# Immersed Actuator Methods

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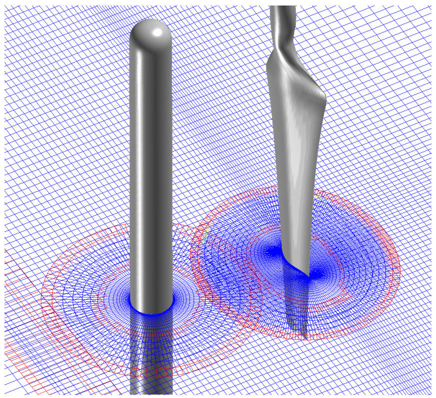
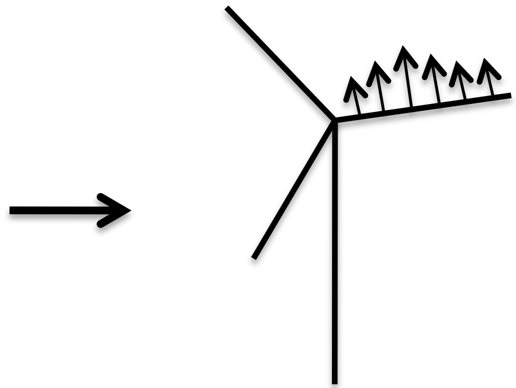
# Introduction

- Actuator methods for wind-turbine wake aerodynamics:
  - Circumvent meshing entire rotor blade
  - Circumvent resolving blade boundary layer
  - Actuator disk, line, surface, …





# Existing methods

- Actuators as volumetric body forces (Sørensen et al.)
  - Cartesian mesh
  - Discrete Dirac functions, regularization parameter
  - Time steps limited by movement of lines
- Actuators as singular surface forces (Masson et al.)
  - Surfaces carrying discontinuities in pressure and velocity
  - Body-fitted meshes
  - Based on inviscid aerodynamic theory
- Actuator shape model (Rethore)
  - Forcing based on intersectional polygon between actuator and mesh

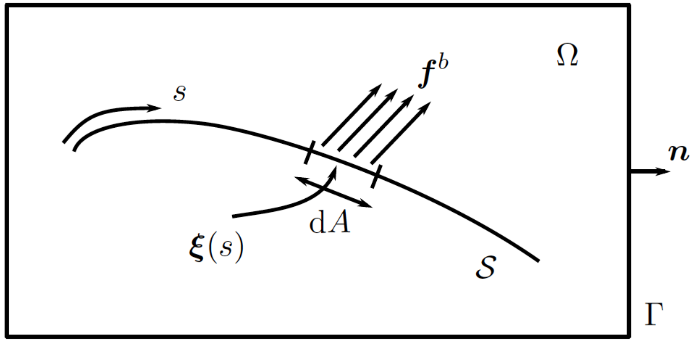
# Forces in the Navier-Stokes equations

- **Surface** forces in **integral** form

$$\int_\Omega \frac{\partial \boldsymbol{u}}{\partial t}\,\mathrm{d}\Omega + \int_\Gamma \boldsymbol{u}\,\boldsymbol{u}\cdot\boldsymbol{n}\,\mathrm{d}\Gamma = \int_\Gamma -p\,\boldsymbol{n}\,\mathrm{d}\Gamma + \int_\Gamma \nu(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T)\cdot\boldsymbol{n}\,\mathrm{d}\Gamma + \boldsymbol{F}$$

$$\boldsymbol{F} = \int_S \boldsymbol{f}^b(\boldsymbol{\xi}(s))\,\mathrm{d}A$$

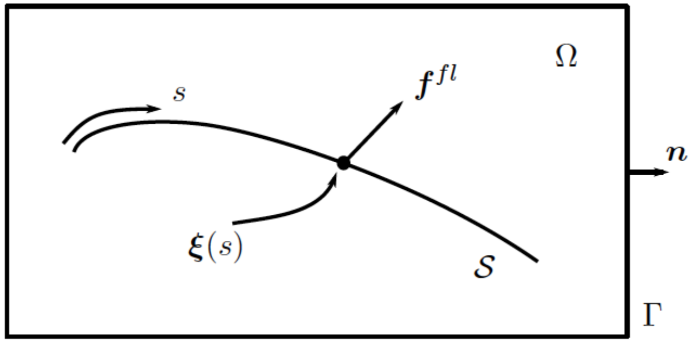
  - Examples: pressure forces, shear forces ($C_p$ , $C_f$ )



