference with the approach of Mautz and Harrington (1971 – 1976) is detailed, where also the presumably earlier work of Garbacz and Turpin is briefly described. In simple words, we consider the spectral decomposition of the impedance operator $\mathcal{Z}$ in $\mathcal{Z}\boldsymbol{J} = \boldsymbol{E}^{\mathrm{ex}}$, thus $\mathcal{Z}\boldsymbol{J} = \nu\boldsymbol{J}$, while they write $\mathcal{Z} = \mathcal{R} + j\mathcal{X}$ and get the weighted eigenvalue-equation $\mathcal{X}\boldsymbol{J} = \lambda\mathcal{R}\boldsymbol{J}$, where $\nu = 1 + j\lambda$. With respect to our formulation, they have two advantages:

- Since $\mathcal{X}$ and $\mathcal{R}$ are real and symmetric, they are self-adjoint, and hence, a countable spectrum is guaranteed.
- Not only the characteristic currents are orthogonal, also their far fields are.

Note that a more detailed comparison can be found in [T1], p. 115. These aspects were considered by us when Stef van Eijndhoven showed me in August 2000 a book of Raj Mittra with an overview paper of Harrington and said that that was what he wanted to do, but a bit different. The idea of expressing "array characteristic modes" in terms of "element characteristic modes" was already shaped a bit, but I had to make first a forward simulator. In a dual-level decomposition approach from element to array, it is not certain that one can maintain orthogonality. The first aspect was discussed by us while doing the numerics and also raised during the ICEAA 2003 conference by prof. Bucci. Standard theory says that also normal operators have a countable spectrum. Numerically we could observe that the impedance operators for a microstrip loop and a microstrip dipole with width-averaged currents, , represented in finite bases, are almost normal. Additionally, the microstrip loop with width-averaged current has an analytically known spectrum (with eigenfunctions $\cos n\varphi$ and $\sin n\varphi$). In 2010, Stef van Eijndhoven proved also the countability of the spectrum of the impedance operator of the microstrip dipole (at least, the 'real part' of the integro-differential operator with logarithmically singular displacement kernel). The work was published in 2012, see [J1], where we supplemented Stef's proof of countability, and eigenvalue upper and lower bounds, by numerical results, a literature review, and an introduction relating the work to [J2], [J3], and [J4]. That was not sufficiently applied for SIAM applied analysis and therefore we published in the end in PEMS, the journal in which J.B. Reade had posted his proof for the logarithmic kernel in 1979.

In the PhD thesis [T1] we linked our work to the following areas (apart from CMA): 1) Analysis and simulation of finite antenna arrays, including also the methods, which generate aggregate or entire-domain basis functions from local basis functions, see [T1], pp. 9 – 12. 2) Sturm-Liouville problems and waveguide modes, which are eigenfunctions of the Helmholtz operator, pp. 114-115. 3) Quantum mechanics with the eigenfunctions and eigenvalues of the Hamilton operator, which are states of a system of particles and the related energy levels[1], see p. 115 of [T1]. 4) Theory of normal and non-normal operators, topological analysis, and pseudo-spectra of linear operators, and the (Multi-level) Fast multipole method, see [1], p. 116, and Refs. [75] by R.V.N. Melnik and [115] by L.N. Trefethen. 5) Theory of Laurent, Toeplitz, and circulant matrices and operatorspp, and their spectra , see e.g. [T1], p. 84, 171-172.

For characteristic figures of eigencurrent expansions on linear arrays and their far fields, we refer to [J3], Figs. 2, 4 and 5. For surface-wave behaviour on finite or truncated periodic arrays (no dielectric substrate waves !), we refer to [J4], Figs. 2 – 5. For the computational performance of the eigencurrent approach with respect to the classical moment method and CBFM, we refer to [J2], Figs. 3 – 6. Most of these, but not all, can also be found in [T1] of course. For completeness, some graphics has been added to the end of this document, see Figs. 2, 3, and 4.

