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Security constrained unit commitment with continuous time-varying reserves

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ABSTRACT

This paper presents a security constrained unit commitment (SCUC) with continuous intra-hour time-varying reserves. The hourly formulation extends the power-based UC formulation and has reserves which vary continuously within the hour, as opposed to the traditional hourly energy-based SCUC that uses constant reserves within the hour. We show that the traditional hourly energy-based formulation cannot ensure N-1 security at all times, since this formulation is not able to take the power trajectories of units within the hour into account. This is remedied by an hourly power-based version which allows the formulation of contingency constraints to guarantee N-1 security at all times within the hour. The proposed formulation uses continuous time-varying reserves which lowers the cost for providing reserves and makes better use of units' flexibility while still ensuring N-1 security. The energy-based and power-based formulations are evaluated using different versions of a 5-min security-constrained economic dispatch (SCED) based on real load data, thus simulating the real time operation of the system under different assumptions for reserve procurement. The results show that the power-based formulation increases security compared to the energy-based formulation, both if reserves are fixed to the values from the SCUC or co-optimized in real time by the SCED.

1. Introduction

The unit commitment (UC) problem is widely recognized as the most efficient method for weekly and day ahead planning in power systems [1–3], and is used by many system operators, especially in the US, for system operation and market clearing [4,5]. N-1 security constraints are commonly used to guarantee that the dispatch schedule given by a UC is robust against contingencies, i.e., that the remaining generators can be re-dispatched within a given time period to compensate for any lost generation or line outages, and to bring the flow on transmission lines within limits [6–10]. This problem is known as security constrained unit commitment (SCUC). Notice that here we use the term SCUC to refer to UC formulations that explicitly model the dispatch schedule under both generator and transmission line outages, i.e., N-1 security, which is the focus of this paper.

1.1. Energy-based SCUC and its limitations

In this section we describe the limitations that arise when using the traditional energy-based UC formulation to provide N-1 security. The traditional way of formulating the UC problem describes the production of units as staircase profiles using discrete energy blocks. For a SCUC, this type of formulation will underestimate the reserves needed to secure against unit outages, as illustrated in Fig. 1. Let us assume that unit 1 has to ramp up by 100 MW during hour t to follow the load, while unit 2 produces at 50 MW and provides reserves to secure against the loss of unit 1. Using the energy-based formulation, the energy production of unit 1 during hour t is 150 MWh. Thus unit 2 must hold 150 MW of reserves to compensate for the possible outage of unit 1. However, as unit 1 is ramping up, its production will be above 150 MW for the second half of the hour and hence the reserves held by unit 2 will be insufficient. The total reserve deficit is shown by the dark shaded triangle in Fig. 1.

A second problem concerns the representation of ramps in the energy-based formulation. Suppose also that unit 1 must provide reserves to secure against the outage of unit 2, and that the maximum ramp rate of unit 1 is 100 MW/h. Since unit 1 is ramping at its maximum ramp capability, it will not be able to provide any reserves during hour t. However, the energy ramp is only 50 MWh/h, and hence the energy-based formulation may schedule unit 1 to provide 50 MW of reserves which it cannot deliver.

Thus the energy-based formulation cannot provide sufficient reserves and ensure that these are available given the power trajectories of units. Furthermore, the energy-based formulation does not specify what

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Nomenclature

Indices

$\mathscr{C}$	contingency scenarios, indexed c
$\mathscr{C}_g$, $\mathscr{C}_l$	generator/line outage contingencies
$\mathscr{G}$	all generator units, indexed g
$\mathscr{G}^f$, $\mathscr{G}^s$	fast/slow-start units
$\mathscr{G}^{ns} \subset \mathscr{G}^f$	units able to provide non-spinning reserves
$\mathscr{G}_n$	units at bus n
$\mathscr{N}$	network buses, indexed n
$\mathscr{T}$	time periods, indexed t

Parameters

C_g^{NL}	no load cost for unit g [$/h]
C_g^{LV}	variable cost for unit g [$/MWh]
C_g^{SU}, C_g^{SD}	startup/shutdown cost for unit g [$]
C_g^{S+}, C_g^{S-}	up/down spinning reserve cost for unit g [$/MW]
C_g^{NS}	non spinning reserve cost for unit g [$/MW]
D_t^e	energy load during hour t [MWh]
D_t^p	power load at end of hour t [MW]
$\overline{F}_l$	capacity limit for line l [MW]
$\overline{P}_g, \underline{P}_g$	maximum/minimum generation level for unit g [MW]
P_{gi}^{SU}	production at start of i^{th} hour of startup period for unit g [MW]
P_{gi}^{SD}	production at start of i^{th} hour of shutdown period for unit g [MW]
$\overline{R}_g^{ns}$	maximum limit for non-spinning reserves for unit g [MW]
RU_g, RD_g	ramp up/down capability of unit g [MW/h]
SU_g, SD_g	startup/shutdown ramp capability of unit g [MW/h]
SU_g^D, SD_g^D	duration of startup/shutdown period for unit g [h]
TU_g, TD_g	minimum up/down time for unit g [h]
τ^s	deployment time for spinning reserves [min]
Γ_{ln}	distribution factor for line l and bus n [p.u.]
Γ_{ln}^c	distribution factor for line l and bus n with line c outaged [p.u.]

Binary variables

u_{gt}	commitment variable for unit g at hour t
v_{gt}	startup variable for unit g at hour t
z_{gt}	shutdown variable for unit g at hour t
u_{cgt}^{ns}	commitment of non-spinning reserves for contingency c and unit g at hour t

Continuous variables

e_{gt}	energy output above $\underline{P}_g$ from unit g during hour t [MWh]
p_{gt}	power output above $\underline{P}_g$ from unit g at end of hour t [MW]
r_{gt}^{s+}, r_{gt}^{s-}	up/down spinning reserves from unit g at hour t [MW]
r_{gt}^{ns}	non spinning reserves from unit g at hour t [MW]
$\delta_{cgt}^{e,s}$	spinning reserve deployment (in energy) from unit g for contingency c during hour t [MWh]
$\delta_{cgt}^{p,s}$	spinning reserve deployment (in power) from unit g for contingency c at end of hour t [MW]
$\delta_{cgt}^{e,ns}$	non-spinning reserve deployment (in energy) from unit g for contingency c during hour t [MWh]
$\delta_{cgt}^{p,ns}$	non-spinning reserve deployment (in power) from unit g for contingency c at end of hour t [MW]

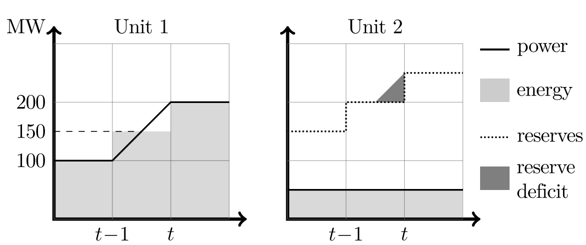


Fig. 1. Energy-based UC with N-1 security. Time labels mark end of hour.

the power trajectories of units are within the hour. For a given energy profile there is an infinite amount of power profiles that may impose different reserve requirements. Fig. 2 shows another power profile for unit 1 which corresponds to the same energy profile as the example in Fig. 1 but requires more reserves to satisfy N-1 security. Hence, the inability of the energy-based formulation to distinguish between different power trajectories means that it is not able to know how much reserves are needed.

