96-CON-R1224 **THE INFLUENCE OF MATERIAL PROPERTIES ON PLASTIC HINGE ROTATIONAL CAPACITY AND STRENGTH**

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SUMMARY

In this article the effects of standardised material stress-strain behaviours on plastic hinge length, moment and rotational capacity are investigated using a specially developed computer program. Material properties are described using three standard post-yield stress-strain characteristics, as given in the Dutch steel code. Two cross-sections are included in this study, an HE 140 A and an IPE 140. The static system and loading investigated is a simple-span beam subjected to three point loading. The influence of residual stresses are taken into account based upon simplified models given in the Dutch steel code. To indicate clearly the influence of material stress-strain behaviour, the effects of local and global instabilities, load introduction widths and shear are not taken into account in the model.

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1 INTRODUCTION

The wide range of structural steels available to designers, and their increasing popularity, has called into question safety aspects related to their use for plastic design. This is especially true for high-strength steels. There is a common misconception amongst designers, and many researchers, that the ultimate stress f_u, and the ratio of ultimate to yield stress f_u/f_y, does not influence plastic hinge moment or rotational capacity. It is widely held that only the yield stress f_y and the maximum strain before failure ε_{max}, is of importance. Because of this misconception steels with behaviour unacceptable for plastic design are suggested to be adequate.

In this article, an attempt has been made to correct this misconception. This is done by isolating and illustrating the effects of material properties using a specially developed computer program. The underlying concept of this program and its results may be clearly understood because the effects of local buckling, global instability, load introduction widths, transverse shear, etc. have not been introduced.

Three standard material behaviours are investigated. Each of these behaviours is taken from models given in the Dutch steel code [1]. A determinant static system is chosen for this study: three point bending of a single-span specimen. This implies that the external moment at each cross-section is known, and the moment gradient (function of the span length) is linear. The span length to beam depth ratios investigated correspond to those found in typical steel-framed buildings. Two class 1, I-shaped, cross-sections were examined, an HE 140 A and an IPE 140. The effects of residual stresses on plastic behaviour are included in this investigation. Residual stresses are defined using relative values taken from the Dutch steel code [1].

2 LITERATURE REVIEW

This literature review is provided to give the reader an overview of recent articles siting the influence of post-yield steel characteristics on the plastic zone length, moment capacity and rotational capacity. It is not intended to review plastic design, its development, or give a definitive list of references in this field of study.

2.1 Technical reports

Spangemacher and Sedlacek, 1982 [2].
A numerical method for investigating the plastic behaviour of full-scale member tests was developed. Such a development was deemed of interest due to the time and cost of full-scale testing. An example calculation is given for a three point bending test. The ratio of the ultimate to yield stress is included as a possible input parameter. Three types of steels were investigated, St 37, StE 460 and StE 690. A comparison of results for each steel type indicated that as steel strength increases the rotational capacity decreases.

CRM report, 1992 [3].
This report contains the results of an experimental investigation on the plastic behaviour of HE 200 B sections made of different steel grades. All tests consisted of three point bending specimen with span lengths of 3 meters. The steel grades investigated ranged from 235 to 460 MPa. One of the parameters investigated is low ultimate to yield strain ratios (f_u/f_y). For this parameter FeE 420 steels with experimentally determined ratios between 1.12 and 1.16 were used. The report concludes that a high ratio (greater than 1.2) is not needed to ensure the adequate behaviour of frames designed according to plastic theory. Further, it is stated that it is possible to develop adequate rotation capacity with a ratio of 1.0 ($f_u = f_y$). It is postulated that the ratio of ultimate to yield strain is of no importance for plastic design, only the maximum strain in the steel is of importance. This postulate is followed by a caution that it would be interesting in the future to perform numerical simulations looking at different values of the stress ratio and maximum strains.

2.2 Code provisions

A short review has been made of mechanical requirements for steels used in combination with plastic designs. These are listed in the following paragraphs.

Eurocode 3, Design of Steel Structures [4].
Three minimum requirements must be satisfied for steels when using plastic design. First, a minimum value of 1.2 is set on the ratio of ultimate to yield stress (f_u/f_y). Second, a minimum elongation at failure of $\varepsilon_{max} = 15$ % is specified for a gauge length of $5.65\sqrt{A_o}$ (where A_o is the original cross-sectional area). Lastly, the ultimate strain (strain corresponding to the ultimate tensile

strength) must be at least 20 times larger than the yield strain. The following steel types are assumed to satisfy these requirements, according to EN 10025: Fe 360, Fe 430, Fe 510, and according to prEN 10113: Fe E 275, Fe E 355.

Dutch steel design code TGB 1990 [1].
Mechanical requirements are specified similar to those listed in the Eurocode 3 [4]. In addition to these requirements, the following rules are given:
- Only steels with yield stresses equal to or lower than 355 MPa may be used.
- The actual ratio of ultimate to yield stress f_u/f_y must be greater than 1.11 (this acknowledges that a nominal value of 1.2 is specified but that individual tests may give lower values).
- Strain at the ultimate strength must be larger than 8%.

3 THEORETICAL MODEL

3.1 General

A special purpose computer program has been developed to isolate and study the effects of post-yield material characteristics on the length, moment and rotational capacities of plastic hinges. In the following paragraphs the method of analysis and assumptions used in the program development are described.

3.2 Cross-sectional description

The cross-section is described using the geometrical parameters shown in FIGURE 1. Each cross-section is divided vertically and horizontally. Flanges are divided 20 times horizontally and 40 times vertically. The transition sections between flange and web are divided into 20 parts, and the remaining web is divided into 100 equal parts.

3.3 Material behaviour

Three stress vs. strain material behaviours are included in this study, as shown in FIGURE 2. Each of these behaviours may be found in the Dutch steel code [1]. These behavioural types are identified as types 2, 2a and 3. This terminology is the same as found in the Dutch code. Type 1 behaviour is not included in the study as no plastic behaviour is allowed. For each behavioural type values of f_y between 200 MPa and 500 MPa are studied. Ratios of f_u/f_y between 1.0 and 1.5 are also investigated.

3.4 Residual stresses

Two standard residual stress distributions have been included in this study, FIGURE 3. These are the simplified residual stress distributions, given in the Dutch steel code, that may be used for standard hot-rolled I sections [1]. The $\sigma_{o.rel} = 0.3$ relative residual stresses model is used for h/b > 1.2, and the $\sigma_{o.rel} = 0.5$ relative residual stresses distribution is used for sections with h/b ≤ 1.2. The resulting residual stresses σ_o can be calculated as follows:

$$\sigma_o = 235\ \sigma_{o.rel} \qquad (1)$$

where:

σ_o is the residual stress in MPa

$\sigma_{o.rel}$ is the relative residual stress

3.5 Numerical solution procedure for the plastic zone

The numerical solution process describe below is illustrated using FIGURE 4. First, numerical integration using a given cross-section and material is evaluated when the strain on the extreme fibre is equal to ε_u and ε_y. Numerical integration when the strain on the extreme fibre is equal to ε_y gives M_{el} and κ_y (equal to the moment at first yield and the corresponding cross-sectional curvature). Numerical integration when the strain of the extreme fibre is equal to ε_u gives M_u and κ_u (equal to the maximum cross-sectional moment and the corresponding cross-sectional curvature). Note in FIGURE 4 that the plastic cross-sectional curvature κ_{pl}, at a given moment M_i (between M_{el} and M_u) can be calculated by subtracting the elastic curvature from the cross-sectional curvature. This can be stated as follows:

$$\kappa_{pl} = \frac{\varepsilon_i}{\frac{1}{2}\,h} - \frac{M_i}{M_{el}}\frac{\varepsilon_y}{\frac{1}{2}\,h} \qquad (2)$$

where:

κ_{pl} is the plastic curvature of the cross-section at moment M_i

ε_i is the strain on the extreme fibres of the cross-section at a moment M_i

M_i is a given cross-sectional moment between M_{el} and M_u

ε_y is the strain on the extreme fibres of the cross-section at a moment M_{el} (strain at first yield)

h is the height of the cross-section

M_{el} is the cross-sectional moment when the strain on the extreme fibre is equal to ε_y

Due to the static system chosen, FIGURE 5, there is a linear relationship between the span length and the cross-sectional moment (i.e. for a statistically determinant system no moment redistribution is possible). The maximum extent of the plastic zone may now be calculated. This is done using the following expression:

$$L_{pl} = L\,\frac{M_u - M_{el}}{M_u} \qquad (3)$$

where:

L is the span length

L_{pl} is the maximum length of the plastic zone

M_u is the cross-sectional moment when the strain in the extreme fibres is equal to ε_u

M_{el} is the cross-sectional moment when the strain in the extreme fibres is equal to ε_y

The strain difference between ε_u and ε_y is divided into 1000 equal divisions ε_i. Cross-sectional integration is performed for each value of ε_i, which is set equal to the strain on the extreme fibre. The location of this strain state on the simple span beam L_i, can be determined in a manner similar to equation 3, see FIGURE 5.

Lastly, the total plastic rotation in the beam can be calculated by summing the individual plastic curvatures and the average length between adjacent cross-sectional calculations. This is stated as follows:

$$\varphi_{pl} = \sum \kappa_{pl,i} \frac{L_{i+1} - L_{i-1}}{2} \qquad (4)$$

where:

φ_{pl}	is the total <u>plastic</u> rotation in the plastic hinge
$\kappa_{pl,i}$	is the plastic curvature in cross-section i
L_{i+1}	is the location of cross-section i+1
L_{i-1}	is the location of cross-section i-1

4 RESULTS OF NUMERICAL CALCULATIONS

4.1 Plastic hinge length

Calculated values of plastic hinge length as a function of f_u/f_y for the four cases given in TABLE 1 have been plotted in FIGURES 6a and 6b. In all cases it may be noted that plastic hinge length increases as a function of f_u/f_y. This increase in not quite linear, and the rate of increase reduces with increases in f_u/f_y. The large apparent difference between plastic hinge lengths with and without residual stresses is due only to the definition of the elastic limit.

Further, it may be noted that material behaviour (types 2, 2a and 3) have little influence on plastic hinge length.

TABLE 1: Description of base cases

Case	Profile type	L/h	f_y MPa	Relative residual stresses model $\sigma_{o.rel}$
1	HE 140 A	10	360	0.0
2	HE 140 A	10	360	0.5
3	IPE 140	10	360	0.0
4	IPE 140	10	360	0.3

4.2 Moment capacity

Calculated ratios of ultimate to plastic hinge moment capacity (M_u/M_{el}) as a function of f_u/f_y for the four case given in TABLE 1 have been plotted in FIGURES 7a and 7b. For all cases it may be noted that moment capacity increases linearly as a function of f_u/f_y. As for the plastic hinge length, the large apparent differences between moment capacity with and without residual stresses are due to the definition of the elastic capacity.

It may be noted that the material behaviour (types 2, 2a and 3), and residual stresses have little influence on plastic hinge moment capacity.

4.3 Rotational capacity

4.3.1 Definition of R_{req}

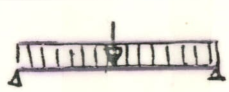
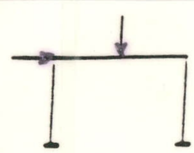
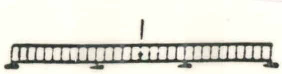
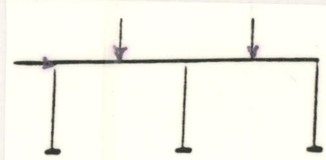
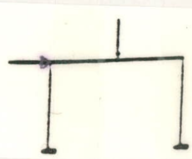
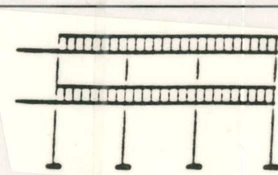
Required rotational capacity, referred to as R_{req}, is a non-dimensional term that is calculated using <u>only</u> the static system, loading cases and elastic flexural stiffness EI.

These values (always greater than 1.0) represent the ratio of the rotation required to form a plastic mechanism to the rotation at the formation of the first hinge. R_{req} is equal to 1.0 if the formation of the first plastic hinge coincides with the development of a collapse mechanism, as is the case for a single-span beam, pinned at both ends with a point load at midspan. As R_{req} increases the required rotational capacity increases. For example, R_{req} for a three-span continuous beam with a uniformly distributed load is equal to 3.0.

For simple cases R_{req} can be hand calculated, and their values are usually given in table form. An example of such values for some simple geometrical and load conditions is given in TABLE 2.

The values of R_{req} given in TABLE 2 may need to be increased to account for the following:
- Settlement
- Global instabilities in members with low slenderness ratios
- Local effects due to holes and discontinuities

TABLE 2: Tabulated values of R_{req}

R_{req}	System	R_{req}	System
1.0		2.5	
3.0		3.0	
1.3		2.5	

4.3.2 Definition of R_{aval}

Available rotation capacity, R_{aval}, depends primarily upon cross-sectional geometry and material stress-strain properties. Normally it is evaluated for the case of a simply supported beam with a concentrated load at midspan. The span length is chosen to represent the distance between adjacent inflection points at the formation of a plastic mechanism for the static system and load combination investigated when determining R_{req}. R_{aval} may be measured by tests or modelled by using the method described in Chapter 3, and is defined as follows:

$$R_{aval} = \frac{\varphi_u}{\varphi_{el}} \tag{5}$$

where:

R_{aval} is the rotation capacity as defined in equation 5 and shown in FIGURE 8
φ_u is the rotation shown in FIGURE 8
φ_{el} is the rotation at M_{el} or the equivalent elastic rotation at the maximum calculated rotation (See FIGURE 8)

It is important to note that this definition allows a direct comparison between R_{req} and R_{aval}. In some references R_{aval} is defined differently; the elastic rotation is subtracted from φ_u in equation 5. R_{aval} is non-dimensional, and can represent two values, shown using the curves depicted in FIGURE 8. Curve "a" is typical of behaviours predicted using calculation models and standardised material behaviours without falling branches. Curve "b" is typical of behaviours observed during strain or deformation controlled tests, in which unloading is allowed to occur.

The elastic rotation, φ_{el}, can be calculated for the simply supported point loaded case shown in FIGURE 5 as follows:

$$\varphi_{el} = \int \frac{M(x)}{EI} dx \tag{6}$$

where:

φ_{el} is the total elastic rotation at the initiation of plastification (the rotation at which the extreme cross-sectional fibres first reach a stress of f_y)
M(x) Moment along the major axis of the beam
EI is the elastic cross-sectional rotational stiffness

In accordance with standard convention the initiation of plastification used in equation (6) ignores residual stresses. The resulting evaluation of equation (6) is thus:

$$\varphi_{el} = \frac{f_y\, W\, L}{2\, EI} \qquad (7)$$

where:
W is the elastic section modulus

4.3.3 Numerical predictions of R_{aval}

The basic requirement for the performance of a plastic design, from the point of view of rotational capacity, can be stated as follows:

$$R_{aval} \geq R_{req} \qquad (8)$$

The program described in chapter 3 was used to calculate values of R_{aval} as a function of three parameters. These are listed as follows:
- Ratios of span length to depth L/h. Values between 5 and 20 were examined.
- Yield stress f_y. Values of f_y between 200 MPa and 500 MPa were examined.
- The ratio of ultimate to yield stress f_u/f_y. Values of f_u/f_y of 1.01, 1.05, 1.10 and 1.2 were examined.