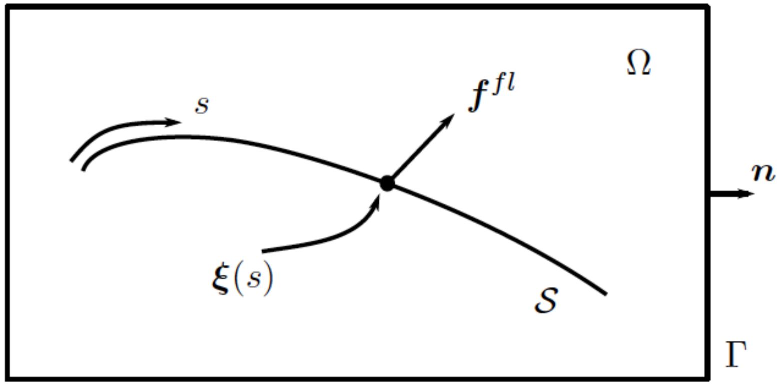

- **Volumetric** forces in **differential** form

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p + \nu\nabla^2\boldsymbol{u} + \boldsymbol{f}^{fl}(\boldsymbol{x})$$

$$\boldsymbol{f}^{fl}(\boldsymbol{x}) = \int_S \boldsymbol{f}^b(\boldsymbol{\xi}(s))\delta(\boldsymbol{\xi}(s) - \boldsymbol{x})\,\mathrm{d}A$$

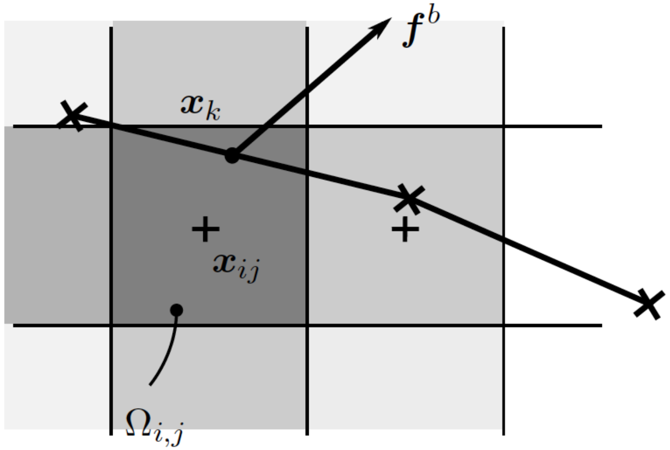


# Standard approach




**continuous**

$$\boldsymbol{f}^{fl}(\boldsymbol{x}) = \int_{\mathcal{S}} \boldsymbol{f}^{b}(\boldsymbol{\xi}(s))\delta(\boldsymbol{\xi}(s) - \boldsymbol{x})\,\mathrm{d}A$$

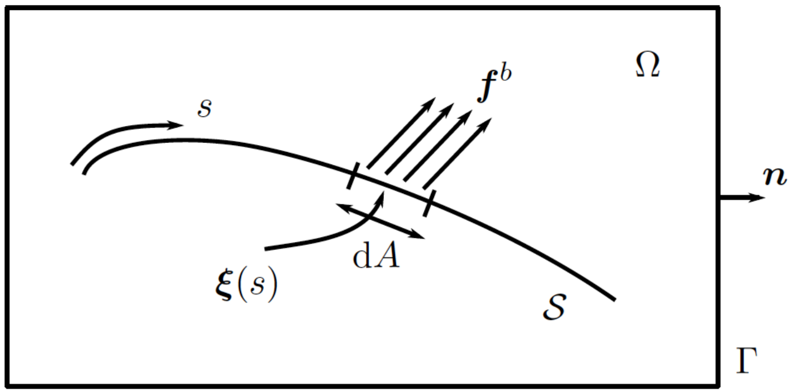



**discrete**

$$\boldsymbol{f}_{ij} = \sum_k \boldsymbol{f}_k d(\boldsymbol{x}_k - \boldsymbol{x}_{ij})$$

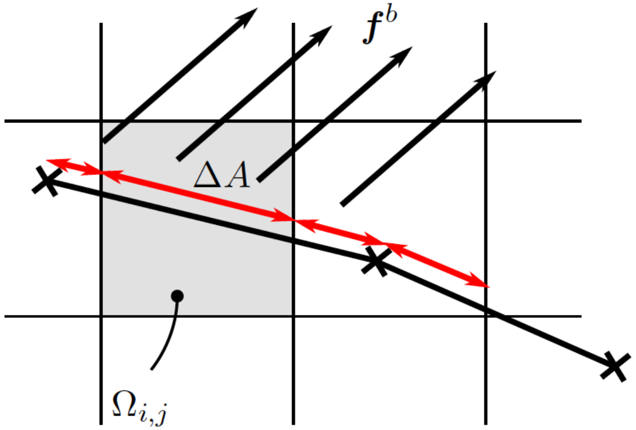
**discrete** Dirac function

# New approach




**continuous**

$$\boldsymbol{F} = \int_{\mathcal{S}} \boldsymbol{f}^b(\boldsymbol{\xi}(s))\,\mathrm{d}A$$




