Economic Analysis of Transmission Expansion Planning With Price-Responsive Demand and Quadratic Losses by Successive LP

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***Abstract*—The growth of demand response programs and renewable generation is changing the economics of transmission. Planners and regulators require tools to address the implications of possible technology, policy, and economic developments for the optimal configuration of transmission grids. We propose a model for economic evaluation and optimization of inter-regional transmission expansion, as well as the optimal response of generators' investments to locational incentives, that accounts for Kirchhoff's laws and three important nonlinearities. The first is consumer response to energy prices, modeled using elastic demand functions. The second is resistance losses. The third is the product of line susceptance and flows in the linearized DC load flow model. We develop a practical method combining Successive Linear Programming with Gauss-Seidel iteration to co-optimize AC and DC transmission and generation capacities in a linearized DC network while considering hundreds of hourly realizations of renewable supply and load. We test our approach for a European electricity market model including 33 countries. The examples indicate that demand response can be a valuable resource that can significantly affect the economics, location, and amounts of transmission and generation investments. Further, representing losses and Kirchhoff's laws is also important in transmission policy analyses.**

***Index Terms*—Demand response, nonlinear optimization, successive linear programming, transmission planning.**

NOMENCLATURE

Sets and Indices:

H	Set of hours, indexed h, each represents a different combination of load and renewable output.
I	Set of buses, indexed i, j.
N	Set of generation firms, indexed n.
K	Set of generation technologies, indexed k.
L	Set of AC corridors, indexed l.
U	Set of DC corridors, indexed u.

Parameters:

A_{ih}, B_{ih}	Inverse demand function parameters ($A_{ih} > 0, B_{ih} < 0$).
CX_l	Annualized capital cost of AC link [€/year].
CY_{ik}	Annualized capital cost of generator [€/MW/year].
CZ_u	Annualized capital cost of HVDC link [€/year].
D_{ih}	Fixed demand under no demand response [MW].
$\overline{F_l}$	Initial capacity of AC line [MW].
MC_{ik}	Marginal cost of generator [€/MWh].
NH_h	Number of hours per year.
P_u, R_l	Percentage of active power losses of DC line u and AC line l loaded at maximum capacity.
S_l	Susceptance of AC line [p.u.].
$\overline{T_u}$	Initial capacity of DC line [MW].
W_{ikh}	Maximum capacity factor of generator.
Y^0_{nik}	Initial installed generation of firm n [MW].
Φ_{il}	Node-line incidence matrix of AC lines.
Ξ_{iu}	Node-line incidence matrix of DC lines.

Variables:

a_{ih}	Net injection [MW].
d_{ih}	Forecast demand [MW].
$\overline{f}_{lh}, \underline{f}_{lh}$	Power flows on AC line [MW].
g_{nikh}	Generation dispatch level [MW].
p^*_{ih}	Locational marginal price [€/MWh].
$\bar{t}_{uh}, \underline{t}_{uh}$	Power flows on DC line [MW].
x_l	Expansion, as percent of increase, on AC corridor.
y_{nik}	Generation capacity addition [MW].
z_u	Expansion, normalized such that 1 is the existing capacity, on HVDC corridor.
θ_{ih}	Phase angle.
ρ_{ih}	Curtailed demand [MW].

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I. Introduction

THIS paper addresses the inclusion of demand response (via real-time prices and demand functions), quadratic transmission losses, and Kirchhoff's laws into a long-run policy model of transmission and generation expansion, and a practical method for solving that model using successive linear programming. These features have not been previously brought together in policy models.

The first feature we include, which is demand response that reacts to real-time or spot prices of electricity, was first proposed in the 1970s by Schweppe [1]. His spot pricing proposal was originally foreseen as a mechanism to balance supply and demand in an energy marketplace. Since then, most electricity markets around the world have implemented some form of spot pricing. However, many of those markets (especially in Europe) have not implemented a nodal version of Schweppe's spot pricing, preferring zonal or even copper plate constructs. Further, all these markets are largely or entirely one-sided, emphasizing generation scheduling and prices that reflect the marginal costs of generating power, with relatively little participation by the demand-side. Contrary to Schweppe's vision of supply and demand being equal partners, consumers tend to be treated as fixed loads, rather than equal parties who can modify their loads and who can submit bids that can affect prices and reflect the value of consumption. This reflects the reality that short-term demand remains price-insensitive because of retail rate regulation or absence of smart meters [2].

Even though Schweppe's original idea remains only partially implemented in current market designs, there has been a growing interest in increasing the flexibility of demand. Some benefits of demand response are load shifting from peak to off-peak hours, reducing the need for peaking generation capacity [3]; improved system reliability due to higher flexibility; and market power mitigation due to increased demand elasticity [4]. Furthermore, demand response can help operators cope with the variability of large amounts of variable renewable resources [5], [6]. Thus, power system planning and policy analysis need to account for how demand response and other smart grid technologies can help reduce the need for costly infrastructure investments [7].

However, transmission planning approaches used today usually take into consideration only two types of demand resources: narrowly focussed interruptible loads and energy efficiency measures [8], rather than loads that can respond to spot prices at any time. We attempt to correct this deficiency by introducing one type of demand response program into a transmission planning model, namely elastic demand functions.

Another way in which present transmission planning approaches are simplified is that they usually assume an exogenous pattern of generation capacity that is not affected by the costs or location of transmission. That is, a scenario of the locations, fuel-types, and amounts of generation capacity is assumed, and then the cost-minimizing transmission configuration needed to deliver that generation is defined [8]–[10]. However, transmission expansion not only can lower dispatch costs, it can also decrease the need for building generation by improved siting, generation mixes, and exploitation of load/resource diversity to lower reserve margin requirements. To rigorously consider those benefits, transmission and generation expansion, along with demand response, should be considered simultaneously in co-optimization models [11]–[14]. For example, one co-optimization study found that up to half of transmission's benefits could be in the form of reduced generation investment [15]. In a vertically-integrated utility environment, co-optimization can be interpreted as a type of integrated resource planning; in an unbundled environment, it is instead a type of anticipatory planning, in which the transmission grid owner projects how the grid configuration might affect the response of generation investment and operations.

In this paper, we propose a model for co-optimizing investments in electricity transmission and generation capacity, taking into account demand response, Kirchhoff's laws, generation intermittency, and quadratic resistance losses. Using a linearized DC power flow to approximate the effect of Kirchhoff's voltage on flows through AC lines, we assume that both transmission and generation investments can take place in small increments. This is of course a simplification, since in reality line and generator capacities come in discrete sizes. However, this simplification allows us to avoid the use of integer variables, which permits much larger models to be solved; furthermore, such a simplification is not an unreasonable approximation when considering broad patterns of transmission and generation many years or even decades in the future. However, since line susceptances are proportional to line capacities (given the assumed voltage and conductor type), enforcing Kirchhoff's voltage law (KVL) results in nonlinear model constraints. This nonlinearity can be avoided by assuming fixed PTDFs (as in [14]) or by using a transportation model and thereby ignore Kirchhoff's voltage law altogether, but the resulting flows and interacting economics of transmission, demand response, and generation could be greatly distorted [16]. Another nonlinearity in our model constraints results from incorporating quadratic losses [17]. Thus, assuming continuous capacity variables still results in a nonlinear model that is difficult to solve.