Ratio of span length to depth L/h
For both cross-sections, HE 140 A and IPE 140, and the values of L/h examined it was observed that R_{aval} was independent of the ratio L/h. This is an important finding, as it implies that for the vast majority of typical structures for which plastic design is commonly used, R_{aval} is independent of the length between adjacent inflection point at the formation of the plastic mechanism.

This simplifies the designers task. The static system and loading conditions need not be considered in the design process: R_{aval} is influenced only by the geometry and material characteristics of the plastically designed member itself.

Yield stress f_y and the ultimate to yield stress ratio f_u/f_y
Yield stress f_y and the ratio of ultimate to yield stress f_u/f_y has an effect upon R_{aval}. These effects are illustrated in two manners. First, in FIGURE 9, R_{aval} is shown as a function of f_u/f_y for a given elastic limit, in this case $f_y = 360$ MPa. It may be seen that for all three material behaviours (types 2, 2a and 3, see FIGURE 2) R_{aval} approaches 1.0 for $f_u/f_y = 1.0$. This is a logical observation, as one would expect limited plastic rotations if the ultimate stress is equal to the yield stress. Upon increasing f_u/f_y from 1.0 to 1.5, it may be observed that R_{aval} increases rapidly. The rate of increase, however, tends to decrease with increasing values of f_u/f_y. For a given value of f_u/f_y material behaviour types 2 and 2a give higher rotational capacities than material behaviour type 3. This would also appear to be in

accordance with expectations as material behaviour type 3 has no yield plateau. The effect of relative residual stress is small for all values of f_u/f_y and material behaviour types studied.

A second, and more useful method of comparison, is shown in FIGURE 10. It may be noted in this figure that for a given value of f_y material behaviour types 2 and 2a give higher values of R_{aval} than material behaviour type 3. This effect is the same as that observered in FIGURE 9.

Further, R_{aval} decreases with <u>increases</u> in f_y. Using FIGURE 10 as an example ($f_u/f_y = 1.10$), one could expect R_{aval} to be near 8.5 for $f_y = 200$ MPa, but only 3.75 at $f_y = 500$ MPa (a reduction of more than 50%). R_{aval} also decreases with <u>decreases</u> in f_u/f_y. As an example, at a constant value of $f_y = 200$ MPa, R_{aval} is equal to 14.5 for $f_u/f_y = 1.20$ but 4.9 for $f_u/f_y = 1.05$. This is a decrease of about 70%. It may be observed that for $f_u/f_y = 1{:}01$ R_{aval} is unacceptably low for all yield stresses.

The influence of f_y and f_u/f_y is more profound than may first be suspected by these figures when one considered the following:

1. At low values of f_y, steels are better modelled using material behaviour type 2 and 2a. At high values of f_y steels are better modelled using material behaviour type 3.
2. At low values of f_y, steels have higher values of f_u/f_y than for high-strength steels. This suggests that for a $f_y = 200$ MPa, R_{aval} is best estimated using $f_u/f_y = 1.20$. At $f_y = 500$ MPa, R_{aval} is best estimated using $f_u/f_y = 1.05$.

It is generally accepted that the ratio f_u/f_y for a given steel type is more variable than its yield stress f_y. For a normal strength steel (200 MPa $\leq f_y \leq$ 355 MPa), if a nominal value of 1.2 is asked, actual values of 1.1 are common. This is reflected in existing code provisions [1]. FIGURE 10 suggests that for normal strength steels actual values of $f_u/f_y = 1.10$ are sufficient to ensure adequate rotational capacity for $R_{req} > 3.0$. This is in agreement with good design practice. For high-strength steels (355 MPa $< f_y \leq$ 500 MPa), actual values of f_u/f_y must be higher than 1.10.

Caution should be exercised when using the above values for suggesting appropriate minimum nominal and actual values of f_u/f_y. The values calculated in this report are meant only to indicate the influence of f_u and f_u/f_y and thus do not include the effects of other parameters such as local and global instabilities, load introduction widths, shear, etc. The effects of such parameters will be to change the magnitude of R_{aval} for any given value of f_u and f_u/f_y. The relative importance of f_u and f_u/f_y, however, <u>will not</u> change.

5 CONCLUSIONS

The following conclusions are valid only for the range of parameters studied and cross-sections investigated. These conclusions represent the results of a study in which the effects of f_u and f_u/f_y are purposely isolated from the effects of local buckling, global instability, load introduction lengths, vertical shear and other variable that are recognised to influence plastic hinge behaviour:

1. There has been confusion in the literature over the definition of R_{req}. This value is calculated using <u>only</u> the static system, loading conditions and elastic rotation stiffness (EI). R_{req} is <u>not</u> influenced by cross-sectional geometry and material stress-strain characteristics.
2. Statements in the existing literature indicating that plastic hinge rotational capacity is not effected by f_u or f_u/f_y are false.
3. Statements in the existing literature indicating that f_u/f_y may be reduced to values approaching 1.0 are unrealistic and dangerous.
4. Available plastic hinge rotational capacity R_{aval}, is <u>independent</u> of L/h for values of L/h between 5 and 20. This corresponds to the vast majority of L/h values for typical steel constructions for which plastic designs are used.
5. Available plastic hinge rotational capacity R_{aval}, is dependent upon both f_u and f_u/f_y. R_{aval} decreases with increases in f_y. R_{aval} increases with increases in f_u/f_y.
6. Actual values of $f_u/f_y = 1.1$ seem to be appropiate for normal strength steels ($200 \text{ MPa} \leq f_y \leq 355 \text{ MPa}$).
7. Actual values of $f_u/f_y > 1.1$ seem to be appropiate for high strength steels ($355 \text{ MPa} < f_y \leq 500 \text{ MPa}$).

6 RECOMMENDATIONS

The following recommendations are set forth:

1. Codes should reflect the influences of f_u and f_u/f_y on R_{aval}.
2. For high strength steels (355 MPa $< f_y \leq$ 500 MPa) a minimum actual value of $f_u/f_y > 1.1$ is needed.
3. To determine this value an in depth study of R_{aval}, including the effects of local and global instabilities, load introduction widths, shear, etc. should be conducted.

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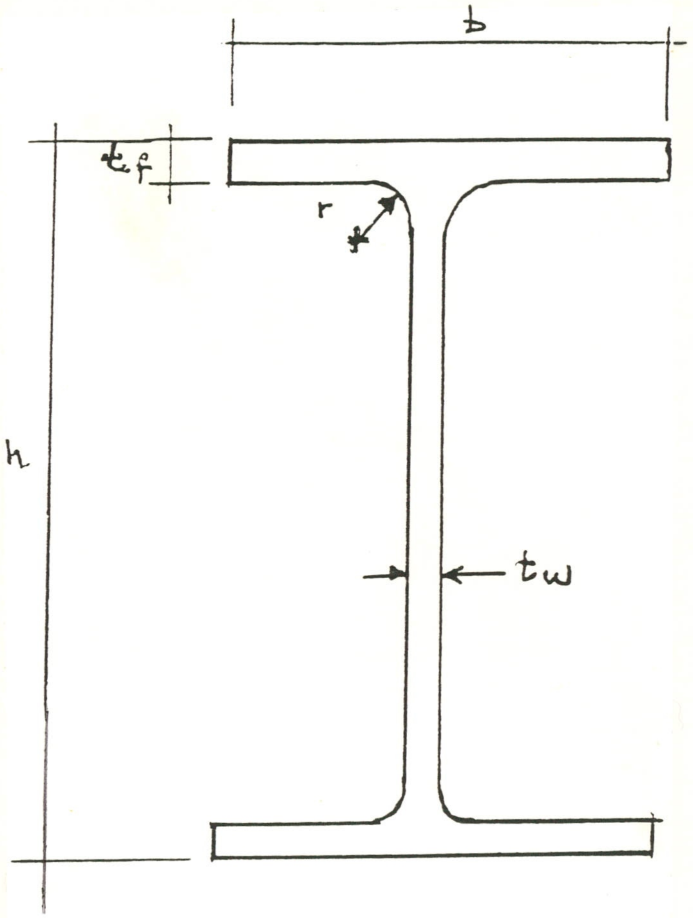