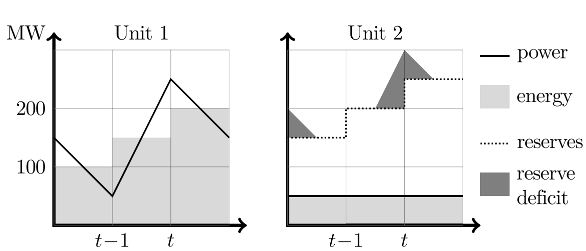


Fig. 2. Energy-based UC with N-1 security and higher reserve deficit.

In summary, formulating the SCUC using the traditional energy-based formulation leads to the following short-comings related to scheduling reserves:

1. It will schedule insufficient reserves to cover the loss of units that are ramping up or down.
2. It may schedule reserves which the units are unable to deliver due to incorrect representation of ramping resulting from ramp constraints being applied to discrete energy-blocks.
3. It does not include power trajectories for units within the hour, meaning that it does not include the information needed to know how much reserves are required to cover the outage of a specific unit.

Apart from the problems mentioned above, the energy-based approach does not guarantee the existence of a power trajectory to satisfy the resulting energy profile. This can, for example, lead to a unit being unable to meet its scheduled energy delivery due to ramp restrictions, as has been discussed in [11,12].

Several UC formulations have been proposed to overcome the limitations of the energy-based formulations [13–16]. These power-based formulations explicitly account for the power trajectories of units and enforce ramp constraints on instantaneous power quantities rather than averaged energy quantities. While it is possible to decrease the short-comings of the energy-based formulation by using shorter time intervals, this comes at the price of increased computational burden. We refer the reader to [17] for a study of how increasing the time resolution effects the performance and computational complexity of UC formulations.

In general it is of interest for system operators to be able to solve the UC problem more efficiently, which has motivated research into UC formulations and solution techniques that reduce computation times without sacrificing modelling accuracy [18–21]. As the methods for solving the UC problem get more efficient, system operators and researchers want to include more features into the formulations, e.g., more accurate modelling of post-contingency transmission constraints [22] and line switching [8], stochastic/robust optimization [23], or ac transmission constraints [24]. Some operators also run UC problems every hour, e.g., the ERCOT hourly reliability unit commitment, which incorporates N-1 security and is run every hour with a maximum planning horizon extending to the end of the next day [25], putting stringent requirements on solution times. At the same time, many of the techniques proposed for reducing the solution time can also be applied to power-based formulations [18–20]. Hence, power-based formulations are of interest if they can improve the modelling accuracy without increasing the computational burden.

1.2. Power-based SCUC and contributions

Different power-based formulations have been proposed in the literature. A formulation with intra-hour resolution was proposed in [14], where units ramp to their specified production levels during the first part of the hour. While correctly representing unit ramp capabilities, this formulation suffers from the problem of increased computational burden due to the intra-hour resolution and several constraints that require linearization, and also only enforces energy-balance during the hour, as opposed to enforcing continuous power balance. Wu *et al.* [13] proposed a formulation with linear ramping during hours, but not accounting for production during startup and shutdown.

A more rigorous approach to power-based unit commitment has been formulated in [15,16,26]. They consider fast-start and slow-start units and their production during the startup and shutdown periods. Moreover, the core UC formulations have been proven to be the tightest possible representation of the feasible region for unit operation, i.e., the convex hull, thereby decreasing the MIP solution time [16].

Recently another approach to power-based UC which uses cubic splines to model unit trajectories has been proposed in [27] and extended in [28] to consider transmission constraints and stochastic optimization.

In this paper, we formulate a SCUC using the power-based formulation in [15,16]. Thus, unlike previous power-based formulations, we consider reserves and post-contingency transmission constraints to ensure system security. Since the formulation includes explicit piecewise linear intra-hour trajectories for units, we are able to formulate security constraints which guarantee that enough reserves are available to cover the outage of any unit or transmission line at all times within the hour, and that the scheduled reserves are feasible. Notice that the assumption of linear production trajectories is key to guaranteeing N-1 security within the whole hour, as this means there is a unique trajectory between the instantaneous power values at the beginning and end of the hour. Thus guaranteeing N-1 security using cubic splines as in [27,28] would likely be more difficult.

Furthermore, the proposed formulation has time-varying reserves within the hour, see Fig. 3. In this way the reserves are specified instantaneously as power values at the end of every hour and vary linearly within the hour, creating a continuous piecewise linear reserve trajectory similarly to the power profiles. As shown in Fig. 3 the reserves

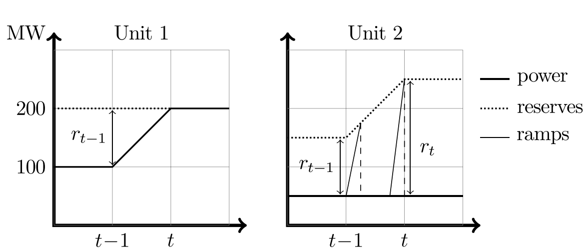


Fig. 3. Power-based UC with variable reserves and N-1 security.

held by unit 2 can now follow the power trajectory of unit 1, guaranteeing N-1 security while reducing the total amount of reserves held. With time-varying reserves it is also possible for a unit to provide reserves even when it starts/ends an hour at maximum capacity, so the capacity of units is used more efficiently. For example, assuming the maximum capacity of unit 1 is 200 MW, this unit can still provide the reserves shown in Fig. 3 during hour t, which would not be possible if reserves were constant throughout the hour.

The formulation of continuously time-varying reserves is a major difference from hourly energy-based formulations where reserves are constant during the hour [7–10]. Previous versions of the power-based UC problem have also used constant reserves [15]. Here we instead propose to let the reserves vary continuously within the hour, and implement the power-based SCUC using this approach.

To evaluate the formulations we use a 5-min resolution security constrained economic dispatch (SCED), thus simulating the real time operation of the power system subject to all possible contingencies, given the commitment decisions and reserve schedules from the SCUC. The methods for procuring reserves vary between different systems. Most U.S. ISOs co-optimize energy and reserves in the real time markets [29], while other operators in North America such as AESO [30] procure reservers day-ahead or earlier in separate markets. The latter is also the case in most European countries [31]. To account for the range in methods by which different systems procure reserves, we use two different SCED formulations. In the first formulation reserves are fixed to the values obtained from the day-ahead SCUC, to represent the more inflexible procurement of reserves, e.g., day ahead, and in the second formulation reserves are re-optimized in the real time SCED, to represent the most flexible case when reserves are procured in real time without cost penalizations for deviating from earlier reserve schedules. In this way we provide results which are relevant for a broad range of market designs.

Thus the main contributions of this paper are:

1. We propose a power-based SCUC formulation that guarantees that the reserves held at any point in time within the hour are sufficient to cover the loss of any unit or transmission line, as opposed to the traditional energy-based formulation.
2. The proposed formulation includes continuous time-varying reserves (Fig. 3), thus using the flexibility of units more efficiently and reducing the cost of providing N-1 security.
3. The energy-based and power-based SCUC formulations are evaluated using different versions of 5-min SCED for simulating the real time operation of the system, and comparing the formulations under assumptions relevant for a wide range of market designs with regards to the procurement of reserves.

1.3. Paper organization

The remainder of the paper is structured as follows: Section 2 formulates the SCUC problems based on energy and power. Section 3 compares the formulations for two different test systems using different versions of 5-min SCED to simulate the real time operation of the system, and evaluates the results in terms of operational cost and security. Section 4 discusses the obtained results and Section 5 concludes.