The large scale of real-world networks and the need to model the nonlinearities resulting from Kirchhoff's laws and quadratic losses, together with the inability of nonlinear solvers to solve large nonconvex problems reliably is a challenge that we attempt to overcome by using successive linear programming (SLP) [18]. Successive linear programming has been widely used in other disciplines to find optimal or high-quality solutions to large-scale industrial problems [19], including ones in the field of power, as discussed in our literature survey below.

In general, an SLP solution strategy consists of solving a sequence of linear programs in which the nonlinear objective function terms and constraints of the original nonlinear model are replaced with first-order approximations around the most recent solution, and then the resulting LP is solved to generate a new solution. The process is iterated until convergence, which can be guaranteed under restrictive conditions that are unfortunately not satisfied by our model. However, as we explain below, rapid convergence is achieved for our model when we combine iterative linearization of the transmission constraints with Gauss-Seidel iterations on load (in which, like the Project Independence Evaluation System (PIES) algorithm [20], the most recent energy balance duals are used as prices and are inserted into demand functions to update the values of load used in the

LP). The main advantage of our approach is the possibility of using out-of-the-box algorithms that can efficiently solve very large linear programs.

We test our approach on a European Electricity Market Model (a version of COMPETES [21]) for the year 2050 including flow-based market coupling of 33 countries, demand response, and intermittency in generation (based on the large-scale renewable penetration assumptions of IRENE-40 [22], [23]). From our examples, we observe, first, that disregarding Kirchhoff's Voltage Law and/or quadratic losses in policy models can distort the recommended transmission and generation additions, and, second, that demand response can be a valuable resource that can significantly affect the economics, location, and amounts of transmission investments.

This paper is organized as follows. In Section II we summarize the existing literature on transmission planning and demand response integration. Section III describes the formulation of our transmission-generation-demand response co-optimization model. The formulation is first introduced as a market equilibrium between independent but interacting transmission, generation, and consumer entities; then an equivalent single optimization model for computing that equilibrium is presented, followed by the combined SLP/Gauss-Seidel computational approach in Section IV. In Section V, we describe our test-case and summarize the results. Conclusion are presented in Section VI. Appendices present first-order (KKT) conditions for the individual market participant's optimization problems, and a multi-year version of our optimization model.

II. Literature Review: Approaches to Modeling Nonlinearities in Network Optimization

There exist a variety of optimization approaches to transmission planning [24]. For computational reasons, AC power flows are often modeled using linearized DC approximations that disregard reactive power and ohmic losses [1]. Network optimization approaches that disregard Kirchhoff's Voltage Laws can be purely linear (e.g., [25]), but this assumption could grossly distort transmission recommendations in networked transmission systems [26], [27]. Mixed-integer formulations improve upon this assumption by including Kirchhoff's Voltage Laws as linear disjunctive constraints [28]; however, this approach presents numerical difficulties when optimizing large-scale transmission networks with multiple investment alternatives. Sophisticated solution algorithms for large-scale planning problems are in Munoz *et al.* [29] and in Munoz and Watson [30]. We use a DC approximation of the application of Kirchhoff's voltage law to AC lines (thus ignoring reactive power flows and voltage constraints), and we represent high voltage DC lines as having controllable flows.

The lossless DC power flow model, which is commonly used in transmission planning models, has been improved by modeling losses assuming they are either proportional to line flow, a piecewise linear function of flow, or a quadratic function of flow [17]. Some models with losses optimize transmission additions using an objective function that minimizes the cost of investments and losses (e.g., [31]–[33]). However, those approaches assume an exogenous cost of losses, ignoring how the generation system is operated and the resulting marginal sources of generation in different hours. Other approaches seek to minimize the cost of transmission investments and operating costs by modeling power losses and generation dispatch explicitly in the system's constraints. Linear approximations (e.g., [34]) ignore the dependence of losses on line loading conditions, an assumption that can be improved using piecewise linear approximations in mixed-integer programming formulations (e.g., [35], [36]). None of those models consider demand response, and thus they disregard the potential cost savings from the implementation of demand programs that can take advantage of short-term price signals. Hence, our model improves on these approaches by combining the quadratic loss formulation (as in [1, Appendix A] or [17]) with elastic demand functions.

As illustrated in [6], the availability of short-term demand response can shift some electric loads from peak to off-peak hours, thereby reducing the need for investments in peaking generation. The benefits of demand response programs in transmission planning have been analyzed treating demand response exogenously, considering various load profile scenarios (e.g., [37]). However, in reality, shifts in electricity consumption would result from the interaction between demand elasticity and spot prices which, in general, can be location specific (locational marginal prices, LMPs). To the authors' knowledge, the only transmission planning models with endogenous consumers' response are [38], for price-responsive demand, and [39], where load-curtailment programs are considered. However, those studies disregard transmission losses. In general, the state of the art in modeling demand response in problems as diverse as generation planning, transmission economics, and unit commitment is to model the load as responding to the spot price using a demand curve [40] and, in one case, using cross-price elasticities to represent the shift in load from one period to another [6]. An important research topic is to incorporate more realistic representations of demand response. These could account, for instance, for required lead time (hours of notice), effects on load in earlier or later periods (sometimes called "rebound", which can occur because of pre-cooling behavior in air conditioned homes or consumer operation of thermal storage), and discretionary recharging of electric vehicles.

We assume a perfectly competitive electricity market and exclude market power in our model. This allows us to solve the transmission and generation planning problems as one optimization model. Real markets, of course, may depart from perfect competition: for instance, generators might behave oligopolistically [41]. Also, they might expand their capacity anticipating how market prices would respond while the network planner might expand transmission capacity anticipating how siting decisions by generators might change. Such interaction would result in multilevel imperfectly competitive Stackelberg models [42]–[44]. However, such multilevel models are computationally very difficult to solve in practical situations. For real life systems, model size increases exponentially with the geographical scope and the number of options for investments decisions. Another difficulty is that market power can be exercised and modeled in many different ways (see [41]), for instance as Nash-Cournot or Nash-supply function equilibrium games, conjectural variation games, or tacit collusion games. Choosing any single type of market power game is necessarily arbitrary, and, by introducing more uncertainty and modeler judgment in the results, makes them more difficult to defend in regulatory or business settings.