**discrete**

$$\boldsymbol{f}_{ij} = \boldsymbol{f}^b \Delta A$$

No discrete Dirac function required, but …

# Discontinuities

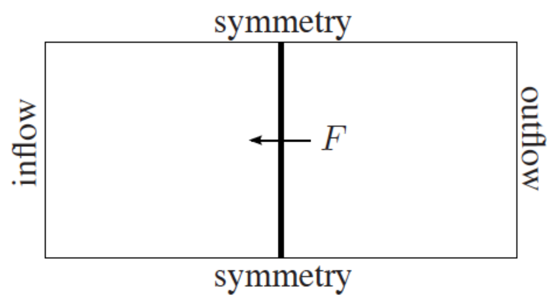
- Surface forces induce discontinuities

velocity is continuous $[\boldsymbol{u}] = 0$ $\quad \left[\mu \dfrac{\partial \boldsymbol{u}}{\partial n}\right] = -f_\tau^b \boldsymbol{\tau}$

pressure jumps $[p] = f_n^b$ $\quad \left[\dfrac{\partial p}{\partial n}\right] = \dfrac{\partial f_\tau^b}{\partial s}$

- Example: 1D actuator disk – pressure jumps due to normal force

$$\Delta p = -\tfrac{1}{2} C_T$$



# Handling discontinuities

- Discrete Dirac functions:
  - Easy implementation
  - Diffuse interfaces
  - Choice of Dirac function and regularization parameter
  - First order accurate in general
- Immersed interface methods (IIM)[1]:
  - **Take jump conditions as corrections into discretization scheme**
  - Second order (or higher) accurate
- **Immersed actuator methods (IAM)**:
  - IIM in finite volume framework
  - Application to actuators

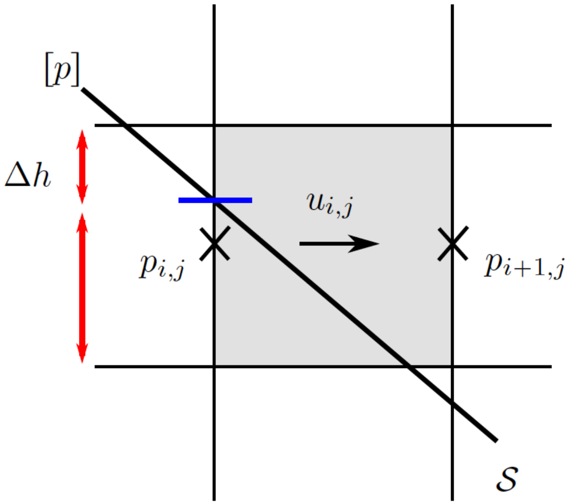
1: Z. Li and K. Ito, The Immersed Interface Method. Numerical Solutions of PDEs Involving Interfaces and Irregular Domains, SIAM 2006.

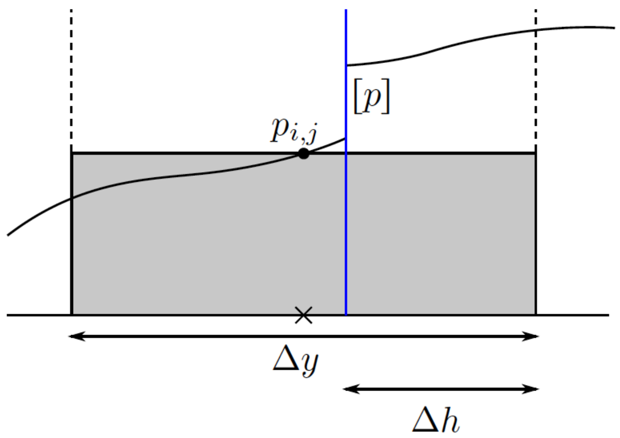
# IAM applied to Navier-Stokes

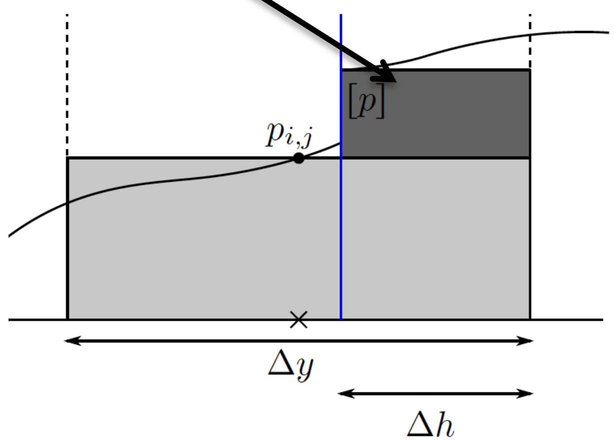
- Apply the IAM to the pressure gradient term:

$$\int_\Omega \frac{\partial \boldsymbol{u}}{\partial t}\,\mathrm{d}\Omega + \int_\Gamma \boldsymbol{u}\,\boldsymbol{u}\cdot\boldsymbol{n}\,\mathrm{d}\Gamma = \int_\Gamma -p\,\boldsymbol{n}\,\mathrm{d}\Gamma + \int_\Gamma \nu(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T)\cdot\boldsymbol{n}\,\mathrm{d}\Gamma + \boldsymbol{F}$$

- Jump correction: $\displaystyle\int p\,n_x\,\mathrm{d}y = p_{i,j}\Delta y + [p]\Delta h$