2. Mathematical formulation

This section presents the SCUC formulations. To clearly differentiate which formulations we are comparing, we specify both the energy-based and power-based formulations. For the energy-based formulation, we specify both a version without and with startup/shutdown trajectories, as these are commonly not included in traditional formulations [8–10], but are included in the power-based formulation.

Note that we make a clear distinction between variables in terms of energy and variables in terms of power, by using notation such as $\delta^{e,s}_{cgt}$

and $\delta^{p,s}_{cgt}$ for the spinning reserve deployment in energy and in power, respectively. Also note that quantities marked with a hat, such as $\widehat{p}_{gt}$, are derived from other variables, but used to simplify the notation. All continuous variables are positive except the spinning reserve deployment $\delta^{e,s}_{cgt}$, $\delta^{p,s}_{cgt}$.

2.1. Energy-based formulation

A typical SCUC formulation in energy, similar to those in [8–10], is presented below.

2.1.1. Objective

The objective is to minimize the total cost for energy, including commitment costs, and reserves:

$$\sum_{g\in\mathscr{G}}\sum_{t\in\mathscr{T}}\left[C^{NL}_g u_{gt}+C^{LV}_g\left(\underline{P}_g u_{gt}+e_{gt}\right)+C^{SU}_g v_{gt}+C^{SD}_g z_{gt}+C^{S+}_g r^{s+}_{gt}+C^{S-}_g r^{s-}_{gt}+C^{NS}_g r^{ns}_{gt}\right] \tag{1}$$

2.1.2. Total energy

The total energy of units is used to enforce load balance and transmission constraints. Most common UC formulations do not include the energy produced during startup and shutdown of units [8,10]:

$$\widehat{e}_{gt}=\underline{P}_g u_{gt}+e_{gt}\quad \forall g,t \tag{2}$$

where only energy produced during the up period (those hours with $u_{gt}=1$) is accounted for.

However, the startup and shutdown trajectories of units may be accounted for by using

$$\widehat{e}_{gt}=\underline{P}_g u_{gt}+e_{gt}+\sum_{i=1}^{SU^D_g} v_{g,\left(t-i+SU^D_g+1\right)}\frac{P^{SU}_{g,i+1}+P^{SU}_{gi}}{2}+\sum_{i=1}^{SD^D_g} z_{g,t-i+1}\frac{P^{SD}_{g,i+1}+P^{SD}_{gi}}{2}\ \forall g\in\mathscr{G}^s,t \tag{3}$$

for slow-start units, and by

$$\widehat{e}_{gt}=\underline{P}_g u_{gt}+e_{gt}\quad \forall\, g\in\mathscr{G}^f,t \tag{4}$$

for fast-start units [26]. The option whether to account for startup and shutdown trajectories then gives two different SCUC formulations, where (2) is used if startup and shutdown trajectories are neglected or (3)-(4) if they are considered. Notice that even if (3)-(4) are used the energy produced during startup and shutdown does not enter into the objective function, since the cost for this energy is internalized in the startup and shutdown costs.

2.1.3. Commitment logic

The binary commitment logic is given by

$$u_{gt}-u_{g,t-1}=v_{gt}-z_{gt}\quad \forall\, g,t. \tag{5}$$

2.1.4. Minimum up and down times

Minimum up and down times are enforced by

$$\sum_{i=t-TU_g+1}^{t} v_{gi}\le u_{gt}\quad \forall\, g,t\in\left[TU_g,T\right] \tag{6}$$

$$\sum_{i=t-TD_g+1}^{t} z_{gi}\le 1-u_{gt}\quad \forall\, g,t\in\left[TD_g,T\right]. \tag{7}$$

Initial conditions for v and z are implemented as described in [21].

2.1.5. Capacity constraints

The capacity constraints are given by

$$e_{gt}+r^{s+}_{gt}\le\left(\overline{P}_g-\underline{P}_g\right)u_{gt}-\left(\overline{P}_g-SU_g\right)v_{gt}-\max\left(SU_g-SD_g,0\right)z_{g,t+1}\quad\forall g,t \tag{8}$$

$$e_{gt}+r^{s+}_{gt}\le\left(\overline{P}_g-\underline{P}_g\right)u_{gt}-\left(\overline{P}_g-SD_g\right)z_{g,t+1}-\max\left(SD_g-SU_g,0\right)v_{gt}\quad\forall g,t \tag{9}$$

$$e_{gt}-r^{s-}\ge 0\quad \forall g,t. \tag{10}$$

These constraints also restrict production during the first and last hour of the up period (those hours with $u_{gt}=1$). For units with $TU_g>1$ constraints (8)-(9) may be merged into one constraint [32]. Notice that (3)-(10) is the tightest possible formulation of the unit commitment logic and capacity constraints as proven in [32]. For slow-start units the startup and shutdown ramps are set as $SU_g=\underline{P}_g+RU_g/2$ and $SD_g=\underline{P}_g+RD_g/2$, which for SU_g is the energy produced during the first hour of the up period (a unit is up if $u_{gt}=1$), assuming that the unit starts the hour at $\underline{P}_g$ and ramps up at its maximum ramp rate, and similarly for SD_g.

2.1.6. Ramp limits

Ramp rate limits are given by

$$e_{gt}-e_{g,t-1}+\frac{60}{\tau^s}r^{s+}_{gt}\le RU_g\quad \forall\, g,t \tag{11}$$

$$e_{g,t-1}-e_{gt}+\frac{60}{\tau^s}r^{s-}_{gt}\le RD_g\quad \forall\, g,t. \tag{12}$$

Notice that the deployment time of spinning reserves is considered in (11)-(12), so that if, e.g., the deployment time is 15 minutes and the amount of reserves 100 MW, then (11)-(12) will require a ramp rate of 400 MW/h.

2.1.7. Spinning reserves

The deployment of spinning reserves for each contingency must be within limits of the procured reserves:

$$-r^{s-}_{gt}\le\delta^{e,s}_{cgt}\le r^{s+}_{gt}\quad \forall\, c,g,t \tag{13}$$

2.1.8. Non-spinning reserves

The constraints for non-spinning reserves are given by

$$\sum_{i=t-TD_g+1}^{t} z_{gi}\le 1-u_{gt}-\frac{r^{ns}_{gt}}{\overline{R}^{ns}_g}\quad \forall g\in\mathscr{G}^{ns},t\in\left[TD_g,T\right] \tag{14}$$

$$\underline{P}_g u^{ns}_{cgt}\le\delta^{e,ns}_{cgt}\le u^{ns}_{cgt}\overline{R}^{ns}_g\quad \forall c,g\in\mathscr{G}^{ns},t \tag{15}$$

$$r^{ns}_{gt}\ge\delta^{e,ns}_{cgt}\quad \forall c,g\in\mathscr{G}^{ns},t, \tag{16}$$

were (14) replaces (7), making sure that non-spinning reserves can only be scheduled for offline units, while respecting minimum downtime requirements. Furthermore, (15) makes sure that the non-spinning reserve contingency deployment of a unit is within its startup ramp capability, and (16) ensures that enough non-spinning reserves are procured to allow for the scheduled reserve deployment.

2.1.9. Load balance

Load balance is ensured both for the base case and for contingencies.