Although real generation markets may indeed depart from perfect competition, perfect competition models provide an essential benchmark for imperfect competition models. Real markets may also suffer from inefficiencies as a result of regulatory intervention or market design. Utilizing perfect competition models still allows policy makers to gain insights into the social implications of a market design or a policy target (e.g., such as the policy models IPM used by USEPA and NEMS used by USDOE/EIA). Furthermore, the experience of reformed markets in the U.S. and Europe indeed shows that market power mitigation instruments are effective and that properly designed reformed markets function competitively [45]–[47]. Thus, our model, which assumes perfect competition, is a reasonable starting point for policy analysis.

Our model is a large-scale nonlinear program. The objective function is nonlinear, involving the maximization of total market surplus, which equals the sum of the (nonlinear) integrals of the demand curves minus the sum of (linear) transmission and generation costs. The constraint set is also nonlinear. Since it includes nonlinear equality constraints, the feasible region is nonconvex, which complicates computation and also means that a local optimum may not be globally optimal. Large-scale nonlinear programs such as this are much more difficult to solve than linear programs, which leads us to consider SLP. In the field of power systems, SLP has often been applied to solve AC optimal power flow problems [48], [49] and reactive power planning problems [50]. However, it has not, to our knowledge, been used for transmission planning in which transmission capacity is a decision variable, much less for transmission-generation-demand response co-optimization. Under certain conditions that our model does not satisfy, the algorithm has been proven to have superlinear convergence [51], and is guaranteed to converge.

III. Model Description

We describe our modeling approach in three steps. First we pose a market equilibrium problem for a single year that assumes perfect competition (price-taking behavior) among all market parties, including the transmission owner, generators, and consumers. Second, we state a single optimization problem that is equivalent to the market equilibrium problem in which the sum of consumer, transmission, and producer surpluses (market surplus) is maximized. Third, we describe the combined SLP-Gauss-Seidel algorithm we use to solve that optimization model.

A. Market Equilibrium Problem

A market equilibrium has two characteristics. First, each market party pursues its own objective (its surplus), and believes that it cannot increase its surplus by deviating from the equilibrium solution. This is modeled by formulating the maximization problem for each party (profit maximization for generators, consumer surplus maximization for consumers, and transmission surplus maximization for the grid operator), and then deriving each problem's first-order (KKT) conditions. The second characteristic is that the market clears: supply equals demand for energy at each node in the network. The concatenation of KKT conditions for all market parties with market clearing equalities yields what is known as a complementarity problem, an increasingly common formulation of energy market equilibrium problems [41]. The complementarity model of this section can be viewed as a variant of short-run electricity market models in the literature (e.g., [17]) that include quadratic losses and capacity expansion, while assuming competitive rather than oligopolistic behavior.

Complementarity problems can be solved either by specialized algorithms or, in special cases, by instead formulating and solving an equivalent single optimization model. Real-world problems lead to large-scale complementary models that are computationally more complex to solve than an optimization problem. We adopt the single optimization problem approach (Section III-B), solving the problem by SLP, which reduces computational times significantly and allows us to address large-scale problems. However, before presenting the single optimization model for the entire market, we first present the optimization problem for each of the market players, in order to make clear the assumptions of the model. The models below and in Section III-B represent costs and revenues for a single year. In Appendix B, we generalize the static representation (at the expense of having a larger model) to a multiyear representation in which the timing of investments is also a decision.

Under perfect competition assumption, each market player is a price taker. Price taking behavior can be modeled by formulating price (which is signalled by an asterisk *) as an exogenous parameter in each market player's problem. First, we consider the generator's problem. Each firm chooses its generation production and capacity in order to maximize its annualized profits. For each firm $n \in N$

$$\underset{y,g}{\text{Max}} \sum_{i,k,h} NH_h(p^*_{ih} - MC_{ik})g_{nikh} - \sum_{i,k} CY_{ik}y_{nik} \quad (1)$$

$$\text{s.t.}\, g_{nikh} \leq W_{ikh}(Y^0_{nik} + y_{nik})(\mu_{nikh})\ \forall i,k,h \quad (2)$$

$$g_{nikh}, y_{nik} \geq 0 \quad \forall i,k,h \quad (3)$$

where constraints (2) and (3) correspond to maximum generation limits and variable non-negativity, respectively. To account for variability of renewable output, W_{ikh} is a coefficient less than or equal to one that varies depending on the hour h, technology k, and location of generator i. This model can readily be generalized to include nonlinear production cost functions, ramp limitations, and other more realistic considerations, with the exception of unit commitment constraints that require binary variables.

Meanwhile, consumers at each location i choose demand levels d_{ih} in each hour by maximizing their net surplus, given by the difference between their valuation of the consumption (which is the integral of their demand curve $P_{ih}(d_{ih}) = A_{ih} + B_{ih}d_{ih}$, summed across hours) and what they pay for electricity (the electricity price p^*_{ih} times consumption). For consumers in region $i \in I$

$$\underset{d}{\text{Max}} \sum_h NH_h \left[d_{ih} \left(A_{ih} + \frac{1}{2} B_{ih} d_{ih} \right) - p^*_{ih} d_{ih} \right] \quad (4)$$

$$\text{s.t. } d_{ih} \geq 0 \, \forall h. \quad (5)$$

Here we assume that cross-price elasticities are zero, thereby accounting only for own-price elasticities. More general formulations can consider cross-price elasticities across hours [6] or pricing rules that average over zones or otherwise deviate from the pure LMP model [52]. Estimating the coefficients of the demand curves requires estimation of demand elasticities, for which there are some relevant econometric studies [53], [54]. We disregard the possibility of loss of load (unserved demand).

The grid planner and operator is modeled as a pool operator: it buys power directly from generators and sells it to consumers.[1] The planner and operator is assumed to be a single entity, although in reality there are multiple operators (leading to seams issues) who can also be separate from grid owners. The operator's objective is to choose optimal investment in transmission capacity maximizing the value of its transmission services (i.e., revenues obtained from this arbitrage) minus the cost of losses and annualized expense of transmission investment subject to feasibility of flows:

$$\underset{f,t,\theta,x,z,a}{\text{Max}} \sum_{i,h} NH_h p^*_{ih} a_{ih} - \left[\sum_l CX_l x_l + \sum_u CZ_u z_u \right] \quad (6)$$

s.t.