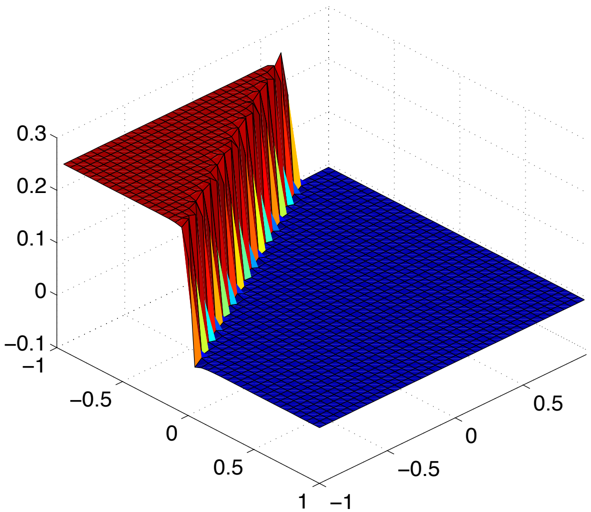


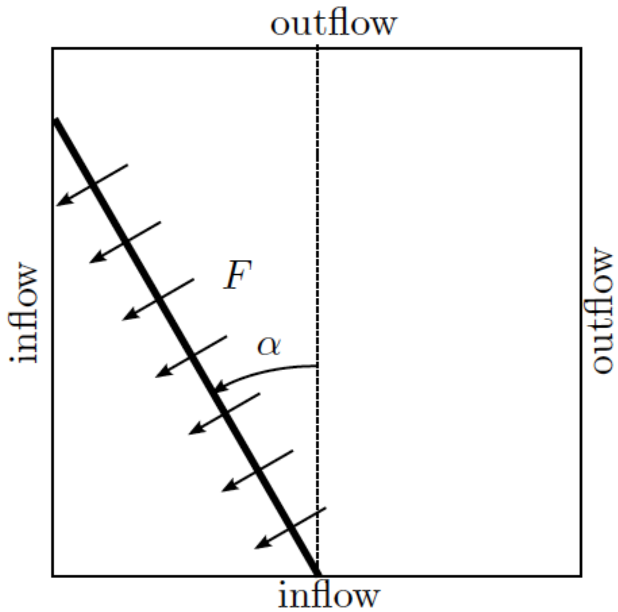



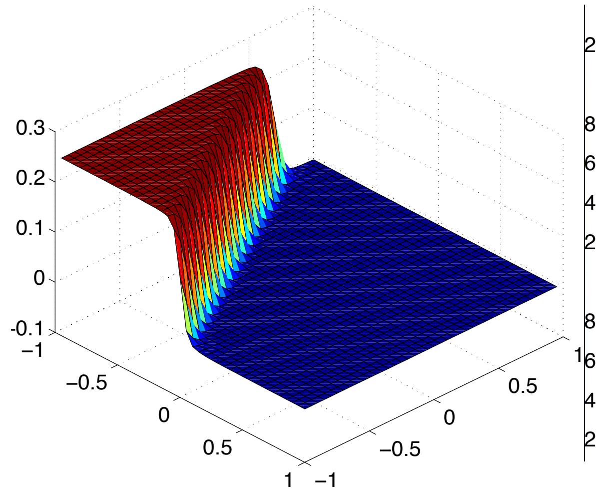
# Quasi-1D actuator disk

- Actuator disk under angle in uniform flow