Base case:

$$\sum_{g\in\mathscr{G}}\widehat{e}_{gt}=D^e_t\quad \forall\, t \tag{17}$$

Line outages:

$$\sum_{g\in\mathscr{G}}\delta^{e,s}_{cgt}+\sum_{g\in\mathscr{G}^{ns}}\delta^{e,ns}_{cgt}=0\quad \forall\, c\in\mathscr{C}_l,t \tag{18}$$

Generator outages:

$$\sum_{g\in\mathcal{G}^{s}\setminus\{c\}} \delta^{e,s}_{cgt} + \sum_{g\in\mathcal{G}^{ns}\setminus\{c\}} \delta^{e,ns}_{cgt} = \widehat{e}_{ct} \quad \forall\, c\in\mathcal{C}_g, t \tag{19}$$

2.1.10. *Transmission constraints*

Similarly to load balance, transmission constraints are also enforced both for the base case and for contingencies.

Base case:

$$-\overline{F}_l \le \sum_{n\in\mathcal{N}} \Gamma_{ln}\left(\sum_{g\in\mathcal{G}_n}\widehat{e}_{gt} - D^e_{nt}\right) \le \overline{F}_l \quad \forall\, l, t \tag{20}$$

Line outages:

$$-\overline{F}_l \le \sum_{n\in\mathcal{N}} \Gamma^c_{ln}\left(\sum_{g\in\mathcal{G}_n}\left(\widehat{e}_{gt} + \delta^{e,s}_{cgt}\right) + \sum_{g\in\mathcal{G}^{ns}_n}\delta^{e,ns}_{cgt} - D^e_{nt}\right) \le \overline{F}_l \quad \forall\, c\in\mathcal{C}_l, l, t \tag{21}$$

Generator outages:

$$-\overline{F}_l \le \sum_{n\in\mathcal{N}} \Gamma_{ln}\left(\sum_{g\in\mathcal{G}_n\setminus\{c\}}\left(\widehat{e}_{gt} + \delta^{e,s}_{cgt}\right) + \sum_{g\in\mathcal{G}^{ns}_n\setminus\{c\}}\delta^{e,ns}_{cgt} - D^e_{nt}\right) \le \overline{F}_l \quad \forall\, c\in\mathcal{C}_g, l, t \tag{22}$$

2.2. *Power-based formulation*

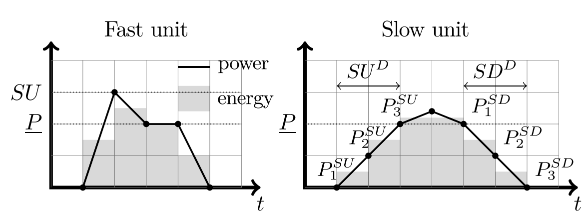
Fig. 4 shows the commitment logic for the power-based formulation. Notice the difference between how fast-start and slow-start units are modelled. Fast-start units start up in one period and may ramp up to a value above $\underline{P}$ at the end of the startup period, or shut down from a value higher than $\underline{P}$. Slow-start units take several periods to start, and begin/end their up periods (the hours with $u_{gt} = 1$) at $\underline{P}$. The power-trajectories of slow-start units during startup and shutdown are inflexible, and the units can't contribute with reserves during this period, unlike fast-start units that may contribute with reserves during startup and shutdown. While the formulation for slow-start units is from [16], the formulation for fast-start units is different from those in [15,16] to allow fast-start units that have positive production $\widehat{p}_{gt}$ for only one period.

Notice that for the power-based formulation the reserves, such as r^{s+}_{gt}, refer to the instantaneous amount of reserves held at the end of hour t, see Fig. 3. This is fundamentally different from the reserves specified in the energy-based formulation, which are assumed to be constant throughout the hour.

2.2.1. *Objective*

The objective is similar as for the energy-based formulation and includes the energy cost, commitment cost, and reserve cost:

$$\sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\left[C^{NL}_g u_{gt} + C^{LV}_g \widehat{e}_{gt} + C^{SU}_g v_{gt} + C^{SD}_g z_{gt} + C^{S+}_g r^{s+}_{gt} + C^{S-}_g r^{s-}_{gt} + C^{NS}_g r^{ns}_{gt}\right] \tag{23}$$



t [h]	1	2	3	4	5	6
u	0	1	1	1	0	0
v	0	1	0	0	0	0
z	0	0	0	0	1	0

t [h]	1	2	3	4	5	6	7	8
u	0	0	0	1	1	0	0	0
v	0	0	0	1	0	0	0	0
z	0	0	0	0	0	1	0	0

Fig. 4. Commitment logic for power-based UC formulation.

2.2.2. *Energy and total power*

The total energy used in the objective (23) is given by

$$\widehat{e}_{gt} = \frac{1}{2}\underline{P}_g\left(u_{g,t-1} + u_{gt}\right) + \frac{1}{2}\left(p_{g,t-1} + p_{gt}\right) \quad \forall g\in\mathcal{G}^f, t \tag{24}$$

for fast-start units, and by

$$\widehat{e}_{gt} = \underline{P}_g u_{gt} + \frac{1}{2}\left(p_{g,t-1} + p_{gt}\right) \quad \forall\, g\in\mathcal{G}^s, t \tag{25}$$

for slow-start units. Notice that while the energy variable $\widehat{e}_{gt}$ for fast-start units includes all the energy produced, the energy for slow-start units excludes the energy produced during startup and shutdown, which is internalized in the startup/shutdown costs.

The total power production used in the load balance and transmission constraints is given by

$$\widehat{p}_{gt} = \underline{P}_g u_{gt} + p_{gt} \quad \forall\, g\in\mathcal{G}^f, t \tag{26}$$

for fast-start units, and by

$$\widehat{p}_{gt} = \underline{P}_g u_{gt} + p_{gt} + \sum_{i=1}^{SU^D_g+1} P^{SU}_{gi} v_{g,t-i+SU^D_g+2} + \sum_{i=2}^{SD^D_g+1} P^{SD}_{gi} z_{g,t-i+2} \quad \forall\, g\in\mathcal{G}^s, t \tag{27}$$

for slow-start units [15].

2.2.3. *Commitment logic*

The commitment logic is given by (5).

2.2.4. *Minimum up and down times*

Minimum up and down times are enforced by (6)-(7).

2.2.5. *Capacity constraints*

Capacity constraints are enforced at the end of every hour. As both generation and reserves vary linearly within hours this means the capacity limits are respected at all times within the hour.

The different formulations for fast-start and slow-start units means that the upper capacity constraints are different. For fast-start units two constraints are required similar to the energy-based formulation:

$$p_{gt} + r^{s+}_{gt} \le \left(\overline{P}_g - \underline{P}_g\right)u_{gt} - \left(\overline{P}_g - SU_g\right)v_{gt} - \max\left(SU_g - SD_g, 0\right)z_{g,t+1} \quad \forall\, g\in\mathcal{G}^f, t \tag{28}$$

$$p_{gt} + r^{s+}_{gt} \le \left(\overline{P}_g - \underline{P}_g\right)u_{gt} - \left(\overline{P}_g - SD_g\right)z_{g,t+1} - \max\left(SD_g - SU_g, 0\right)v_{gt} \quad \forall g\in\mathcal{G}^f, t \tag{29}$$

while for slow-start units the constraint is the same as in [15]:

$$p_{gt} + r^{s+}_{gt} \le \left(\overline{P}_g - \underline{P}_g\right)\left(u_{gt} - z_{g,t+1}\right) \quad \forall\, g\in\mathcal{G}^s, t \tag{30}$$

The lower capacity constraint is the same for all units:

$$p_{gt} - r^{s-}_{gt} \ge 0 \quad \forall\, g, t \tag{31}$$

Notice that the formulation for slow-start units, (5)-(7), (25), (27), (30)-(31), is the tightest possible [16].