$$\begin{aligned} &\sum_l \Phi_{il} \left[\left(\overline{f}_{lh} - r_l(x_l) \left(\frac{1+\Phi_{il}}{2} \right) \overline{f}^2_{lh} \right) \right. \\ &\quad \left. - \left(\underline{f}_{lh} - r_l(x_l) \left(\frac{1-\Phi_{il}}{2} \right) \underline{f}^2_{lh} \right) \right] \\ &\quad + \sum_u \Xi_{iu} \left[\left(\overline{t}_{uh} - o_u(z_u) \left(\frac{1+\Xi_{iu}}{2} \right) \overline{t}^2_{uh} \right) \right. \\ &\quad \left. - \left(\underline{t}_{uh} - o_u(z_u) \left(\frac{1-\Xi_{iu}}{2} \right) \underline{t}^2_{uh} \right) \right] \\ &\quad - a_{ih} = 0 \; (\chi_{ih}) \, \forall i, h \end{aligned} \quad (7)$$

$$\sum_i a_{ih} = 0 \; (\psi_h) \, \forall h \quad (8)$$

$$\overline{f}_{lh} - \underline{f}_{lh} - S_l(1+x_l) \sum_{i \in I} \Phi_{il} \theta_{ih} = 0 \; (\lambda_{lh}) \quad \forall l, h \quad (9)$$

$$\overline{f}_{lh} - \overline{F_l}(1+x_l) \leq 0 \; (\xi^+_{lh}) \quad \forall l, h \quad (10)$$

$$\underline{f}_{lh} - \overline{F_l}(1+x_l) \leq 0 \; (\xi^-_{lh}) \quad \forall l, h \quad (11)$$

$$\overline{t}_{uh} - \overline{T_u} z_u \leq 0 \; (\beta^+_{uh}) \, \forall u, h \quad (12)$$

$$\underline{t}_{uh} - \overline{T_u} z_u \leq 0 \; (\beta^-_{uh}) \, \forall u, h \quad (13)$$

$$\overline{t}_{uh}, \underline{t}_{uh}, \overline{f}_{lh}, \underline{f}_{lh}, x_l, z_u \geq 0 \quad \forall l, u, h. \quad (14)$$

Constraints (7) and (9) are Kirchhoff's Current and Voltage Laws, respectively. (10)–(11) are maximum and minimum flow limits for AC lines, (12)–(13) are maximum and minimum flows limits for DC lines,[2] and (14) is the non-negativity constraint.

Since the transmission operator is a price taker, maximization of the value of its transmission services efficiently allocates the transmission capacity. There also exists an alternative formulation of the operator's problem where its objective is to distribute the power among consumers to maximize social welfare, given firm's generation decisions [52]. Under perfect competition assumption, both formulations yield the same equilibrium [55], [56], which is also the market surplus maximization in the perfect competition case.

Note that we only consider AC upgrades in existing corridors and ignore the option of building new AC links that would create new parallel flows in the system. The advantage of this assumption is the continuity of the nonlinear optimization problem, which in our experience increases the likelihood of convergence of the successive linear programming approach of the following section to a KKT point. A system with new AC corridors that create new loops in the system presents a physical discontinuity for extremely small line upgrades, since constraint (9) should not restrict the angle difference in the absence of transmission investments. As a result, the derivative of the constraint with respect to capacity does not exist when AC line capacity is zero, although this derivative is required by the SLP algorithm.

Just for illustration purposes, we assume in our application that the quadratic loss coefficients are inversely proportional to the line capacities, approximated as $r_l(x_l) = (R_l)/(\overline{F_l}(1+x_l))$ for AC lines, and as $o_u(z_u) = (P_u)/(\delta + \overline{T_u} z_u)$ for DC lines. The factors R_l and P_u correspond to the fraction of active power losses when the lines are loaded at their maximum capacity (e.g., 5%) and depend on line lengths and characteristics. The term $\delta > 0$ in the denominator of the definition of $o_u(z_u)$ is used to avoid $o_u(z_u) \to \infty$ if $z_u \to 0$, since we are considering investment alternatives for new DC corridors. This correction term δ results in a slight underestimation of the losses for $\overline{T_u} z_u \ll 1$. Note that our definitions of the quadratic loss coefficients for both AC and DC lines imply that doubling the capacity will cut transmission losses in half for a given MW flow over the line. It also implies that all AC line additions have the same voltage and conductor characteristics (per MW of capacity) as the existing line(s) in its corridor.

To complete the equilibrium model we need to define the KKT conditions for the above surplus maximization problems, and then add market clearing conditions. The KKT conditions (21)–(39) are given in Appendix A for the generators, consumers, and the grid planner and operator.

Finally, the market clearing conditions (15) in the equilibrium model correspond to the balance between transmission imports/exports, transmission losses, generation, and demand for each bus at every hour:

$$a_{ih} + \sum_{n \in N} \sum_{k \in K_i} g_{nikh} = d_{ih} \; (p^*_{ih}) \, \forall i, h. \quad (15)$$

[1]The alternative to pool market is a market mechanism based on bilateral transactions between producers and consumers. Competitive generators purchase transmission services for these transactions from the system operator who prices scarce transmission capacity to ration it efficiently. Most electricity markets can be classified as of being pool, bilateral or its variants. Under perfect competition assumption, both markets yield the same equilibria [55].

[2]To keep the model compact, we write the maximum flow constraints for DC lines as if the initial installed capacity was zero. If there is an existing DC line (i.e., $z_u = 1$ already), this can be modeled by setting the associated investment cost to zero without the need to reformulate the model.

The market prices are endogenous to the whole system where the Lagrange multipliers of these conditions (p^*_{ih}) correspond to the market's hours weighted LMPs. At equilibrium, the generation, demand, and prices are in balance, satisfying all the KKT and market clearing conditions. Prices adjust to clear the markets, reflecting for instance the effects of demand response, transmission additions, and generation operation on prices. However, a key point about the competitive market model is that individual market players do not recognize that their actions affect the price; price is viewed as exogenous by each player, but endogenous to the market as a whole.

The KKT and market clearing conditions together define a square system of nonlinear complementarity and/or equality conditions, in which the number of conditions equals the number of variables [41]. This system could be solved for the market equilibrium by using commercial complementarity solvers such as PATH [57]. However, nonlinear complementary problems for large-scale systems are computationally challenging, being limited to thousands or tens of thousands of variables in practice. Thus we take another approach to reduce computational complexity, described next, based on successive linear programming, which can be solved for problems with millions of variables.

B. Equivalent Nonlinear Optimization Problem

We obtain the equilibrium by formulating and solving a single nonlinear optimization problem (NLP) whose KKT conditions are equivalent to the equilibrium problem of Section III-A (defined by the KKT conditions of all the market agents plus market clearing conditions in Appendix A). Formulation of an NLP may not be possible for general complementarity problems, but it is often feasible for problems formulated under an assumption of perfect competition (see, e.g., [41], [58]).

Because the optimal solution of the below model must satisfy the model's KKT conditions, therefore the solution is also a market equilibrium; the reverse also applies, in that a solution satisfying the market equilibrium conditions above maximizes social welfare (assuming that second order conditions are satisfied). The model is as follows:

$$\begin{aligned}\underset{f,t,\theta,x,z,a,y,g,d}{\text{Max}} \ & \sum_{i,h} NH_h \left[d_{ih}\left(A_{ih} + \frac{1}{2}B_{ih}d_{ih}\right) \right. \\ & \left. - \sum_{n,k} MC_{ik} g_{nikh} \right] \\ & - \left[\sum_l CX_l x_l + \sum_u CZ_u z_u + \sum_{n,i,k} CY_{ik} y_{nik} \right] \quad (16)\end{aligned}$$

$$\begin{aligned}&\text{s.t. } (2), (3) \forall n, (5) \forall i, (7) - (14), \\ &a_{ih} + \sum_{n\in N}\sum_{k\in K_i} g_{nikh} = d_{ih} \quad \forall i, h. \quad (17)\end{aligned}$$

The objective can be interpreted as total market surplus. This is the sum of the objectives for generators, consumers, and the grid operator; note that all revenue terms (involving energy price p^*_{ih}) from the objective functions of the individual player problems cancel, leaving only the integral of the demand functions minus all transmission and generation costs.