My view of CMA and Eigencurrent/Eigenmode Analysis

Most people in the CMA community refer to the Mautz and Harrington formulation, which has been presented in many papers by them between approximately 1971 and 1976, with revival in the eighties for e.g. aperture problems and electrically-small conducting bodies. However, there are various other spectral decomposition works, which can be mentioned and used: characteristic currents for wires (Garbacz, Turpin, end sixties), SEM and EEM (Baum, 1976, Ramm, 1980, 1982), BI-RME (Conciauro, Bressan, Perregrini, Arcioni, Bozzi, and others, 1994 onwards), non-linear eigenproblems (Guillaume, 1999), spectral analysis of resonating and non-resonating structures (e.g. Chernokozhin, Shestopalov, 2001, Shestopalov, Okuno, Kotik, 2003, and Svishchov, 2008), spectral analysis of the reduced kernel in Floquet expansions (Monni, 2004, van Eijndhoven, Monni, Bekers, 2007), scatttering operator eigendecomposition (Morvan et al. 2005, 2016 or later), eigen decomposition (Fischer, Yagle, and Volakis, 2005), resonances in arrays of concentric strips (Li, Scharstein, 2006), dispersion characteristics of arrays at the interface between two half spaces (Neto, Gerini, Bruni, Maci, 2007), scattering by infinite and semi-finite arrays (Thompson, Linton, Porter, 2008), pseudo-potentials (Psarros, Fikioris, 2009), eigenvectors of the Fourier transform (Horn, 2010), combination of eigencurrents and LEGO (of A.M. van de Water, et al., 2005) approaches (Lancellotti et al., 2009 onwards).

Secondly, one could pay more attention the fundamental differences between e.g CMA and the eigencurent approach. Some differences have been mentioned before and in [T1]. We can also mention the following. There are two types of resonances: radiation resonances and standing-wave resonances. In CMA they turn up as two different “classes” of eigenvalues. A radiation resonance turns up as $\lambda = 0$ and thus $\nu = 1$, while a standing-wave resonance turns up as $\nu = 0$ and thus $\lambda = j$ (λ and ν have parameter dependence: frequency and geometry parameters). A standing-wave resonance in the eigencurrent approach turns also up for $\nu = 0$, but a radiation resonance turns up as some real ν (not necessarily 1).

Thirdly, the relation to operator and matrix structures such as Toeplitz, circulant, and real symmetric can be investigated to get more theoretical grip on the matter. Also, the relation with the Riemann hypothesis can be investigated. Recently, I was pointed to the following website by my colleague Jeroen Boschma: https://phys.org/news/2022-01-quantum-zeta-epiphany-physicist-approach.html . Particularly the text about interaction and exchanging an infinite set of … particles … looks like expression array eigencurrents in terms of element eigencurrents. Moreover, it also matches the link we described in [T1] in 2004 with the book of Gasiorowicz, see above. CMA has also a link with the Riemann hypothesis, but via its eigenvalues $\nu = 1 + j\lambda$, which show points on the critical line when multiplied by $1/2$. So, it seems one does not need to dive into quantum mechanics, but can also look at e.g. electromagnetics and antennas at RF (both deal with the wave equation).

After little references from the CMA world over the past 13 to 18 years, we have been prominently referenced in the paper Characteristic Mode Analysis of Mutual Coupling (Feb. 2022) by S. Ghosal, R. Sinha, A. De, and A. Chakrabarty (Refs. [1] and [2]). If we want to accelerate research, we should view the methods mentioned above as belonging to the same family of mathematical-physical problems driven by the

[1] Like our dual-level spectral decomposition, these eigenvalues are also perturbed with respect to some unperturbed state of non-interacting particles. At least when considering Ref. [39] of [T1] (Gasiorowicz), these perturbations seem to be small and one uses asymptotics to determine the perturbed eigenvalues. In our case, the perturbations seem to be larger due to the larger coupling levels. Also, the Hamilton operator is self-adjoint in contrast to our impedance operator.