2.2.6. *Ramp limits*

Since reserves vary within the hour, it must be ensured that the reserves respect the ramp capabilities of units both at the beginning and at the end of the hour, as illustrated in Fig. 3. The ramp constraints for the beginning of the hour are given by

$$p_{gt} - p_{g,t-1} + \left(\frac{60}{\tau^s} - 1\right) r^{s+}_{g,t-1} + r^{s+}_{gt} \leq RU_g \ \forall g, t \tag{32}$$

$$p_{g,t-1} - p_{gt} + \left(\frac{60}{\tau^s} - 1\right) r^{s-}_{g,t-1} + r^{s-}_{gt} \leq RD_g \ \forall g, t \tag{33}$$

where we note that these constraints are enforced using the reserves τ^s minutes into the hour, to account for the reserve deployment time. The ramp constraints for the end of the hour are given by

$$p_{gt} - p_{g,t-1} + \frac{60}{\tau^s} r^{s+}_{gt} \leq RU_g \quad \forall g, t \tag{34}$$

$$p_{g,t-1} - p_{gt} + \frac{60}{\tau^s} r^{s-}_{gt} \leq RD_g \quad \forall g, t \tag{35}$$

Together the ramp constraints at the beginning and end of the hour ensure that the spinning reserves can be activated at any time within the hour while respecting the ramp limits of units. Fig. 5 shows an example of how the ramp constraints limit the reserves. As power production is constant during both hour $t-1$ and hour t the instantaneous upward spinning reserves held at $t-1$ are limited by the ramp rate to $r^{s+}_{t-1} \leq \frac{\tau^s}{60} RU$. Now, for the next hour from t to $t+1$ the unit is ramping at its maximum capability so that $p_{g,t+1} - p_{gt} = RU_g$. Thus constraint (32) evaluated at $t+1$ reduces to

$$RU_g + \left(\frac{60}{\tau^s} - 1\right) r^{s+}_t + r^{s+}_{t+1} \leq RU_g \Rightarrow r^{s+}_t = 0$$

and the reserves that the unit can provide therefore decreases linearly from r^{s+}_{t-1} to 0 during hour t, as shown in Fig. 5.

2.2.7. Spinning reserves

The spinning reserve deployment has to be within the limits of the procured reserves:

$$-r^{s-}_{gt} \leq \delta^{p,s}_{cgt} \leq r^{s+}_{gt} \quad \forall c, g, t \tag{36}$$

2.2.8. Non-spinning reserves

When a unit providing non-spinning reserves is deployed, it must ramp up at least to its minimum output level $\underline{P}_g$. This is true also during the hours when the non-spinning reserves increase from (decrease to) zero, as shown in Fig. 6 for hour 2 and 4. During hour 4, non-spinning reserves are supplied even though $u^{ns}_{cgt} = 0$ for this period, due to the assumed piecewise linear trajectories. To prevent the trajectories for the non-spinning reserves and the energy-dispatch of the units from overlapping, the constraint (40) is added. Apart from this modification the constraints are the same as for the energy-based formulation, and are given by

$$\sum_{i=t-TD_g+1}^{t} z_{gi} \leq 1 - u_{gt} - \frac{r^{ns}_{gt}}{\overline{R}^{ns}_g} \quad \forall g \in \mathscr{G}^{ns}, t \in [TD_g, T] \tag{37}$$

$$\underline{P}_g u^{ns}_{cgt} \leq \delta^{p,ns}_{cgt} \leq u^{ns}_{cgt} \overline{R}^{ns}_g \quad \forall c, g \in \mathscr{G}^{ns}, t \tag{38}$$

$$r^{ns}_{gt} \geq \delta^{p,ns}_{cgt} \quad \forall c, g \in \mathscr{G}^{ns}, t \tag{39}$$

$$u^{ns}_{cg,t-1} + u_{gt} \leq 1 \quad \forall c, g \in \mathscr{G}^{ns}, t \tag{40}$$

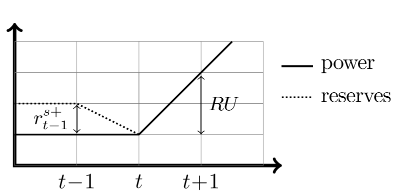
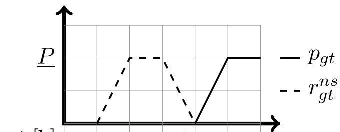


Fig. 5. Unit with reserves restricted by ramp limits.



t [h]	1	2	3	4	5	6
u_{gt}	0	0	0	0	1	1
u^{ns}_{cgt}	0	1	1	0	0	0

Fig. 6. Provision of non-spinning reserves.

where (37) replaces (7) for $g \in \mathscr{G}^{ns}$.

2.2.9. Load balance

Power balance is ensured at the end of each hour, both for the base case and for contingencies. Since both production of units and reserve deployments are piece-wise linear trajectories this ensures load balance is satisfied at all times.

Base case:

$$\sum_g \widehat{p}_{gt} = D^p_t \quad \forall t \tag{41}$$

Line outages:

$$\sum_g \delta^{p,s}_{cgt} + \sum_{g \in \mathscr{G}^{ns}} \delta^{p,ns}_{cgt} = 0 \quad \forall c \in \mathscr{C}_l, t \tag{42}$$

Generator outages:

$$\sum_{g \in \mathscr{G} \setminus \{c\}} \delta^{p,s}_{cgt} + \sum_{g \in \mathscr{G}^{ns} \setminus \{c\}} \delta^{p,ns}_{cgt} = \widehat{p}_{ct} \quad \forall c \in \mathscr{C}_g, t \tag{43}$$

Notice that $\widehat{p}_{ct}$ is the total power-production of the outaged unit, which means that N-1 security also covers the startup and shutdown trajectories of slow-start units. However, slow-start units are not able to contribute with reserves during startup and shutdown.

2.2.10. Transmission constraints

Transmission constraints are enforced at the end of the hour, for the base case and for contingencies. This ensures all transmission constraints are satisfied within the hour.

Base case:

$$-\overline{F}_l \leq \sum_{n \in \mathscr{N}} \Gamma_{ln} \left(\sum_{g \in \mathscr{G}_n} \widehat{p}_{gt} - D^p_{nt} \right) \leq \overline{F}_l \quad \forall l, t \tag{44}$$

Line outages:

$$-\overline{F}_l \leq \sum_{n \in \mathscr{N}} \Gamma^c_{ln} \left(\sum_{g \in \mathscr{G}_n} \left(\widehat{p}_{gt} + \delta^{p,s}_{cgt} \right) + \sum_{g \in \mathscr{G}^{ns}_n} \delta^{p,ns}_{cgt} - D^p_{nt} \right) \leq \overline{F}_l \quad \forall c \in \mathscr{C}_l, l, t \tag{45}$$

Generator outages:

$$-\overline{F}_l \leq \sum_{n \in \mathscr{N}} \Gamma_{ln} \left(\sum_{g \in \mathscr{G}_n \setminus \{c\}} \left(\widehat{p}_{gt} + \delta^{p,s}_{cgt} \right) + \sum_{g \in \mathscr{G}^{ns}_n \setminus \{c\}} \delta^{p,ns}_{cgt} - D^p_{nt} \right) \leq \overline{F}_l \quad \forall c \in \mathscr{C}_g, l, t \tag{46}$$

3. Results

3.1. Evaluation procedure

To evaluate the SCUC formulations, we run a security-constrained economic dispatch (SCED), or, equivalently, a multi-period security-constrained dc optimal power flow, similar to the formulations in [33, 34]. The SCED has 5-min time resolution and thus simulates the real time operation of the system. In the SCED the binary commitment decisions are fixed to the values obtained from the SCUC stage, meaning that it has no binary variables, resulting in an LP problem. All contingencies are included in the SCED, thus capturing the response of the system to any contingency occurring at any time within an hour during the simulated period. Since the methods for procuring reserves vary between different systems, we use two different SCED formulations. In the first formulation, the reserve schedules are fixed to the values from the day-ahead SCUC, and in the second formulation reserves are re-optimized in the real time SCED, meaning that only the commitment decisions from the SCUC are fixed. Thus we present results which are relevant for a wide range of market designs.