FIGURE 1: Cross-sectional description for the computer analysis

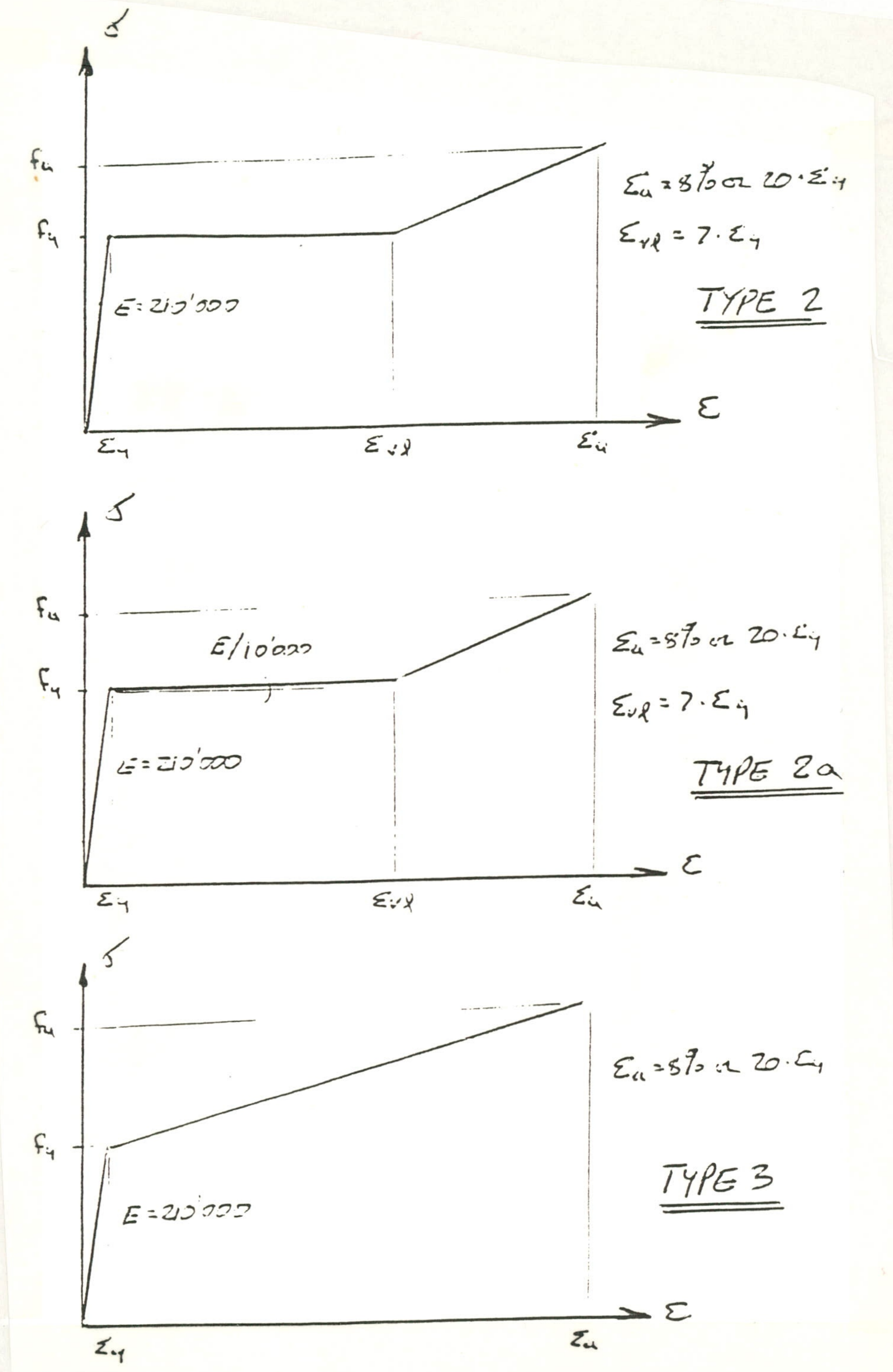


FIGURE 2: Standardised stress vs. strain behaviours taken from the Dutch code [1]

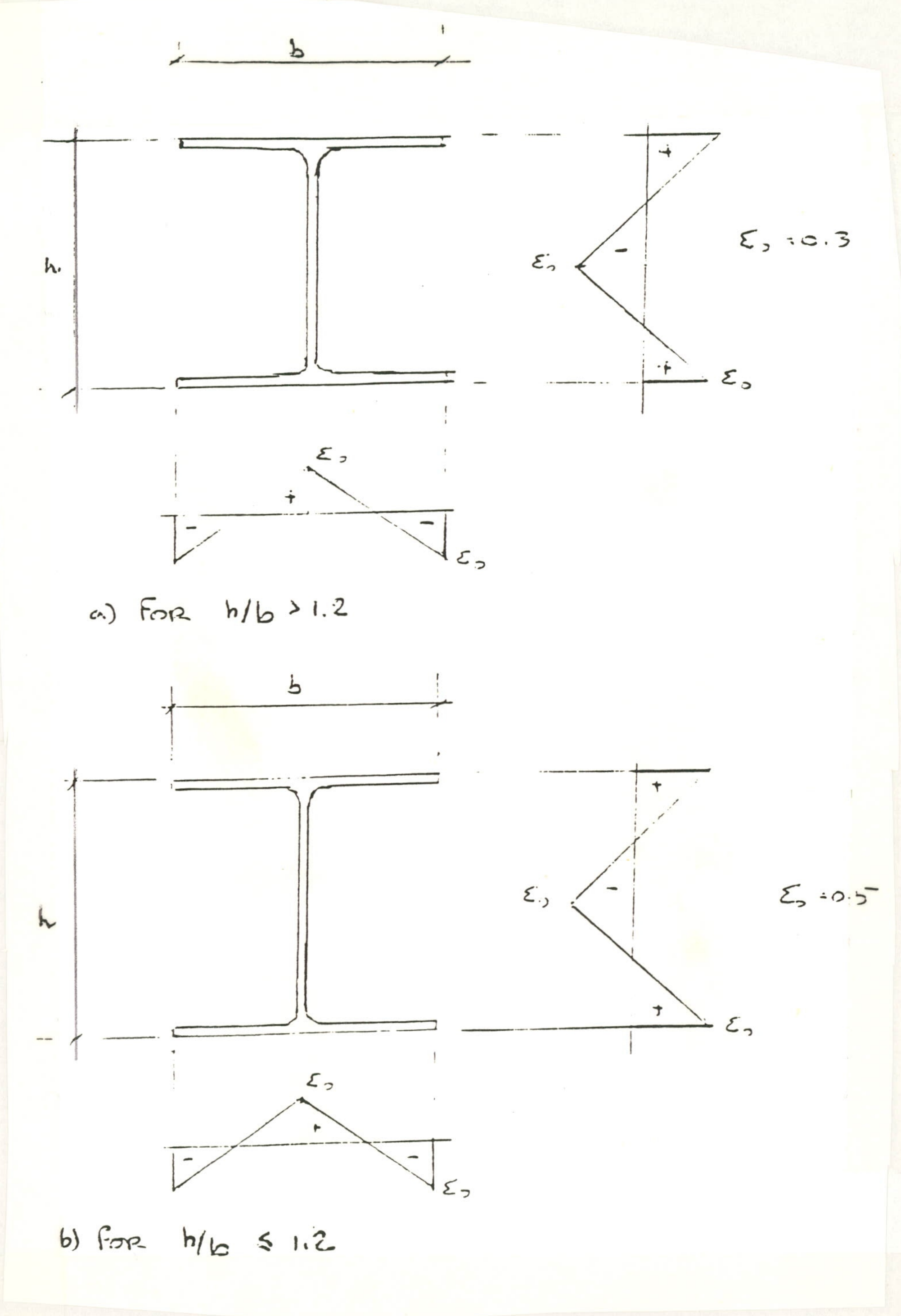


FIGURE 3: Relative residual stress models taken from the Dutch code [1]