- IAM gives exact solution



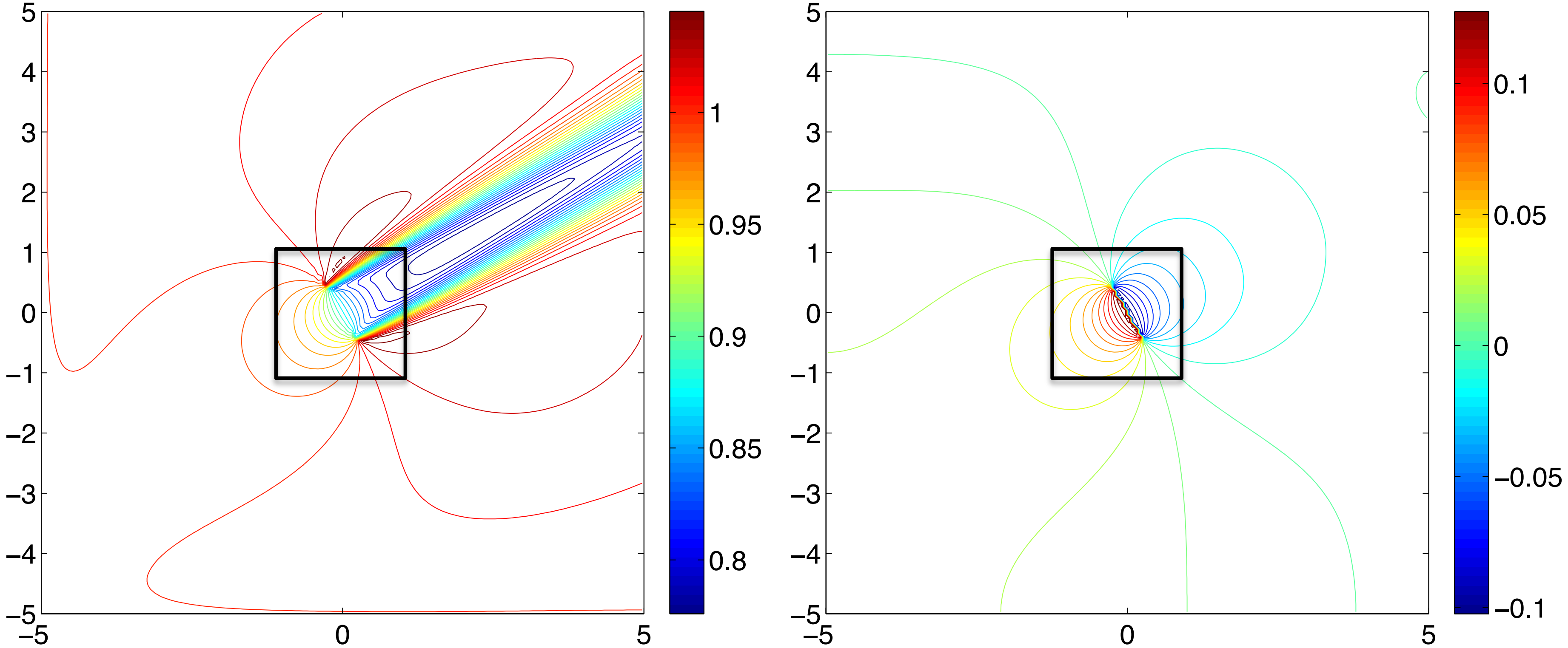

no corrections



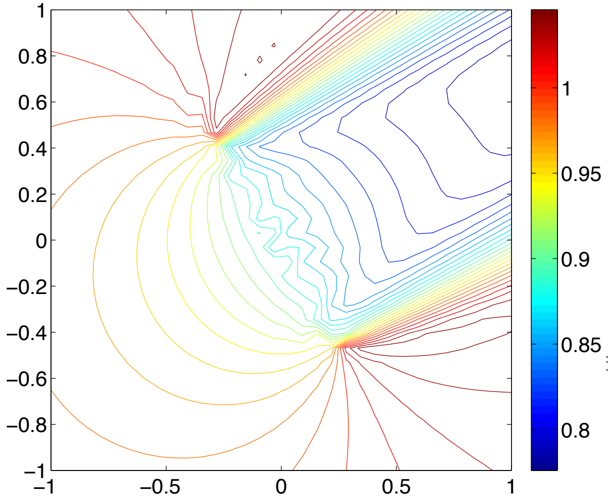




discrete Dirac, Gaussian

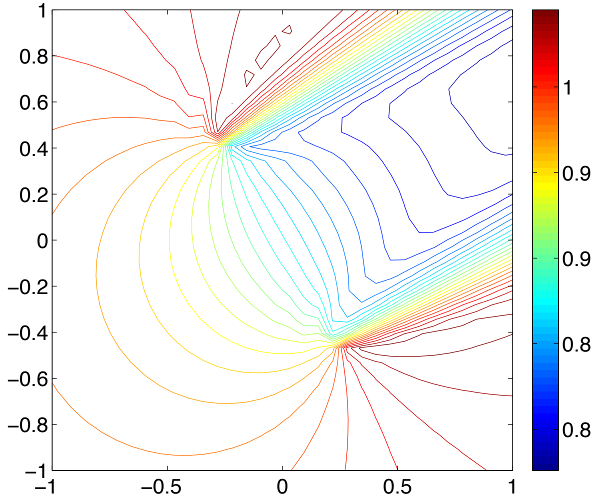