Notice that the reserve deployments are variable in both SCED formulations. However, in the version with fixed reserves the deployment must be within the limits of the committed reserve schedules from the SCUC. For the power-based formulation the fixed time-varying intra-hour reserves are obtained by linear interpolation between the hourly values, while the reserves are constant during the hour for the energy-based formulation. For the version with reserve co-optimization, both reserves and deployments are re-optimized during the real time SCED.

To ensure feasibility of the SCED, load shedding is allowed both in the base case and for all contingencies. For the SCED with fixed reserves, it is also necessary to allow relaxation of the scheduled reserves in the capacity and ramp constraints of the units. As explained in Section 1.1 the energy-based formulation does not represent unit ramps correctly, and hence may result in an infeasible combination of reserves and commitment decisions [12,15]. For this purpose slack variables are introduced which can lower the amount of reserves provided. These variables thus represent the amount of scheduled reserves which proved to be infeasible to supply in real time operation, and are penalized in the objective to reduce their use to a minimum. Though it is expected that these variables are not necessary in the case of the power-based formulation, which correctly represents the ramping of units, these variables are included to use the same SCED for evaluating both SCUC formulations.

The SCUC and SCED formulations were implemented in Python and solved with Gurobi 8.01 on a PC with Intel Core i7-4790 CPU @ 3.6 GHz and 32 GB of RAM. The relative MIP gap for the SCUC formulations was set to 10^{-4}.

3.2. Test systems

The formulations are tested using the IEEE 24-bus reliability test system and the IEEE-118 bus test system, with, respectively, 14 and 54 generator units. The 24-bus system was obtained from [35] but with generator data based on [36], as shown in Table 1. All generator outages, and those line outages which do not lead to islanding are included as contingencies. The 118-bus system was obtained from [37] and the start-up and shut-down ramps were set as mentioned in Section 2.1.5. To further stress the system the capacity of all lines was reduced by 10%. Generators with a capacity larger than 100 MW (17 generators) were included as contingencies along with the 11 most loaded lines (in a load flow for the peak load of 4 GW) which do not lead to islanding, for a total of 28 contingencies.

For both test systems, 5-min load data from CAISO was used for the load profile, see Fig. 7. This was scaled to a peak load of 2.5 GW for the 24-bus system and 4 GW for the 118 bus system. The energy load was calculated as the average hourly load and the power load profile was obtained by minimizing the error between the piecewise linear load profile and the 5 min load data, subject to a constraint that the total energy obtained when integrating the piecewise linear load profile is the same as in the original data. In this way, the energy content of the 5-min load profile and the two hourly load profiles is identical. The normalized hourly load profiles are shown in Table 2.

The deployment time for reserves is 10 minutes, which corresponds to the deployment time of contingency reserves for most US market [29]. The cost for unserved load is 5000 \$/MWh for the base case and 1000 \$/MWh for contingencies, and the cost for infeasible reserves in the SCED without co-optimization of reserves is 20 000 \$/MWh.

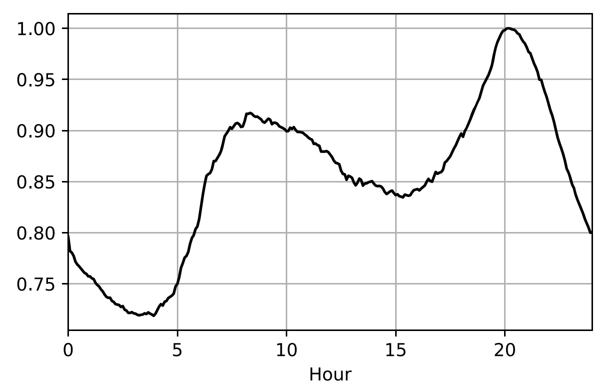


Fig. 7. CAISO normalized load profile for April 16, 2019.

Table 1
Generator Data.

Gen	Technical limits										Costs			
	Bus	$\overline{P}$	$\underline{P}$	TU	TD	RU	SU^D	SD^D	SU	$\overline{R}^{ns}$	C^{NL}	C^{LV}	C^{SU}	C^{SD}
1*	1	40.0	16.0	1	2	180.0	1	1	40.0	0.0	909.1	29.2	68.0	0.0
2	1	152.0	80.0	3	3	240.0	2	1	152.0	0.0	526.8	18.6	1617.7	818.7
3*	2	40.0	16.0	1	2	180.0	1	1	40.0	0.0	909.1	28.6	68.0	0.0
4	2	152.0	80.0	3	3	240.0	2	1	152.0	0.0	526.8	18.2	1586.7	801.6
5	7	300.0	120.0	4	5	1260.0	3	2	300.0	0.0	919.8	17.6	3421.6	2320.6
6	13	591.0	312.0	4	5	930.0	3	2	591.0	0.0	1448.6	17.2	8530.4	5900.5
7*†	15	60.0	27.0	1	2	300.0	1	1	60.0	60.0	1827.4	29.5	142.9	0.0
8	15	155.0	54.2	24	16	70.0	4	2	89.2	0.0	415.5	23.6	3760.2	1406.4
9	16	155.0	54.2	24	16	70.0	4	2	89.2	0.0	415.6	24.1	3812.9	1435.4
10	18	400.0	100.0	168	24	280.0	6	3	240.0	0.0	188.3	6.8	39168.4	1114.1
11	21	400.0	100.0	168	24	280.0	6	3	240.0	0.0	188.3	7.2	39296.8	1184.7
12	22	300.0	156.0	2	5	720.0	3	2	300.0	0.0	3756.6	28.3	6877.7	4855.1
13	23	310.0	108.5	24	16	140.0	4	2	178.5	0.0	831.1	23.8	7569.8	2840.0
14	23	350.0	140.0	8	6	140.0	4	2	210.0	0.0	303.8	26.2	15464.2	4036.8

Notes: 1. Unit types: * - fast start unit, †- unit providing non-spinning reserves.
2. Symmetric ramp capabilities: $RD = RU$ and $SD = SU$.
3. Reserve costs: $C^{S+} = 0.6C^{LV}$, $C^{S-} = 1.1C^{LV}$, $C^{NS} = 0.7C^{LV}$.

3.3. Results for the IEEE 24-bus system

Here we compare the performance of three SCUC formulations, two energy-based and the power-based formulations:

1. EN - energy-based without startup/shutdown trajectories, equations (1)-(2), (5)-(22).
2. ENs - energy-based with startup/shutdown trajectories, equations (1), (3)-(22).
3. PW - power-based which inherently includes startup/shutdown trajectories, equations (23)-(46), (5)-(7).