IV. SLP/Gauss-Siedel Solution Approach

A. Decomposition Into Supply and Demand Models

SLP can be applied directly to the above nonlinear program. However, we instead have had more success in achieving rapid convergence by using the PIES [20] approach of dividing the overall supply-demand equilibrium problem into separate supply and demand models, and iterating between the two. In the PIES approach, the supply model is a linear program that, given a tentative set of energy demands, determines 1) how those demands are to be met from the available supply as well as 2) a set of prices equal to the marginal costs (duals) from the supply-demand balances in the model. The demand model in PIES is simply a statistically estimated (or, in our case, assumed) set of demand functions that, given prices from the supply function, calculates a new set of quantities demanded (loads) to be used in the next iteration of the supply model. In our application, the supply model is a transmission and generation cost minimization model, subject to fixed demands D_{ih} that can be curtailed at cost VOLL:

$$\begin{aligned}\underset{f,t,\theta,x,z,a,y,g,\rho}{\text{Min}} & \sum_l CX_l x_l + \sum_u CZ_u z_u + \sum_{n,i,k} CY_{ik} y_{nik} \\ & + \sum_{n,i,k,h} NH_h MC_{ik} g_{nikh} + \sum_{i,h} NH_h VOLL \rho_{ih} \quad (18)\end{aligned}$$

$$\begin{aligned}&\text{s.t.}(2), (3)\forall n, (7) - (14), \\ &a_{ih} + \sum_{n\in N}\sum_{k\in K_i} g_{nikh} + \rho_{ih} - D_{ih} = 0 \quad \forall i, h \quad (19) \\ &\rho_{ih} \geq 0 \quad \forall i, h \quad (20)\end{aligned}$$

where ρ_{ih} is the curtailed load, which we include to ensure that there is a feasible solution of the overall model. For fixed demand levels D_{ih}, the first order optimality conditions of this problem are equivalent to the KKT conditions of the generator's and grid operator's problems given in Appendix A, and the market clearing conditions (19) for inelastic demand. Thus, a solution of the above NLP can be taken as a perfectly competitive market equilibrium subject to the assumed fixed loads. The duals for the energy balance (19) for each i and h can then be inserted in the demand functions to calculate new D_{ih} for use in the next iteration of the supply model. In this way, we can iterate between this NLP and the demand functions, and if the procedure converges, it is an equilibrium.

B. Successive Linear Programming

Large-scale nonlinear optimization problems, like the above supply NLP, are difficult to solve. To overcome this, we describe here an approach using successive linear programming (SLP). In SLP, we solve the NLP via a sequence of linear programs where all the nonlinear functions are linearized by using their first-order Taylor series approximations. Abusing notation, if $F(x) = 0$ corresponds to the nonlinear constraints (i.e., (7) and (9)) in the NLP, the corresponding linear approximations at vicinity of a point x^k can be expressed as

$$F(x^k) + \nabla F(x^k)\Delta x = 0, \text{ where } \Delta x = x - x^k.$$

We impose a fixed step size $|\Delta x| \leq \gamma$, similar to Method II of [59], which converges quickly for our problem. We include demand response by combining the SLP for the supply model with Gauss-Seidel iteration using the inverse demand function. The SLP algorithm combined with Gauss-Siedel proceeds as follows:

Step 0: Provide a starting point x^0 to initialize the algorithm. In our case, we generate the starting point by first solving the planning problem ignoring demand response and losses and only enforcing KVLs for the existing AC transmission lines.

Step 1: For given x^{k-1} and D^{k-1}, solve the linear optimization problem subject to $F(x^{k-1}) + \nabla F(x^{k-1})(x^k - x^{k-1}) = 0$, yielding the primal solution x^k and dual solution p^k (i.e., electricity prices).

Step 2: Update demand $D^k = P^{-1}(p^k)$ where $P^{-1}()$ is the "vector-valued" inverse demand function.

Step 3: Check convergence of the demand and objective function value of NLP (18) using tolerance ε. If convergence is achieved, accept the solution (x^k, p^k, D^k), else set $k = k + 1$ and go to Step 1.

The superlinear convergence of SLP is guaranteed under certain conditions [39] that, unfortunately, our nonconvex NLP (16) does not satisfy. However, the approach has worked well in our application and converged to a solution. When the algorithm converges to a solution, convergence to a KKT point is guaranteed.[3] Since our NLP is nonconvex, a point satisfying the KKT conditions is not sufficient for global optimality. Convergence to a local optimum is guaranteed if the second order conditions are satisfied for some ε-neighborhood of that point [18]. Global solution is however not guaranteed since it is a nonconvex problem.

V. Case Study: 2050 EU Renewables Development

A. Assumptions

We apply the approach of Section IV-B to the European market model COMPETES [21] which includes 33 countries.[4] COMPETES assumes an integrated EU market where the trade flows between countries are constrained by Net Transfer Capacities (NTC). Network parameters are based upon [23]. The model also includes wind and solar intermittency. Hourly wind data are estimated from 2004 profiles given by [60] and hourly solar data are estimated from the profiles given by [61]. Pre-calculated hourly intermittent variable renewable and hydro generation is taken as must-run in the model.

As initial capacities, we use the existing generation capacities from the WEPPS 2010 database [62] and the ten-year network development plan of ENTSO-E [63]. For 2050, we consider the Renewables Scenario (RES) of IRENE-40 [22], [23]. In the RES scenario, the installed capacities of renewables and nuclear are taken as exogenous since investments/decommissioning for these technologies are assumed to be policy driven. Ambitious GHG targets and strong policy support are assumed to drive the deployment of renewable technologies. This includes large clustered offshore and onshore wind farms in the northwest, solar and wind in the south, and hydropower and biomass in central and northern Europe. Installed nuclear power capacity decreases to 115 GWe in 2050. In the inelastic demand case, electricity generation from various renewable energy sources amounts to 80% of total electricity generation in 2050, consistent with ECF's 80% renewable scenario [64]. We assume that only existing conventional power plants commissioned in/after 2010 are refurbished and operate in 2050, whereas older power plants are all decommissioned. Annual investment costs are estimated based on capital costs and economic lifetime assumptions in [21] for generation technologies and in [23], [65] for transmission technologies. For demand response, we assume that the price elasticity of demand is -0.05.