wave equation and handled by spectral analysis. Note that during my PhD work we were unaware of the work of M. Cabedo Fabres et al. who started to revive CMA around 2002, the same year as we started investigating eigencurrents. In retrospect, we should have referenced from our 2009 paper in IEEE Trans. A&P paper, but we did not browse the entire literature anymore, also because we related particularly to the aggregate function approaches, of which we referenced many. Moreover, the connection with quantum mechanics was in the original version of that paper, but a reviewer did not like that. In 2015 and recently we browsed again the literature. The recent search gave at least 12 papers, next to rather extensive special issues in e.g. 2016 (the year after our literature review). High-level impression from those 12 papers:

- CMA is being used for simulations of a connected array. At TNO The Hague we had the connected array of A. Neto, D. Cavallo, et al. around 2007.
- CMA is being used for simulations of large platforms, also by Chinese groups.
- CMA is being used for simulations of dipole arrays. Presumably this effort by T. Lonsky, P. Hazdra, and J. Kracek (2018) is one of the closest to our research in the years 2002 till 2004, and further elaborated from 2005 till 2009.

The following list of journal papers, conference papers, conference abstracts, and presentations contains only work related to the Eigencurrent Approach and Characteristic Mode Analysis (CMA) / Characteristic Mode Theory (CMT).

Statement of Interest

Citing from the preface of [T1]: “One of the strongest unifying concepts in mathematics is the concept of eigenvalue. As L.N. Trefethen wrote in […] wrote: ‘They [Eigenvalues] give an operator a personality.’ Represented in the complex plane they are much easier to digest by the human brain than the abstract notion of an operator that describes a certain process or phenomenon. Moreover, eigenvalues may provide insight into physical phenomena like resonance, stability, and rate of increase or decay. More specifically, in mechanics, eigenvalues may determine under which conditions a bridge will collapse or a music instrument will give a proper sound. In electromagnetism, they may determine whether a certain signal is propagating. …”.

Based on the previous cite and our summary and view, our interests can be summarized as follows:

- The phenomenology of eigencurrents and characteristic modes in 1D and 2D arrays, including resonance effects and high/super-gain / high-Q operation.
- Bridging the differences between these two formulations as already started in [1], as well as the difference between these two and other types of analyses described before, including spatial versus Fourier-domain formulations.
- Studying aspects like mode ordering, parameter dependence (frequency, geometry), as done in [1] for 1D arrays.
- Using the knowledge build up in the previous items for optimization / synthesis problems (on e.g. beam-pattern characteristics) and for potential combination of CMA / eigencurrent analysis with signal representation, optimization, and processing.
- Extending our views on bridges between CMA / eigencurrent analysis, aspects of mutual coupling, quantum mechanics, and the Riemann zeta function.

References

Journal papers

[J1] D.J. Bekers and S.J.L. van Eijndhoven, “Spectral Properties of Integral Differential Operators Applied in Linear Antenna Modeling,” *Proceedings of the Edinburgh Mathematical Society*, Vol. 55, No. 2, pp. 333 – 354, June 2012.

[J2] D.J. Bekers, S.J.L. van Eijndhoven, and A. G. Tijhuis, “An Eigencurrent Approach for the Analysis of Finite Antenna Arrays,” *IEEE Transactions on Antennas and Propagation*, Vol. 57, No. 12, pp. 3772 – 3782, Dec. 2009.

[J3] D.J. Bekers, S.J.L. van Eijndhoven, and A. G. Tijhuis, “An eigencurrent description of finite arrays of electromagnetically characterized elements,” *Radio Science*, 44, 2009, RS2S90, doi:10.1029/2007RS003797 (Invited).

[J4] D.J. Bekers, S.J.L. van Eijndhoven, A.A.F. van de Ven, P-P. Borsboom, and A.G. Tijhuis, “Eigencurrent Analysis of Resonant Behavior in Finite Antenna Arrays,” *IEEE Transactions on Microwave Theory and Techniques*, Vol. 54, No. 6, Part 2, pp. 2821 – 2829, June 2006.