The first part of Table 3 shows the results of the SCUC and the remaining part shows the results from the evaluation using the different versions of the SCED. The overall performance of the SCUC formulations may be compared by the objective values obtained in the SCED evaluation. This shows that the power-based formulation performs much better in the SCED with fixed reserves and slightly better if reserves are co-optimized in the SCED. However, comparing the SCED objective values does not separate the contributions from the cost-efficiency and security of the formulations. We evaluate these aspects separately in the following.

3.3.1. Operation cost

Among the formulations EN has the highest total SCUC cost, about 0.6% higher than ENs and PW which have very similar costs. However, the cost of the energy-based SCUCs are based on energy profiles which do not represent feasible unit trajectories, and thus comparing the total SCUC cost does not give a true representation of the operation cost. Rather, the cost must be compared based on the actual real time dispatch cost from the SCED. For this reason the total cost is shown for the SCED evaluations. This value is the sum of the commitment decisions taken in the SCUC, the cost for reserves, and the SCED dispatch cost. For the SCED with fixed reserves the SCUC reserve cost is used, while the reserve cost determined in the SCED is used in the case of co-optimization of reserves. Thus note that costs for load shedding and violation of fixed reserves are excluded from the total cost.

With fixed reserves, PW has a lower cost than both EN and ENs, by 0.4% and 0.3%, respectively. Notice that the SCUC and SCED dispatch costs for PW are very close, while ENs underestimates the dispatch costs in the SCUC compared to the actual dispatch costs in the SCED. The reason is that ENs has an incorrect representation of unit ramps, and hence significant re-dispatch is required to obtain a feasible solution in the SCED. EN, on the other hand, overestimates the dispatch costs because it does not include the energy during the startup and shutdown process. This energy shows up in the SCED and allows other units to lower their production, thus reducing the dispatch cost.

If reserves are co-optimied in the SCED, the final costs for all formulations are very similar.

Table 2
Hourly load.

Hour	Energy	Power	Hour	Energy	Power
0	0.7961	0.7961	13	0.8616	0.8500
1	0.7675	0.7520	14	0.8491	0.8483
2	0.7439	0.7341	15	0.8417	0.8356
3	0.7260	0.7197	16	0.8380	0.8407
4	0.7203	0.7208	17	0.8507	0.8588
5	0.7356	0.7484	18	0.8778	0.8926
6	0.7860	0.8208	19	0.9178	0.9415
7	0.8594	0.8888	20	0.9749	1.0000
8	0.9012	0.9126	21	0.9940	0.9860
9	0.9130	0.9116	22	0.9539	0.9269
10	0.9059	0.9020	23	0.8859	0.8512
11	0.8988	0.8942	24	0.8214	0.7963
12	0.8834	0.8749			

Table 3
Evaluation of formulations for 24-bus test system.

SCUC	EN	ENs	PW
Total cost	1.0000	0.9935	0.9939
Commitment cost	0.1439	0.1437	0.1437
Number of startups	3	3	3
Dispatch cost	0.7607	0.7539	0.7552
Reserve cost	0.0954	0.0959	0.0951
Reserves [MWh]†	9600.00	9600.00	9600.00
Solution time [s]	8.0	35.0	24.0
SCED - fixed reserves			
Objective	0.6337	0.6827	0.3183
Total cost*	0.9979	0.9964	0.9938
Dispatch cost	0.7586	0.7568	0.7551
Load shedding cost	0.0956	0.1779	0.0000
Load shedding [MWh]	0.00	0.00	0.00
Contingency load shedding [MWh]	102.81	191.42	0.00
Number of ramp violations	59	47	0.00
Infeasible reserves [MWh]†	11.82	9.93	0.00
Total redispatch [MWh]	1,624.43	1,587.29	388.07
SCED - co-optimized reserves			
Objective	0.4152	0.4188	0.4144
Total cost**	0.9949	0.9947	0.9949
Dispatch cost	0.7560	0.7553	0.7553
Load shedding cost	0.0044	0.0044	0.0000
Load shedding [MWh]	0.00	0.00	0.00
Contingency load shedding [MWh]	4.76	4.76	0.00
Reserve cost	0.0949	0.0958	0.0959
Reserves [MWh]†	9,600.00	9,600.00	9,600.00
Total redispatch [MWh]	1,033.76	814.75	638.11

All costs are scaled to total SCUC cost for EN: 1075820 $.
*Excluding costs for load shedding and infeasible reserves.
**Excluding costs for load shedding.
†Energy-content, e.g. 1 MW of reserves for 1 h gives 1 MWh.

3.3.2. Security

The security of the formulations is measured by the amount of load shedding and its associated cost, and the reserve violations in the SCED. The load shedding will occur in cases where the units cannot be redispatched to match the 5-min load profile or when N-1 security cannot be guaranteed for all times within the hour. Note that load shedding in the base case will also help the SCED to guarantee N-1 security, as this allows the output of units to be decreased, thus reducing the size of possible outages. In reality deviations between the day-ahead scheduling and real time operation could be handled by regulation reserves, and emergency actions will be taken for contingencies, so the amount of load shedding can be seen as a measure of the need for these resources.

Compared to EN and ENs, PW shows a higher level of security. This is especially the case with fixed reserves. In this case, EN and ENs have 103 MWh and 191 MWh of contingency load shedding, respectively, while PW does not have any load shedding and thus completely guarantees N-1 security. Also, for both EN and ENs there is about 10 MWh of infeasible reserves, while the reserves scheduled by PW are completely feasible. If reserves are co-optimized in the SCED the difference between the formulations decreases, as the contingency load shedding required for EN and ENs reduces to 5 MWh.

3.3.3. Redispatch

Another indication of the performance of the formulations is the amount of redispatch required in the SCED. In many electricity markets redispatch leads to extra costs, and hence it is desirable that the day-ahead planning matches the real time operation as closely as possible, especially in the absence of uncertainty from, e.g., renewable generation.

The redispatch was calculated as the total difference in energy between the SCUC and SCED generation profiles. For illustrative purposes, Fig. 8 shows the SCUC and SCED dispatch for unit 14 from the ENs and PW models. The redispatch is the total area between the SCUC and SCED curves, which is smaller for PW than for ENs. As seen in Table 3, PW reduces the redispatch required compared to the energy-based formulations by more than 75% when reserves are fixed and by more than 20% when reserves are co-optimized.

3.4. Results for the IEEE 118-bus system

Table 4 shows the evaluation of the SCUC formulations for the IEEE 118 bus test system. Only the formulations which include start-up and shut-down trajectories are included for this case. Similarly to the 24-bus test system, PW gives a lower objective value than ENs for the SCED with fixed reserves, explained by lower penalizations for security violations. The difference remains also when reserves are co-optimized in the SCED. Thus, even if reserves are co-optimized, PW improves security as the load shedding cost decreases by 73%. Unlike for the 24-bus system, the total cost for PW is now 0.2% higher than for ENs if reserves are fixed and 0.4% higher if reserves are co-optimized.

Notice that for this test system PW does not provide full N-1 security in the 5-min SCED as there is both load shedding and reserve violations if reserves are fixed. This is due to the difference between the piecewise linear load profile assumed in the SCUC formulation and the real 5-min load profile used in the SCED. This is confirmed by the following test. The SCED with fixed reserves was solved using the piecewise linear load interpolated to 5-min values, and Table 4 also shows the evaluation for this case. For PW there was no load shedding and no reserve violations, showing that it provides full N-1 security for a piecewise linear hourly load profile, unlike ENs.