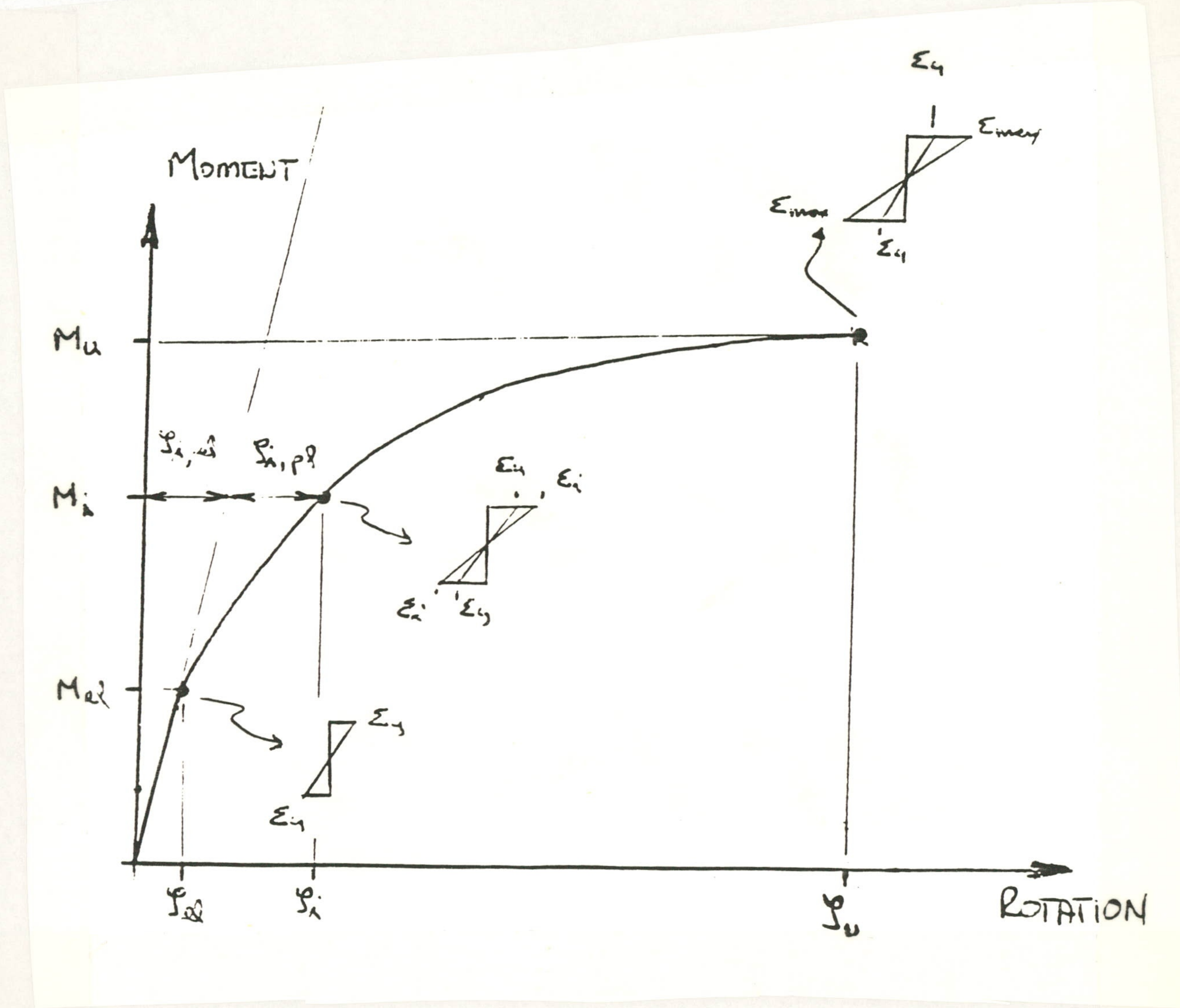


FIGURE 4: Illustration of the numerical solution process (in cross-section)

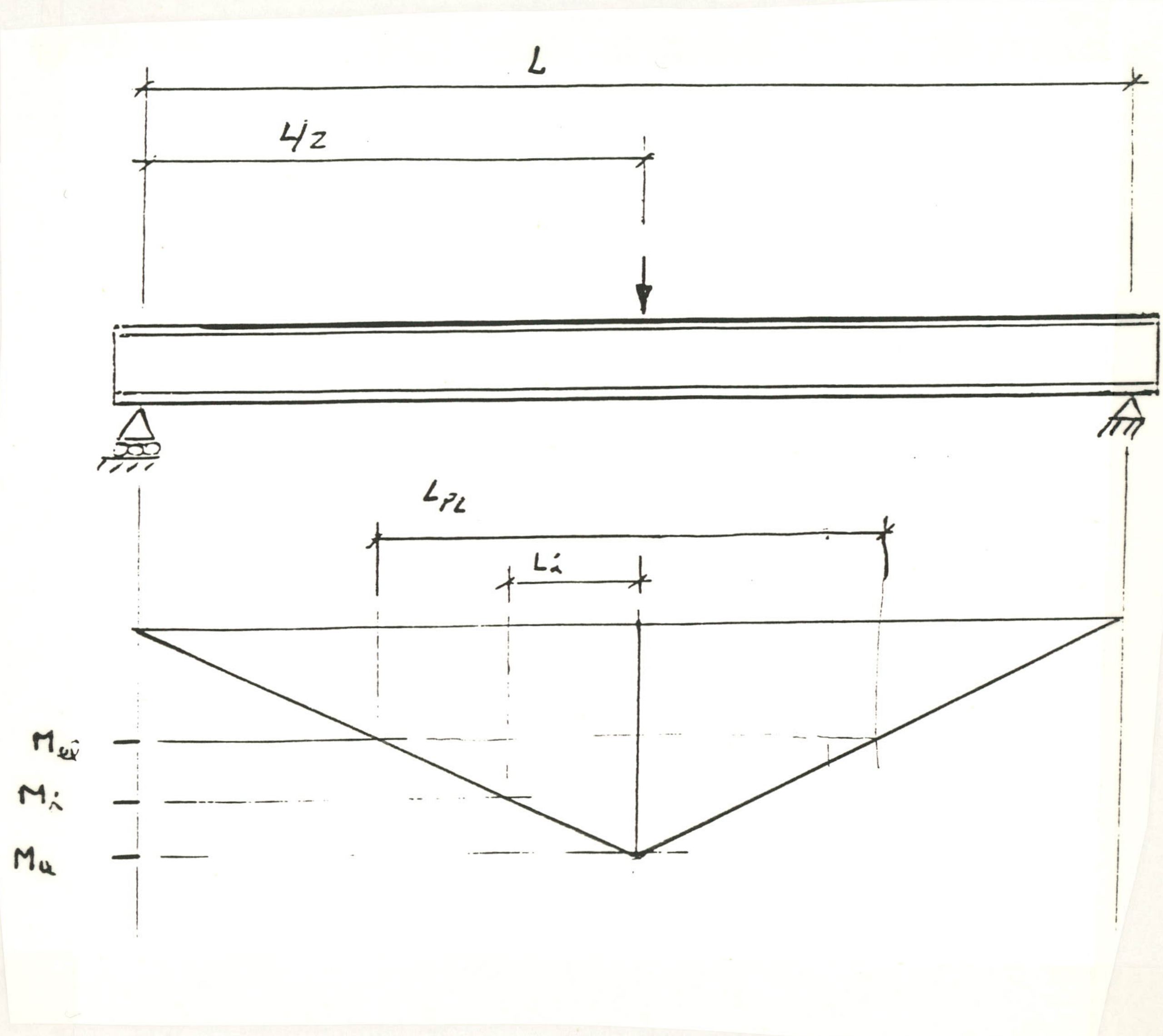


FIGURE 5: Illustration of the solution process (along the span)