# 2D actuator disk



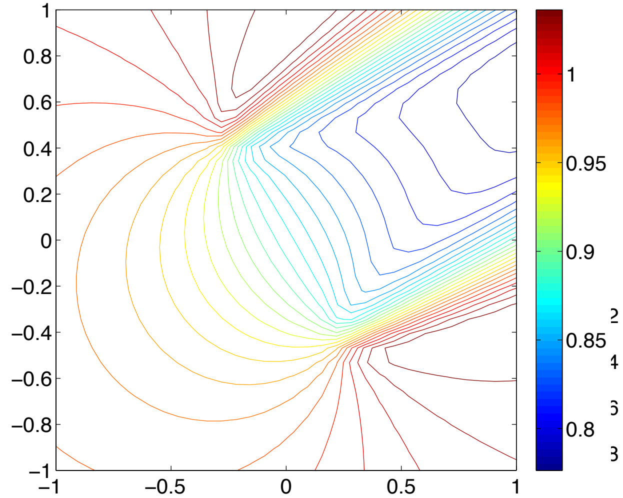
# 2D actuator disk




no correction

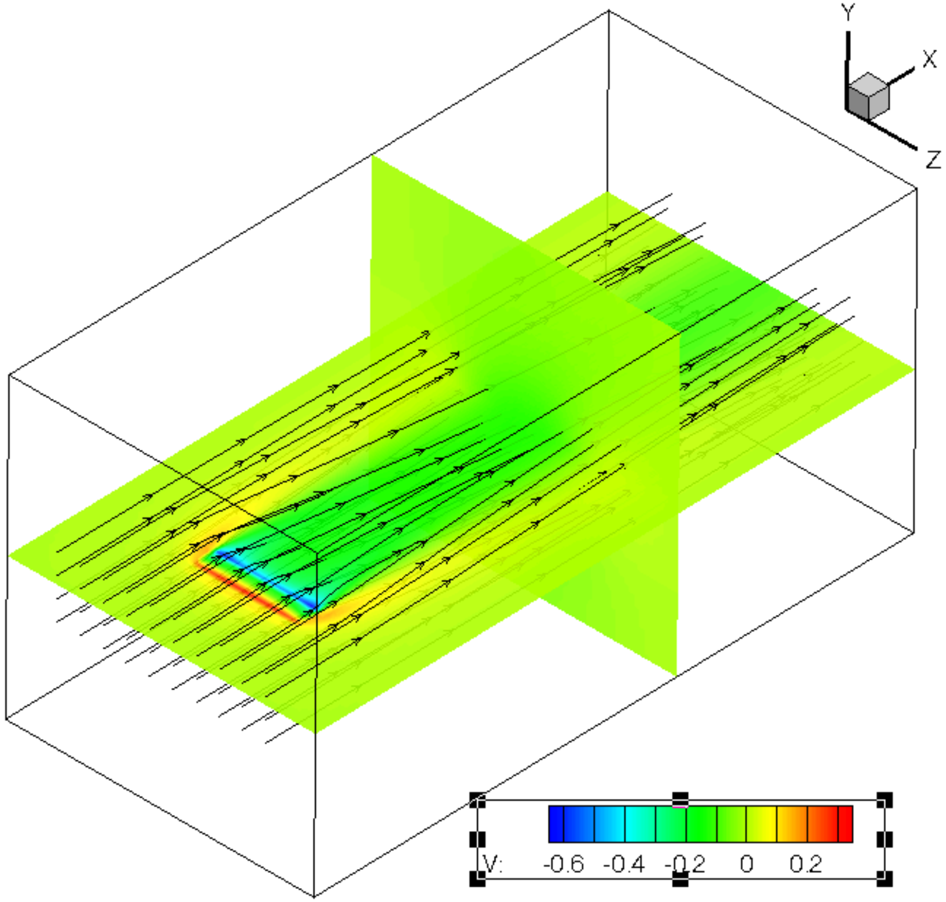


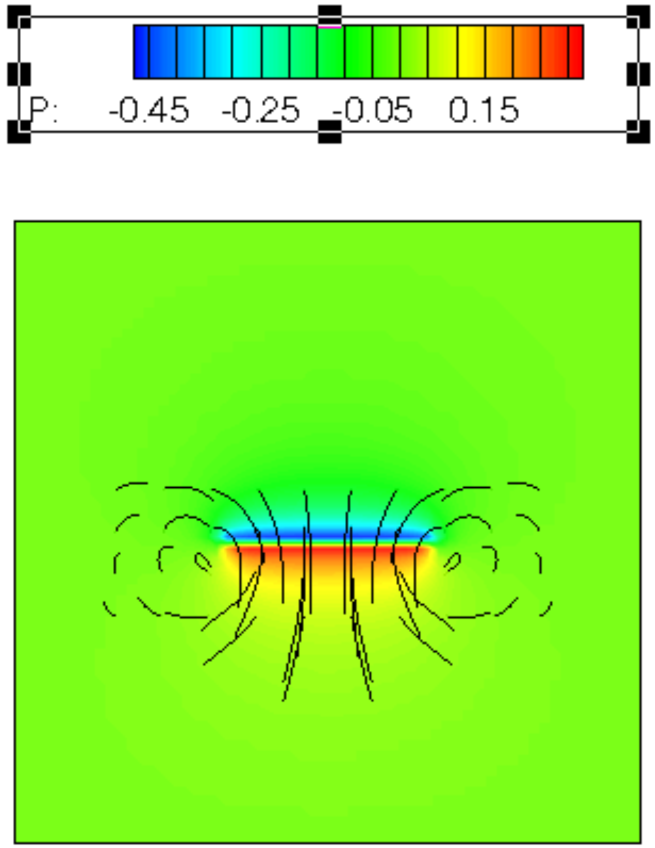

IAM




discrete Dirac (Gaussian)