4. Discussion

In Section 1.1 we identified several short-comings of the traditional energy-based formulation with regards to providing N-1 security in a SCUC formulation with reserves and contingency-constraints. The energy-based formulation does not represent units ramps correctly, and thus cannot guarantee 1) that enough reserves are provided to satisfy N-1 security during the whole hour, or 2) that the scheduled reserves are feasible to provide with respect to the ramp rates of units. It also does not contain information about instantaneous power trajectories and thus does not have the information needed to be able to satisfy N-1 security. In contrast, the proposed power-based formulation solves these issues, under the assumption that demand varies linearly during the hour.

The case studies confirm these results. With a piecewise linear hourly load the commitment decisions and reserve schedules from the power-based SCUC can be used in the 5-min SCED and provide full N-1 security. The energy-based formulation, on the other hand, produces commitment decisions and reserve schedules that require significant amounts of load shedding as well as relaxation of the scheduled reserves in the SCED to be feasible. When using actual 5-min load data in the SCED the power-based formulation cannot guarantee N-1 security completely. However, also in this case the power-based formulation performs better in terms of security than the energy-based formulation.

We also evaluate the formulations using a 5-min SCED where reserves are co-optimized with the dispatch, meaning that only the hourly commitment decisions are fixed to the values obtained from the day-ahead SCUC. In this case the power-based formulation still performs better in terms of security than the energy-based formulation, but the difference becomes much smaller. However, this evaluation overestimates the flexibility of the system and should thus be seen as a low estimate for the difference between the formulations. There are several reasons for this. In the evaluation, all units can be redispatched and the reserve schedules completely altered, as long as it is within the technical capabilities of the units. However, in reality, even in markets where energy and reserves are co-optimized in real time, there will be limited flexibility to deviate from the planned dispatch. For example, in PJM's market inflexible spinning reserves are procured by the ancillary service optimizer before the delivery hour, and the real time market is used to procure the remaining reserves needed. Regarding redispatch, some units may use self-scheduling and thus not be available for redispatch in

Table 4
Evaluation of formulations for 118-bus test system.

SCUC	ENs	PW
Total cost	1.0000	1.0065
Commitment cost	0.0251	0.0250
Number of startups	20	21
Dispatch cost	0.9230	0.9303
Reserve cost	0.0519	0.0512
Reserves [MWh]†	4859.80	4762.71
Solution time [s]	1331	7009
SCED - fixed reserves		
Objective	2.4498	0.9587
Total cost*	1.0032	1.0055
Dispatch cost	0.9263	0.9293
Load shedding cost	0.7997	0.4827
Load shedding [MWh]	143.14	95.98
Contingency load shedding [MWh]	79.50	0.00
Number of reserve violations	335	10
Infeasible reserves [MWh]†	61.17	3.00
Total redispatch [MWh]	3794.52	606.51
SCED - co-optimized reserves		
Objective	0.6320	0.5319
Total cost**	1.0292	1.0333
Dispatch cost	0.9591	0.9635
Load shedding cost	0.1287	0.0344
Load shedding [MWh]	23.81	4.99
Contingency load shedding [MWh]	8.93	9.25
Reserve cost	0.0450	0.0449
Reserves [MWh]†	4288.28	4262.76
Total redispatch [MWh]	19533.30	17,229.84
SCED - fixed reserves, piecewise linear load		
Objective	1.7763	0.4193
Total cost*	1.0048	1.0063
Dispatch cost	0.9279	0.9301
Load shedding cost	0.2291	0.0000
Load shedding [MWh]	29.87	0.00
Contingency load shedding [MWh]	78.45	0.00
Number of reserve violations	558	0
Infeasible reserves [MWh]†	55.78	0.00
Total redispatch [MWh]	3,911.48	295.92

All costs scaled to total SCUC cost for ENs: 994343 $.
*Excluding costs for load shedding and infeasible reserves.
**Excluding costs for load shedding.
†Energy-content: 1 MW of reserves for 1 h gives 1 MWh.

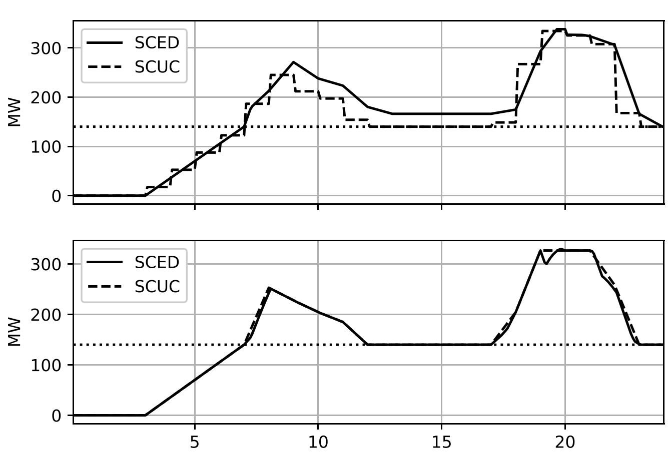


Fig. 8. Dispatch of unit 14 from the SCUC and SCED with fixed reserves, for ENs (top) and PW (bottom). The redispatch of the unit is computed as the area between the SCUC and SCED dispatches, considering only the periods when the unit is producing above its minimum output (marked by the dotted line).

real time [38].

In general, the evaluation procedure used here underestimates the effects caused by the inaccuracies present in the SCUC scheduling stage. The reason is that the SCED optimizes the dispatch for the whole 24h period with perfect foresight, whereas in reality redispatch will be performed with a look-ahead of a few hours using imperfect forecasts. It also assumes that all units can be re-dispatched in real time, which is not always the case in real systems.

5. Conclusion

This paper presents a security-constrained power-based unit commitment (SCUC) with intra-hour time-varying reserves. The proposed formulation has significant advantages over a traditional energy-based SCUC. Unlike the hourly energy-based formulation, the hourly power-based SCUC formulation proposed here can guarantee N-1 security within the whole hour. Also, by allowing reserves to vary continuously during the hour, units are used more efficiently and the amount of reserves needed for N-1 security can be reduced.

The power-based and energy-based SCUC formulations are compared for two different test systems, using different versions of a 5-min security-constrained economic dispatch (SCED), thus simulating the real time operation of power systems. The SCED is implemented both with fixed reserves and variable reserves, to provide results relevant for a wide range of market designs with respect to reserve procurement. The power-based formulation is found to significantly increase the security of the dispatch, while only slightly increasing the total commitment and energy costs. The case studies showed that the cost increase was never more than 0.4%, while the load shedding required was reduced by at least 40%, and in several cases completely eliminated. The increase in security is significant both if reserves are fixed to the schedules determined in the SCUC, or re-optimized in the SCED.

For a piecewise linear demand profile, the power-based formulation, unlike the energy-based formulation, completely guarantees N-1 security, meaning that all scheduled reserves are feasible and that no load shedding is required in the SCED stage. An interesting subject for future research is to find a formulation which completely guarantees N-1 security also for a continuous demand profile with intra-hour demand variations.

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CRediT authorship contribution statement

Elis Nycander: Methodology, Software, Investigation, Writing - original draft. **Germán Morales-España:** Conceptualization, Methodology, Writing - review & editing. **Lennart Söder:** Writing - review & editing, Supervision, Project administration.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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