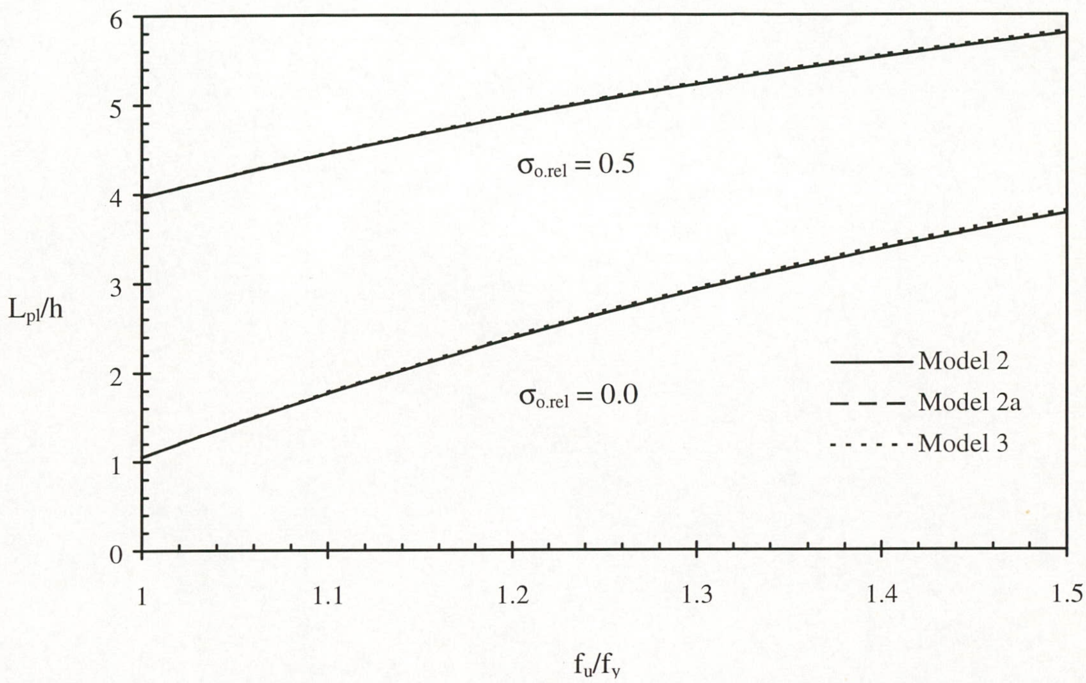


FIGURE 6a: Plastic hinge length as a function of f_u/f_y
- $f_y = 360$ MPa
- for an HE 140 A
- $L/h = 10$.

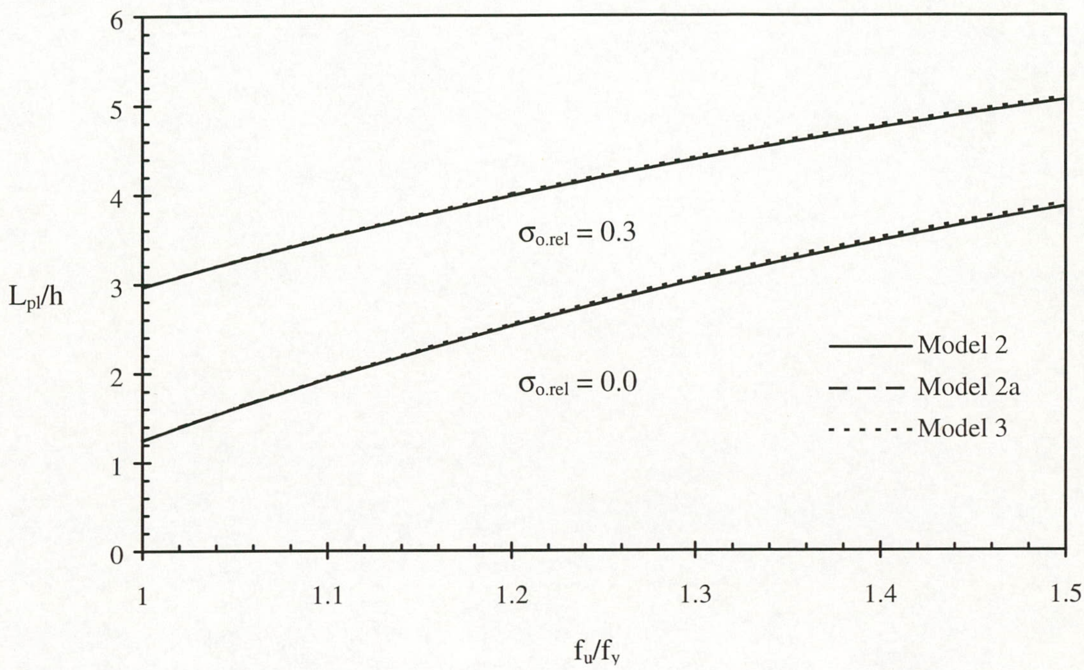


FIGURE 6b: Plastic hinge length as a function of f_u/f_y
- $f_y = 360$ MPa
- for an IPE 140
- $L/h = 10$.

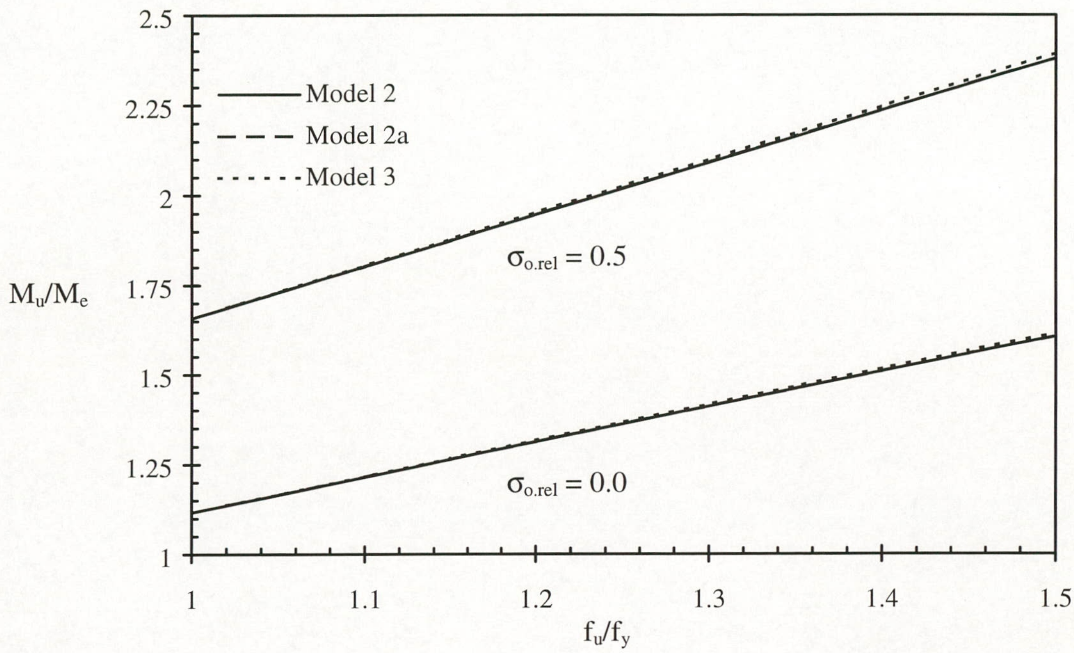


FIGURE 7a: Plastic hinge moment capacity as a function of f_u/f_y
- $f_y = 360$ MPa
- for an HE 140 A
- L/h = 10.