Conference papers

[C1] D.J.Bekers, S.J.L. van Eijndhoven, A.G. Tijhuis, “Frequency and Element Independent Behavior of Array Eigencurrents and its Application in Finite-Array Analysis,” *EMTS 2007* (International URSI - Commission B Electromagnetic Theory Symposium), Ottawa (Canada), 26 - 28 July 2007. (3 pp.) (Invited)

[C2] G. Addamo, D. Bekers, A.G. Tijhuis, B.P. de Hon, R. Orta, R. Tascone, “An Eigencurrent Approach for the Analysis of Leaky Coaxial Cables,” *Proceedings of the European Conference on Antennas and Propagation (EuCAP)*, Nice (France), 6 - 10 November 2006. (4 pp.)

[C3] D.J. Bekers, S.J.L. van Eijndhoven, A.A.F. van de Ven, P-P. Borsboom, A.G. Tijhuis, “Analysis of Resonant Behavior in Planar Line Arrays of Rings by the Eigencurrent Approach,” *Proceedings of the European Microwave Week*, Paris (France), October 2005. (4 pp.)

[C4] D.J. Bekers, S.J.L. van Eijndhoven, A.A.F. van de Ven, P-P. Borsboom, A.G. Tijhuis, “Modeling and Analysis of Finite Phased Arrays of Microstrip Antennas - An Eigenvector Approach,” *Proceedings of the International Conference on Electromagnetism in Advanced Applications,* Torino (Italy), September 2003, ISBN 88-8202-008-8. (4 pp.)

[C5] (abstract) D.J. Bekers, “Model Order Reduction for Large Antenna Arrays: The Eigencurrent Approach,” *Workshop on Model Order Reduction, Coupled Problems, and Optimization*, Leiden (The Netherlands), 19 -- 23 September 2005.

[C6] (abstract) D.J. Bekers, S.J.L. van Eijndhoven, A.A.F. van de Ven, P-P. Borsboom, A.G. Tijhuis, “An Eigencurrent Approach for Simulation and Explanation of Finite-Array Behavior,” *ESA Antenna Workshop on Space Antenna Systems and Technologies*, Noordwijk (The Netherlands), 31 May -- 3 June 2005.

Thesis

[T1] D.J. Bekers, *Finite Antenna Arrays: An Eigencurrent Approach.* PhD Thesis, Technische Universiteit Eindhoven, Eindhoven (The Netherlands), 2004, ISBN 90-386-1012-2. (269 pp.) [Online: https://pure.tue.nl/ws/portalfiles/portal/1982756/200411410.pdf]

Short biography

Dave J. Bekers (Senior Member, IEEE) received the M.Sc. degree in mathematics and the P.D.Eng. and Ph.D. degrees from the Technische Universiteit Eindhoven, Eindhoven, The Netherlands, in 1999, 2001, and 2004, respectively. He carried out his final project for the post-graduate program “Mathematics for Industry” and his Ph.D. project with Thales Nederland, Hengelo, The Netherlands, in the field of array antennas. Since November 2004, he has been with TNO, The Hague, The Netherlands, organization for applied scientific research, where he first worked in (antenna) design, modeling, simulation, and optimization, and guided a few projects in the field of mm-wave and THz antenna technology. In the past ten years, he worked mainly in the fields of radar signal processing and phased-array antennas on topics, such as beamforming, target detection, parameter estimation, and novel waveforms. Some important side topics were in the field of magnetic signatures (coil-layout optimization for degaussing) and decision making. Additionally, he had the technical guidance at TNO for a few activities carried out within D-RACE, the Dutch Radar Centre of Expertise, a strategic alliance between TNO and Thales Nederland B.V.

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Stef J.L. van Eijndhoven (former director of MfI at TU/e, former director of Data Science PDEng program at JADS and TU/e), Alphons A.F. van de Ven (former associate professor of continuum physics at the Applied Analysis group of the Mathematics and Computer Science Department of the TU/e), Anton G. Tijhuis (former chair of electromagnetics at the electrical engineering department of the TU/e), Peter-Paul Borsmboom (former member of the antenna section of Thales Nederland), Evert W. Kolk (former R&D lead at the antenna section of Thales Nederland, Willem Dijkstra (internship student of MfI at the TU/e), Arnold Kooiker, (internship MSc. Student of the University of Twente).