# 3D wing



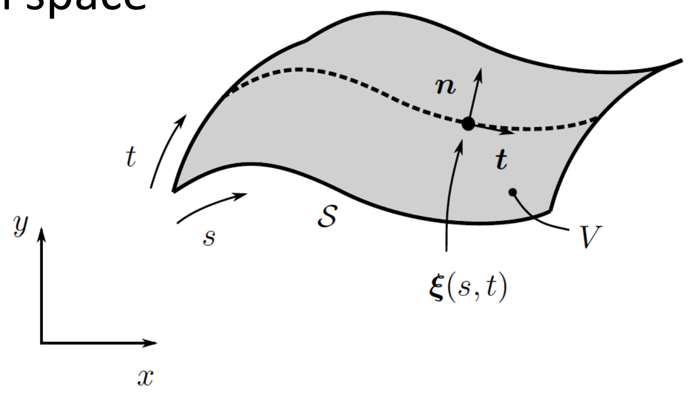
# Moving actuators

- Integral form in space *and time*

$$\left[\int_\Omega \boldsymbol{u}\,\mathrm{d}\Omega\right]_{t^n}^{t^{n+1}} = \int_{t^n}^{t^{n+1}} (\text{conv} + \text{diff} + \text{press})\,\mathrm{d}t + \int_{t^n}^{t^{n+1}} \boldsymbol{F}(t)\,\mathrm{d}t$$

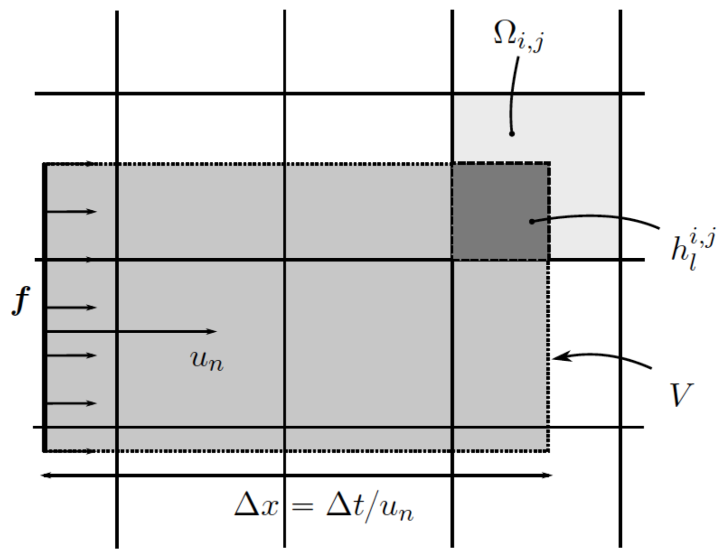
$$\int_{t^n}^{t^{n+1}} \boldsymbol{F}(t)\,\mathrm{d}t = \iint\limits_V \frac{\boldsymbol{f}^b(\boldsymbol{x})}{|u_n(\boldsymbol{x})|}\,\mathrm{d}V$$

- Integral in time is transformed into an integral in space



# Moving actuators

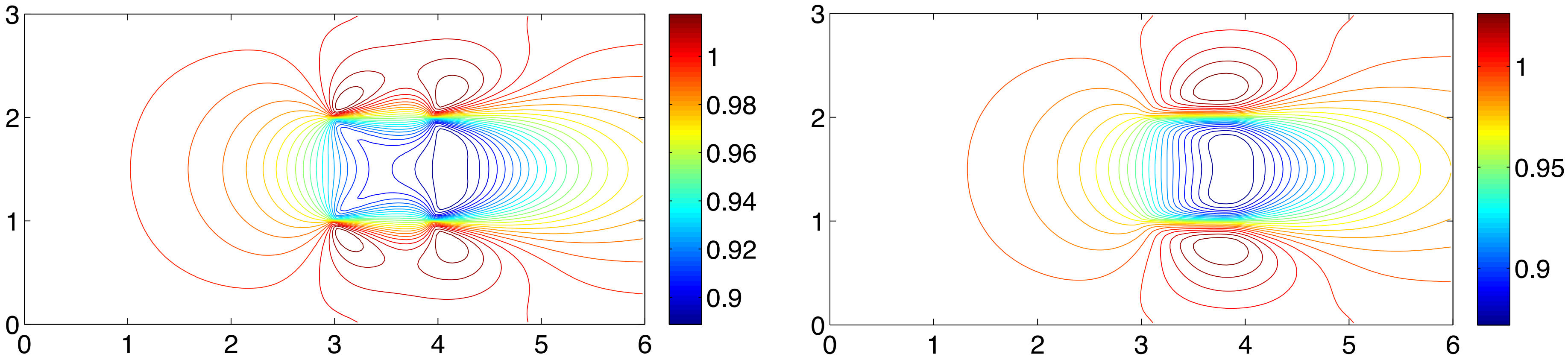
- Discretization based on $\displaystyle\int_{t^n}^{t^{n+1}} \boldsymbol{F}(t)\,\mathrm{d}t = \iint\limits_V \frac{\boldsymbol{f}^b(\boldsymbol{x})}{|u_n(\boldsymbol{x})|}\,\mathrm{d}V$
- A finite volume carries a force contribution based on intersectional area
- Total momentum is conserved in a discrete sense
- For general $\boldsymbol{f}^b(\boldsymbol{x})$ and $u_n(\boldsymbol{x})$ we need numerical quadrature method
- We can take much larger time steps