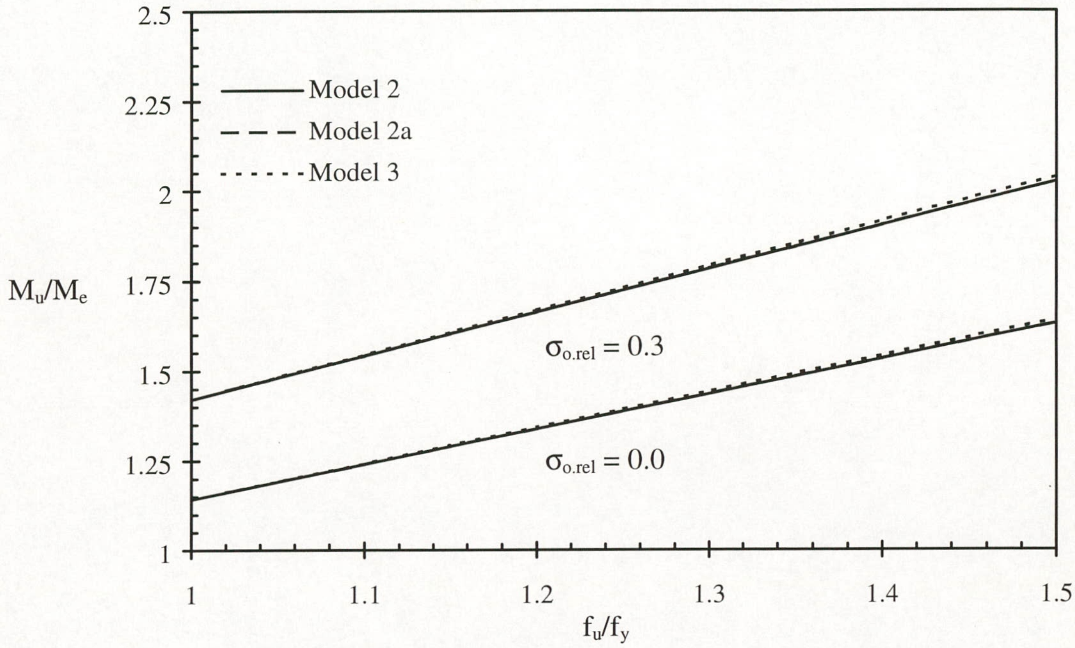
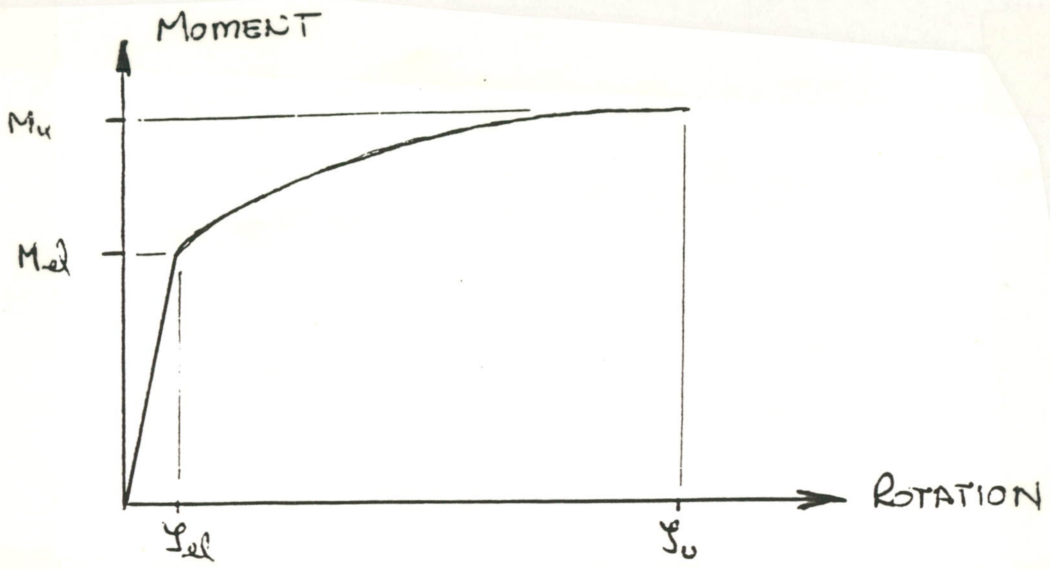
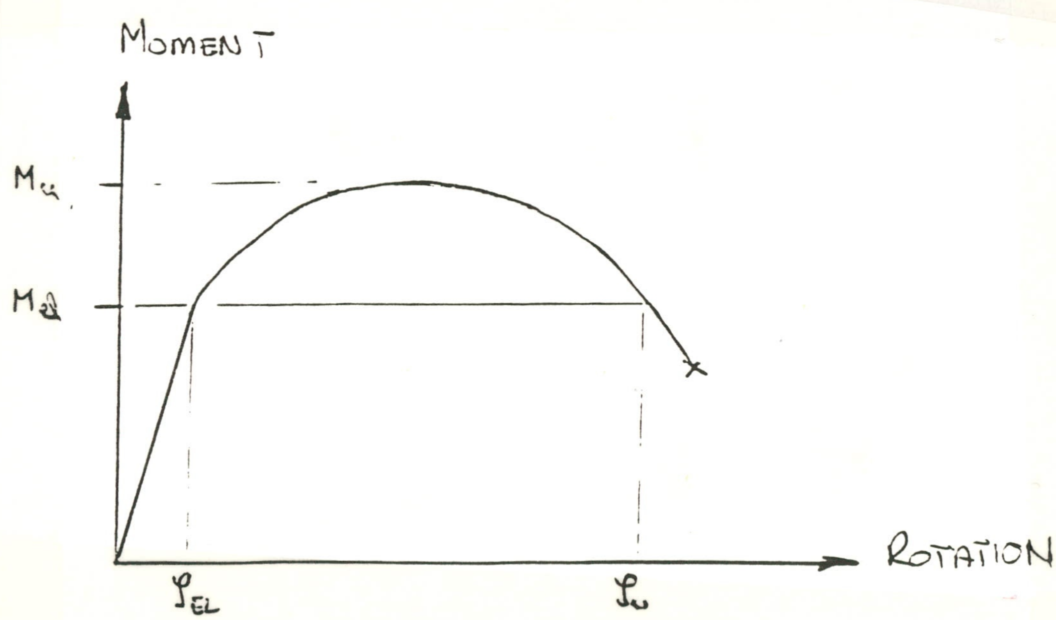


FIGURE 7b: Plastic hinge moment capacity as a function of f_u/f_y
- $f_y = 360$ MPa
- for an IPE 140
- $L/h = 10$.