We use a sample of 50 representative hours selected using *k-means* clustering of loads and variable renewable generation [66]. The COMPETES model is solved using CPLEX 12.5 in AIMMS. Solutions are iterated up to 500 times to ensure convergence. The algorithm stops early if the moving average of the objective of the previous ten solutions falls within 0.001% of the current solution. Solutions are obtained within four hours on an Intel i5-2450M processor. The two primary cases considered below are the system with and without demand response. In addition, we have also considered the impact of omitting quadratic losses and Kirchhoff's voltage law as sensitivity analyses.

TABLE I
Annualized Costs With and Without Demand Response (DR)

Case	Costs [Billion €/yr]				
	Operations	New Gen	New AC	New DC	Total
No DR	104.49	13.21	1.18	2.72	121.59
DR	92.63	10.62	1.20	2.71	107.16

B. Costs Savings With Demand Response

In both cases the total annualized cost is dominated by the plant operations (85% of cost), with 10% of the costs made up by generation capacity and the remainder consisting of transmission capacity costs (see Table I).

In Table I, total cost is reduced by 11.9% with the addition of demand response. A € 14.4 B cost reduction is derived from generation savings; 82% of this reduction is from savings on operations, while the remaining 18% is from generation capacity reductions. The reduction in operations cost is mainly a result of shifting demand from peak hours. Net total electricity energy consumed decreases only by 0.15%. Transmission costs remain effectively constant between cases with a small net increase in costs resulting from a reduction in DC transmission of € 6.9 M combined with an increase in AC transmission of € 27.8 M. Demand response changes LMPs, as expected, with the increased prices during periods of low wind causing a shift of demand to times of more wind and lower loads.

[3]At convergence, $\Delta x \approx 0$ is optimal and x^k solves the linear approximation of the NLP problem. By [18, Theorem 4.3.7], x^k is a KKT point of the NLP problem (18) (i.e., generators' and TSO's problem). In addition, the convergence of D^k satisfies the KKT conditions of the consumers' problem and NLP (16).

[4]COMPETES includes 26 EU members (excluding Malta) and 7 non-EU countries (i.e., Norway, Switzerland, and Balkan countries). Every country is represented by a single node, except Luxembourg which is included in Germany, and Denmark, which split in two nodes due to its participation in two nonsynchronous networks.

TABLE II
METRICS OF CAPACITY SHIFTS IN INVESTMENT DECISIONS

Metric	Generation Capacity	AC Transmission	DC Transmission
Normalized Absolute Sum of Differences	26.10%	10.32%	3.88%
Normalized Δ Total Additions	-26.20%	3.81%	-1.45%

C. *Changes in Investment Decisions With Demand Response*

The differences between investment decisions made with and without demand response are measured by two metrics (see Table II). The first metric, which is the normalized absolute sum of differences, captures changes on a line-by-line basis, while the second metric, the normalized change in total additions, measures how the decisions change in the aggregate. If the magnitude of the two metrics is the same, it indicates that the general pattern of siting of investments remains constant but the magnitude of investment at those sites changes, which is the case with generation capacity. Each metric is normalized by the total additions in the solution without demand response. In the case of AC transmission decisions, the magnitudes of the two metrics differ significantly, indicating that there are significant shifts in the siting of lines. DC transmission decisions shift but not as much as AC decisions. However, there is no spatial shifting of generation capacity, with demand response simply reducing the amount of generation investment everywhere.

D. *Spatial Changes in Transmission Investments*

The shifts noted in transmission siting are portrayed in Figs. 1 and 2. With demand response, while the costs remain relatively constant, there is a net increase in transmission capacity, with AC capacity increasing while DC capacity decreases. Looking only at the locations of increased capacity relative to the no demand response case (Fig. 1), 32.5 GW of AC capacity and 5.5 GW of DC capacity are added. Most of the increased line capacity is located in Northern Europe. Fig. 2 shows corridors in which transmission investment is lower in the demand response case, with reductions of 15 GW of AC and 12.1 GW of DC capacity. The corridors with the most reductions are connected to buses where generation capacities are also reduced. The differences of greatest magnitude are reductions between Italy and Switzerland and additions between Belgium and the Netherlands. However, those corridors are heavily invested in within both cases. The largest corridor to be developed only in one case is a 400-MW addition between Albania and Serbia under demand response.

E. *Effect of Modeling Simplifications*

The impact of not including losses or KVLs is explored by starting with a base version of the COMPETES model without resistance losses or KVLs, and then adding those features and examining how the costs change (see Table III). In terms of percentages in each category of cost, transmission investments are affected far more than generation investments and operating costs. For some cost categories, losses have a bigger impact, while for others KVL is more important. The direction of impacts even differs. For instance, even though adding losses increases generation expenses, adding KVLs surprisingly decreases those costs (although the transmission cost increase more than makes up for that decrease).

Also, it turns out that losses and KVLs interact, with cost increases from adding both differing from the sum of their

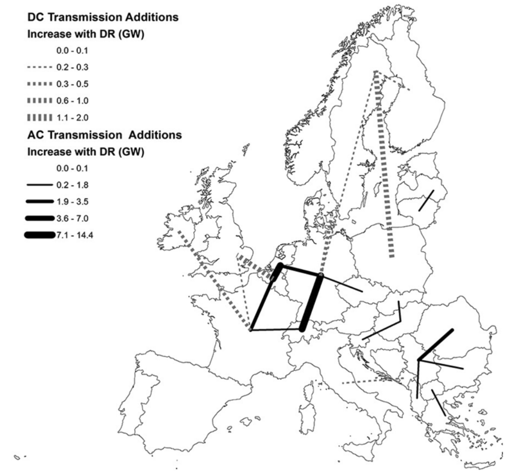

Fig. 1. Increases in investments in AC lines (solid) and DC lines (dashed) in COMPETES network as a result of adding demand response to model.

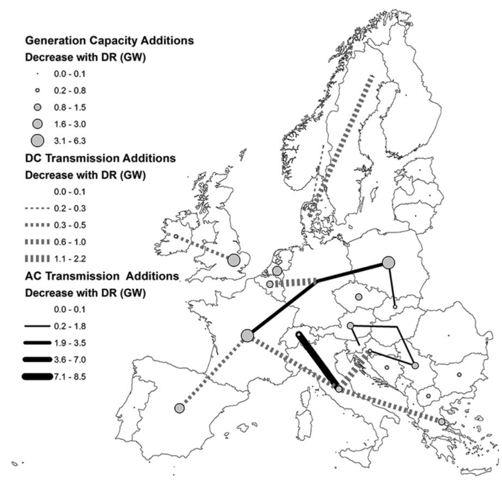

Fig. 2. Reductions in investments in AC lines (solid), DC lines (dashed), and generation capacity (circles) in COMPETES network as a result of introducing demand response.