Also, I would like to thank my colleagues and former colleagues at TNO and several other institutes, universities, and companies, among others, TU/e, Thales Nederland, EPFL Lausanne, Chalmers

Gothenburg, KU Leuven, University of Pisa, DRDC, ANSYS Lyon, Erasmus MC, ESA ESTEC, SRON, TU Delft, Thales Alenia, University of Twente, DTU Copenhagen, BESI, Speyer Germany.

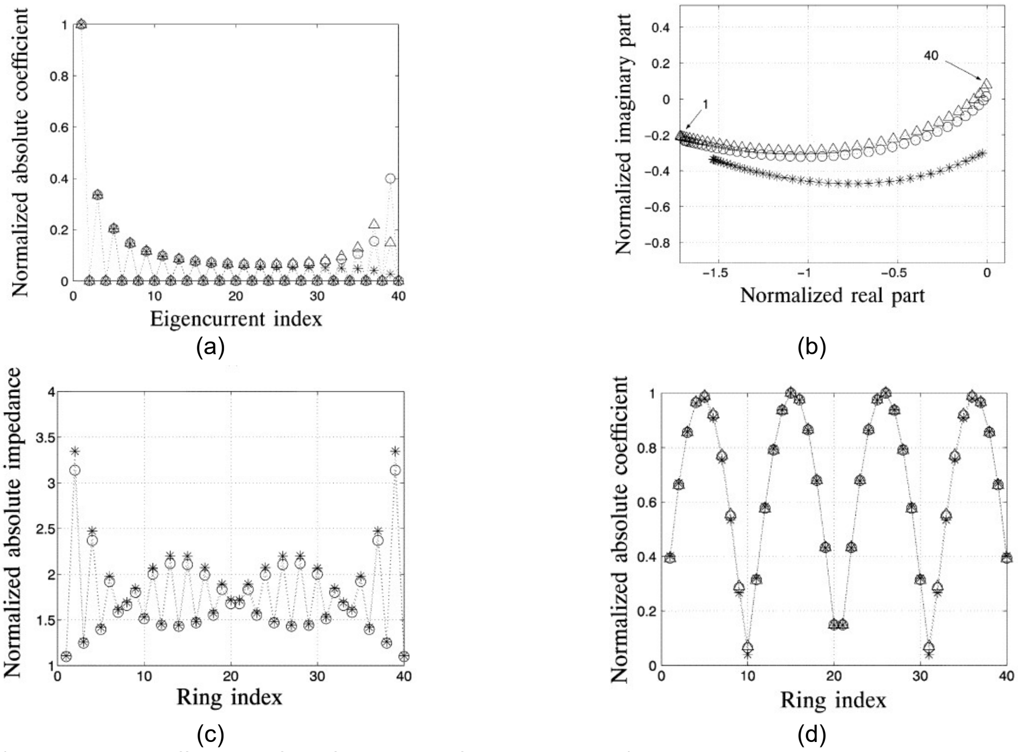
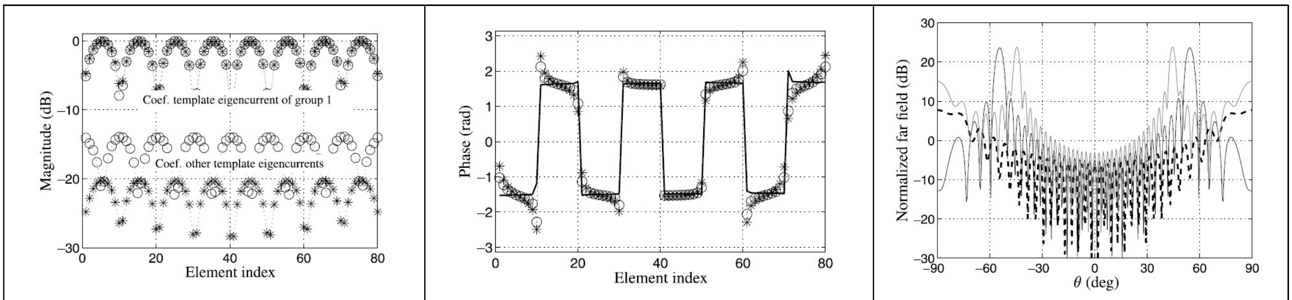
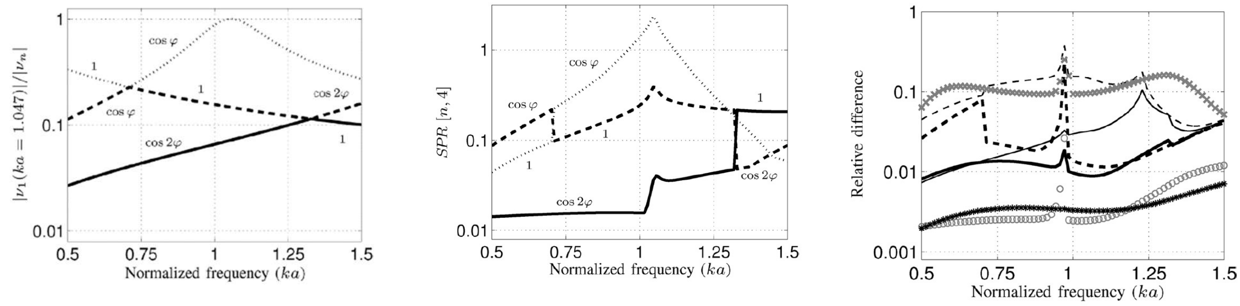