a) Definition of R_{aval} based upon standard material models and force controlled tests



b) Definition of R_{aval} based upon deformation controlled tests

FIGURE 8: Definitions of R_{aval}

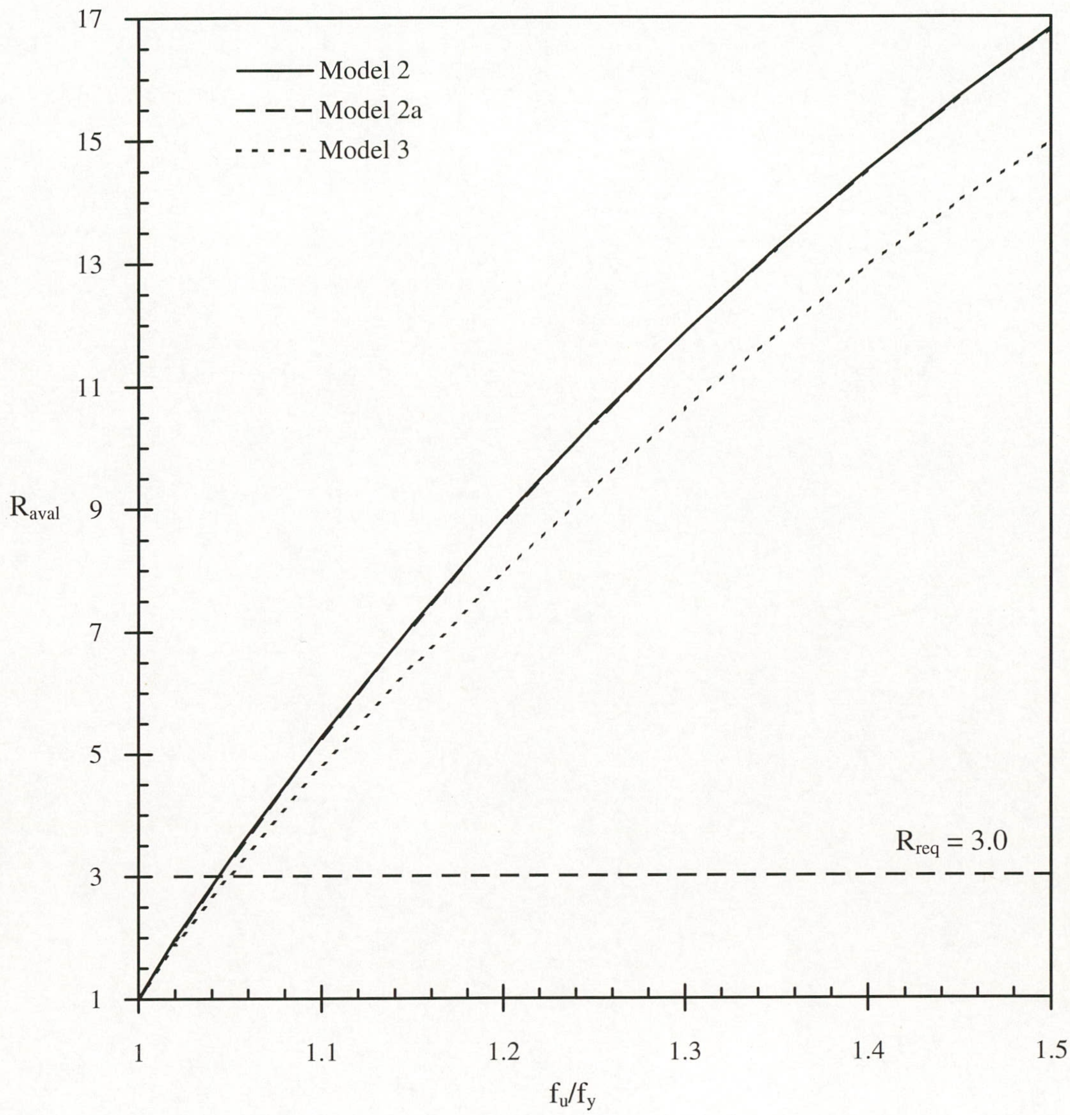


FIGURE 9: R_{aval} as a function of f_u/f_y:
- $f_y = 360$ MPa
- for an HE 140 A

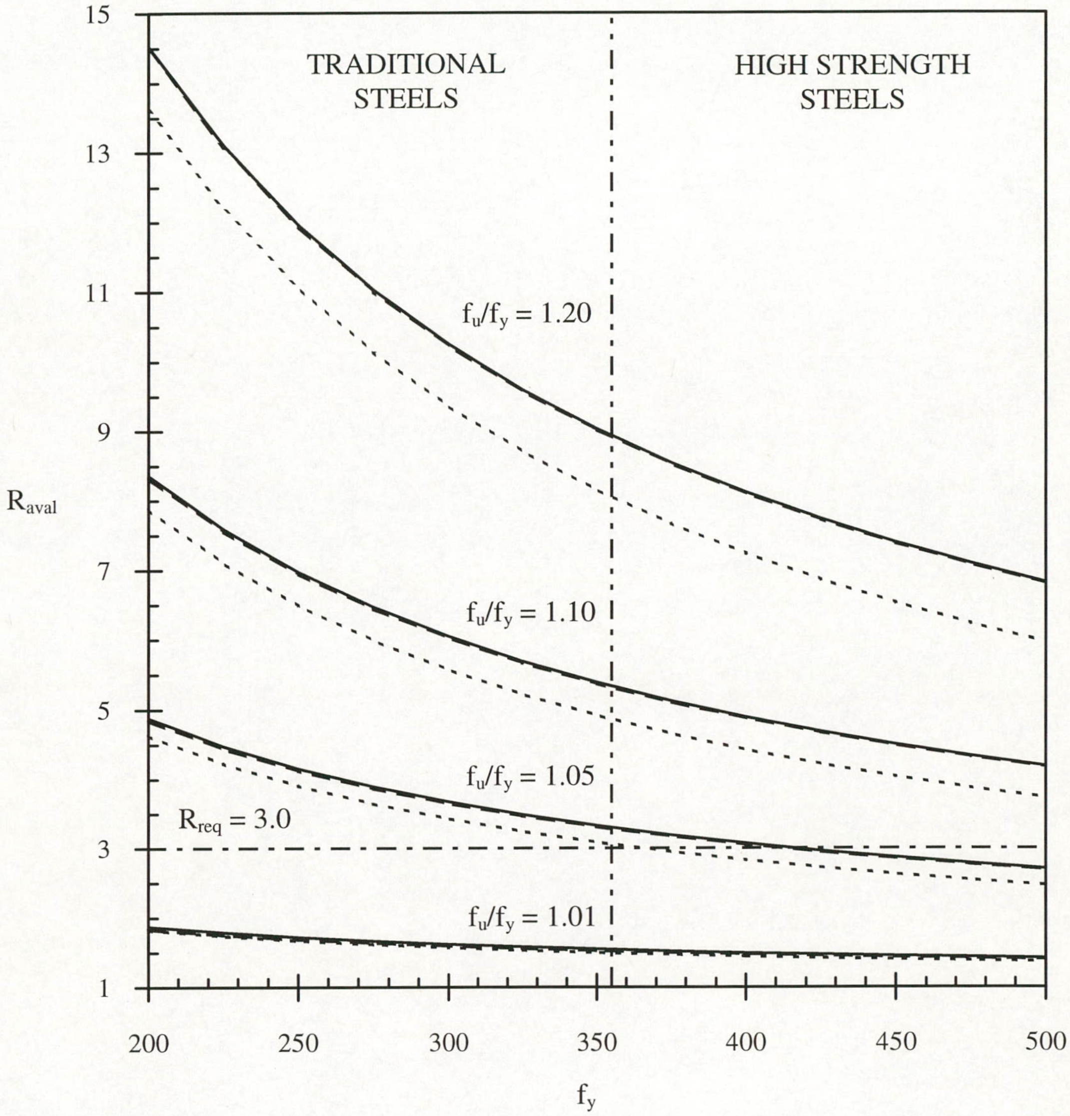


FIGURE 10: R_{aval} as a function of f_y:
- For an HE 140 A