TABLE III
ANNUALIZED INCREASE IN COSTS FROM INCLUDING LOSSES AND KVLS IN THE MODEL

Case	Comparison		Gen Operating	Gen Capital	AC Capital	DC Capital
DR	Adding Losses	ΔM€	1779.51	693.84	557.04	32.37
		Δ%	1.92%	6.54%	46.30%	1.20%
	Adding KVL	ΔM€	-214.36	4.79	158.43	347.40
		Δ%	-0.23%	0.05%	13.17%	12.83%
	Adding KVL and Losses	ΔM€	1973.05	586.64	337.44	496.01
		Δ%	2.13%	5.53%	28.05%	18.31%
No DR	Adding Losses	ΔM€	1669.63	274.54	581.68	2.11
		Δ%	1.60%	2.08%	49.50%	0.08%
	Adding KVL	ΔM€	-94.28	1.35	190.95	325.43
		Δ%	-0.09%	0.01%	16.25%	11.98%
	Adding KVL and Losses	ΔM€	1653.71	277.28	339.73	562.49
		Δ%	1.58%	2.10%	28.91%	20.71%

Note: Percentages relative to cost category in base case (either DR or no DR).

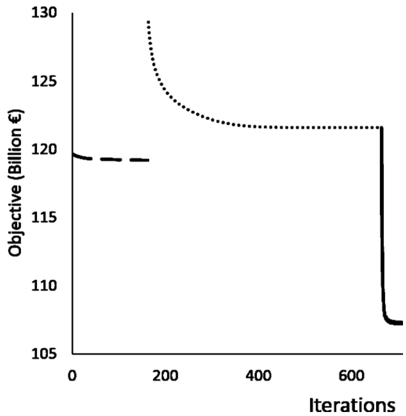

Fig. 3. Change in the objective as the model iterates for three cases: (a) with KVLs, no losses, and no demand response, (b) with KVLs, losses, and no demand response, and (c) with KVLs, losses and demand response.

individual impacts when added separately to the base case. For instance, adding losses makes more of a difference in DC line investment costs in a model with KVL than in a model without; and adding KVL makes more of a difference in a lossy model than a lossless one. Further, DC investments increase while AC investments decrease when adding both because enforcing KVLs results in power traveling further on AC lines, magnifying their losses, thereby putting new AC investments at a disadvantage compared to DC lines.

F. Convergence

As we mention in Section IV, convergence is not guaranteed for our nonconvex NLP. Nonetheless, the approach has worked well and converged to a solution numerically. Fig. 3 illustrates the convergence of the objective for three cases: 1) with KVLs, no losses, and no demand response, 2) with KVLs, losses, and no demand response, and 3) with KVLs, losses and demand response. The algorithm for each case stops if the current solution is within 0.001% of the average of the 10 previous solutions. The solution of the case with no losses and no demand response is used as the starting point of the case with losses and no demand response, whose solution is then utilized as the starting point of the case with losses and demand response.

The case with KVLs, no losses, and no demand response as well as the case with KVLs, losses, and no demand response converge stably to a solution. The case with demand response converges to a solution quite quickly but with slight oscillations. The final 50 iterations have a standard deviation of 0.032%.

VI. Conclusion

The inclusion of demand response, losses, generation co-optimization, and Kirchhoff's voltage law helps transmission policy and planning models to more realistically model the economics of investment. Due to the complexity and computational burden that those features add, they are frequently excluded from models, potentially distorting cost estimates and investment recommendations. We develop a practical method combining Successive Linear Programming with Gauss-Seidel iteration to co-optimize AC and DC transmission and generation capacities while considering demand response and system nonlinearities such as KVLs and resistance losses. We test our approach for an electricity market model COMPETES which represents transmission among 33 European countries. The results indicate that demand response can be a valuable resource that can significantly reduce generation operating and investment costs. Although the cost of transmission investments is affected only slightly, the siting of transmission investment decisions changes significantly. Thus, the potential benefits of demand response should be taken into account in long-term transmission planning.

Appendix A
KKT Conditions

The KKT conditions for each generation firm $n \in N$:

$$0 \le y_{nik} \perp -CY_{ik} + \sum_h NH_h \mu_{nikh} W_{ikh} \le 0 \; \forall i, k \quad (21)$$

$$0 \le g_{nikh} \perp p^*_{ih} - MC_{ik} - \mu_{nikh} \le 0 \; \forall i, k, h \quad (22)$$

$$0 \le \mu_{nikh} \perp g_{nikh} - W_{ikh} \left(Y^0_{nik} + y_{nik} \right) \le 0 \; \forall i, k, h. \quad (23)$$

The KKT conditions for the consumers in region $i \in I$:

$$0 \le d_{ih} \perp A_{ih} + B_{ih} d_{ih} - p^*_{ih} \le 0 \; \forall h. \quad (24)$$

The KKT conditions for the transmission grid operator:

$$0 \le \overline{f}_{lh} \perp -\sum_i \chi_{ih} \Phi_{il} \left[1 - r_l(x_l)(1 + \Phi_{il}) \overline{f}_{lh} \right] - \lambda_{lh} - \xi^+_{lh} \le 0 \; \forall l, h \quad (25)$$

$$0 \le \underline{f}_{lh} \perp \sum_i \chi_{ih} \Phi_{il} \left[1 - r_l(x_l)(1 - \Phi_{il}) \underline{f}_{lh} \right] + \lambda_{lh} - \xi^-_{lh} \le 0 \; \forall l, h \quad (26)$$

$$0 \le \bar{t}_{uh} \perp -\sum_i \chi_{ih} \Xi_{iu} \left[1 - o_u(z_u)(1 + \Xi_{iu}) \bar{t}_{uh} \right] - \beta^+_{uh} \le 0 \; \forall u, h \quad (27)$$

$$0 \le \underline{t}_{uh} \perp \sum_i \chi_{ih} \Xi_{iu} \left[1 - o_u(z_u)(1 - \Xi_{iu}) \underline{t}_{uh} \right] - \beta^-_{uh} \le 0 \; \forall u, h \quad (28)$$

$$\theta_{ih} \text{free} \perp \sum_l \lambda_{lh} S_l (1 + x_l) \Phi_{il} = 0 \; \forall i, h \quad (29)$$

$$\chi_{ih} \text{free} \perp \sum_l \Phi_{il} \left(\overline{f}_{lh} - r_l(x_l) \left(\frac{1 + \Phi_{il}}{2} \right) \overline{f}^2_{lh} \right) - \sum_l \Phi_{il} \left(\underline{f}_{lh} - r_l(x_l) \left(\frac{1 - \Phi_{il}}{2} \right) \underline{f}^2_{lh} \right) + \sum_u \Xi_{iu} \left(\bar{t}_{uh} - o_u(z_u) \left(\frac{1 + \Xi_{iu}}{2} \right) \bar{t}^2_{uh} \right) - \sum_u \Xi_{iu} \left(\underline{t}_{uh} - o_u(z_u) \left(\frac{1 - \Xi_{iu}}{2} \right) \underline{t}^2_{uh} \right) - a_{ih} = 0 \; \forall i, h \quad (30)$$