Fig. 2: (a) Expansion coefficients of the first group of eigencurrents (related to the dominant eigenvalue and eigencurrent of a single loop) of a uniformly-spaced linear array of microstrip loops for three different frequencies ($ka = 1.0378$ (triangle), $ka = 1.0441$ (circle), $ka = 1.0786$) (square). (b) Corresponding eigenvalues in the complex plane. (c) (Normalized) Amplitude of the impedances on the 40 loops. (d) Normalized amplitudes of the dominant expansion coefficients of the first group of eigencurrents. Graphics from [J4].



(a)	(b)	(c)

Fig. 3: (a) Magnitudes and (b) phases of the expansion coefficients (in dB) of array eigencurrent 73 in Group 1 for linear arrays of 80 loops (circles) and 80 dipoles (asterisks) with spacing $d = 0.38\,\lambda$. The coefficients with respect tothe template eigencurrents u_1 and u_2 are indicated by circles and asterisks, respectively. Phases are shown for the template eigencurrent of the group only; pth coefficient multiplied by $\exp(j(p-1)\pi)$. Solid black curve, unprocessed phases of array eigencurrent eight for both linear arrays. (c) Normalized far-field patterns (Ecomponent) of array eigencurrent 33 in Group 1 for the linear array of 40 loops in Figure 3. Dashed, $d = 0.38\,\lambda$; solid black, $d = 0.5\,\lambda$; solid grey, $d = 0.58\,\lambda$. Normalization: maximum E_ϕ magnitude, in the xz plane, of the template eigencurrent of Group 1.



(a) (b) (c)

Fig. 4: (a) $|\nu_1(ka = 1.047)|/|\nu_n|$ for $n = 1$ (dotted curve), $n = 2$ (dashed curve), $n = 3$ (solid curve) on a logarithmic scale as a function of the frequency for the loop in the array corresponding to Fig. 3 in [J2]. The corresponding eigencurrents are indicated on the curves. (b) The eigenvalue spreads of the first, second, and third groups denoted by $SPR[1,4]$ (dotted curve), $SPR[2,4]$ (dashed curve), and $SPR[3,4]$ (solid curve) for the same array. (c) Relative difference between eigencurrent and moment solutions, denoted by $DIF[N_{\mathrm{cpl}}]$, as a function of the frequency for the same array, for $0°$ (thick curves) and $45°$ (thin curves) of scan and for $N_{\mathrm{cpl}} = 1$ (dashed curves) and $N_{\mathrm{cpl}} = 2$ (solid curves), where N_{cpl} is the number of groups of eigenvalues/eigencurrents with mutual coupling. Symbols: $DIF[3]$ for $0°$ of scan (black *) and the relative difference between the MoM and the CBFM for two (gray x) and three (gray ○) CBFs for $0°$ of scan. Graphics from [J2].