$$a_{ih} free \perp p^*_{ih} + \chi_{ih} - \psi_h = 0 \; \forall i, h \quad (31)$$

$$\psi_h free \perp \sum_i a_{ih} = 0 \; \forall h \quad (32)$$

$$\lambda_{lh} free \perp \overline{f}_{lh} - \underline{f}_{lh} - S_l(1+x_l)\sum_i \Phi_{il}\theta_{ih} = 0 \,\forall l,h \tag{33}$$

$$0 \le \xi^+_{lh} \perp \overline{f}_{lh} - \overline{F_l}(1+x_l) \le 0 \,\forall l,h \tag{34}$$

$$0 \le \xi^-_{lh} \perp \underline{f}_{lh} - \overline{F_l}(1+x_l) \le 0 \,\forall l,h \tag{35}$$

$$0 \le \beta^+_{uh} \perp \overline{t}_{uh} - \overline{T_u} z_u \le 0 \,\forall u,h \tag{36}$$

$$0 \le \beta^-_{uh} \perp \underline{t}_{uh} - \overline{T_u} z_u \le 0 \,\forall u,h \tag{37}$$

$$\begin{aligned} 0 \le x_l \perp &-CX_l \\ &+ \sum_{i,h} NH_h \chi_{ih} \Phi_{il} r'_l(x_l) \\ &\times \left[\left(\frac{1+\Phi_{il}}{2}\right)\overline{f}^2_{lh} - \left(\frac{1-\Phi_{il}}{2}\right)\underline{f}^2_{lh}\right] \\ &+ \sum_h NH_h \lambda_{lh} S_l \sum_{i\in I}\Phi_{il}\theta_{ih} \\ &+ \sum_h NH_h\left[\xi^+_{lh}\overline{F_l} + \xi^-_{lh}\overline{F_l}\right] \le 0 \,\forall l \end{aligned} \tag{38}$$

$$\begin{aligned} 0 \le z_u \perp &-CZ_u \\ &+ \sum_{i,h} NH_h \chi_{ih} \Xi_{iu} o'_l(z_u) \\ &\times \left[\left(\frac{1+\Xi_{iu}}{2}\right)\overline{t}^2_{uh} - \left(\frac{1-\Xi_{iu}}{2}\right)\underline{t}^2_{uh}\right] \\ &+ \sum_h NH_h\left[\beta^+_{uh}\overline{T_u} + \beta^-_{uh}\overline{T_u}\right] \le 0 \,\forall u. \end{aligned} \tag{39}$$

These KKT conditions, together with market clearing conditions in Section III-A, are the same as the KKT conditions for the single nonlinear optimization model of Section III-B.

Appendix B
Equivalent NLP for Multiyear Model

For the multiyear problem, the notation of the parameters and the variables is almost identical to the static representation except we denote their dependency with respect to the model decision stages $v, v' \in v_0, \ldots, v_T$. Each decision stage represents all modeled hours in one or more years and has an equal length of $M \ge 1$ years. At the beginning of each stage (e.g., v_0), the generators and the TSO make their investment decisions. The additional capacity resulting from these investment decisions becomes available at the beginning of the next stage (e.g., v_1). Next, we define the additional parameters and variables for the multiyear problem.

Parameters:

CIX_{lv}	Investment cost of AC link at stage v[€]
CIY_{ikv}	Investment cost of generator at stage v [€/MW]
CIZ_{uv}	Investment cost of HVDC link at stage v[€]
σ	Yearly interest rate [1/yr]
α_v	Discount factor at stage $v[\alpha_v = (1)/((1+\sigma)^{M(v-v_0)})]$
LY_{nk}	Construction time of generator [stages]
LX_l	Construction time of AC link [stages]
LZ_u	Construction time of HVDC link [stages]

Variables:

Δy_{nikv}	Incremental expansion of generation capacity at stage v [MW]
Δx_{lv}	Incremental expansion of AC line at stage v
Δz_{uv}	Incremental expansion of HVDC line at stage v

In general, multistage models can lead to mathematical problems with equilibrium constraints (MPEC) for each market player if it exercises market power. But since we assume a perfectly competitive electricity market with price-taking generators and TSOs (as [34]), this allows us to solve the multistage transmission and generation planning problems as one optimization model. Let GC_n, TC, WP_i be the discounted total investment and operational cost of generator n, discounted total cost of transmission investments, and the discounted willingness to pay of consumers at node i, respectively. Then we can obtain the equilibrium by formulating and solving a single nonlinear optimization problem:

$$\underset{f,t,\theta,\Delta x,\Delta z,a,\Delta y,g,d}{\text{Max}} \sum_i WP_i - \sum_n GC_n - TC \tag{40}$$

$$\begin{aligned} &\text{s.t. } (2),(3) \quad \forall n, \forall v > v_0 \\ &(5) \quad \forall i, \forall v > v_0 \\ &(7)-(14),(17) \quad \forall v > v_0 \end{aligned}$$

$$y_{nikv} = \sum_{v'=v_0}^{v-LY_{nk}} \Delta y_{nikv'} \quad \forall n,i,k,v \tag{41}$$

$$x_{lv} = \sum_{v'=v_0}^{v-LX_l} \Delta x_{lv'} \quad \forall l,v \tag{42}$$

$$z_{uv} = \sum_{v'=v_0}^{v-LZ_u} \Delta z_{uv'} \quad \forall u,v \tag{43}$$

where

$$\begin{aligned} GC_n = &\sum_{v=v_0}^{v=v_{T-1}} \alpha_v \sum_{i,k} CIY_{ikv}\Delta y_{nikv} \\ &+ \sum_{v=v_1}^{v=v_T}\alpha_v\left(\sum_{m=1}^{M}\frac{1}{(1+\sigma)^{m-1}}\right)\sum_{i,h,k} NH_h MC_{ikv} g_{nikhv}. \end{aligned}$$

$$TC = \sum_{v=v_0}^{v=v_{T-1}} \alpha_v\left(\sum_l CIX_{lv}\Delta x_{lv} + \sum_u CIZ_{uv}\Delta z_{uv}\right).$$

$$\begin{aligned} WP_i = &\sum_{v=v_1}^{v=v_T}\alpha_v\left(\sum_{m=1}^{M}\frac{1}{(1+\sigma)^{m-1}}\right)\sum_h NH_h \\ &\times\left[d_{ihv}\left(A_{ihv} + \frac{1}{2}B_{ihv}d_{ihv}\right)\right]. \end{aligned}$$

The problem includes investment accounting constraints (41)–(42) that ensure that the capacity available in a given stage equals the sum of capacity built in previous stages, accounting for lag times in construction. SLP can be applied directly to the above nonlinear program by utilizing linear approximations to the nonlinear function WP_i in the objective and to the nonlinear constraints (7) and (9) for each decision stage v. However, the resulting linear optimization problem is a computationally intensive large-scale model. We have solved linearized multistage models that are similar to the above (e.g., [12], [34]).

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