# Moving actuators

- Classical time discretization, e.g.: $\displaystyle\int_{t^n}^{t^{n+1}} \boldsymbol{F}(t)\,\mathrm{d}t \approx \frac{1}{2}\Delta t\left(\boldsymbol{F}(t^n) + \boldsymbol{F}(t^{n+1})\right)$

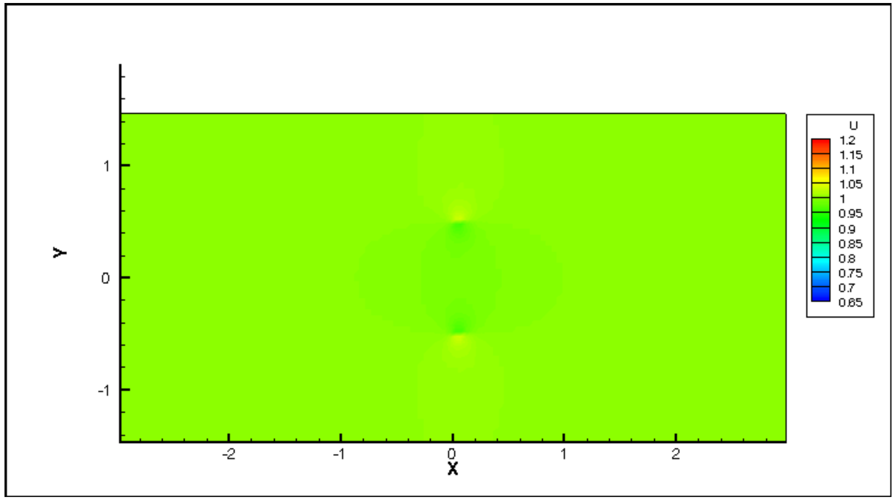
- Comparison of new method with classical method; one time step



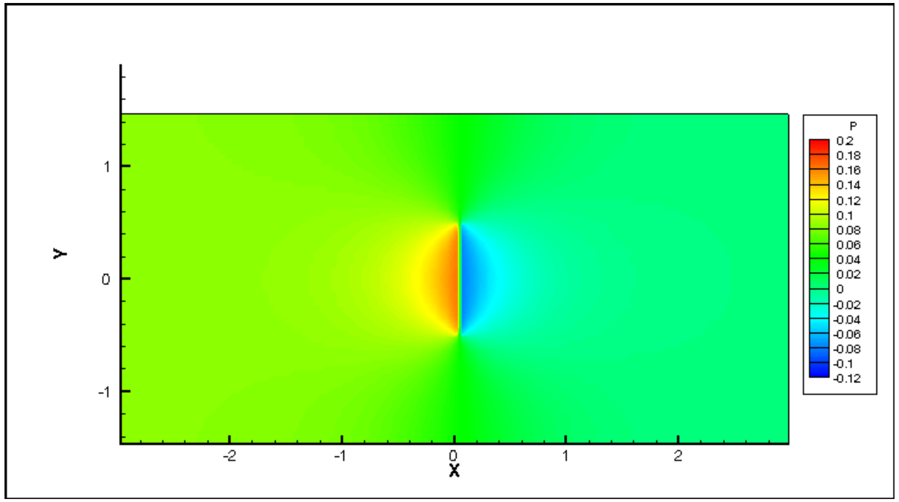

classical new

# Moving actuators

- A **translating** actuator disk with prescribed motion and uniform loading



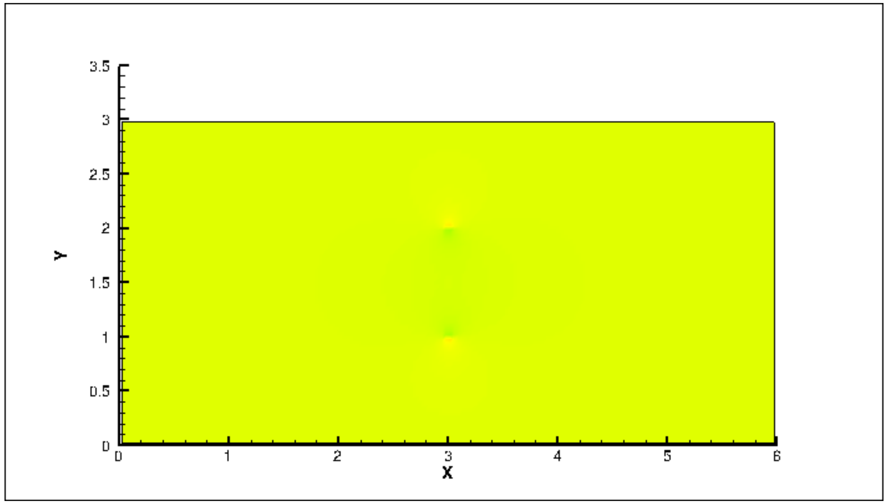

velocity



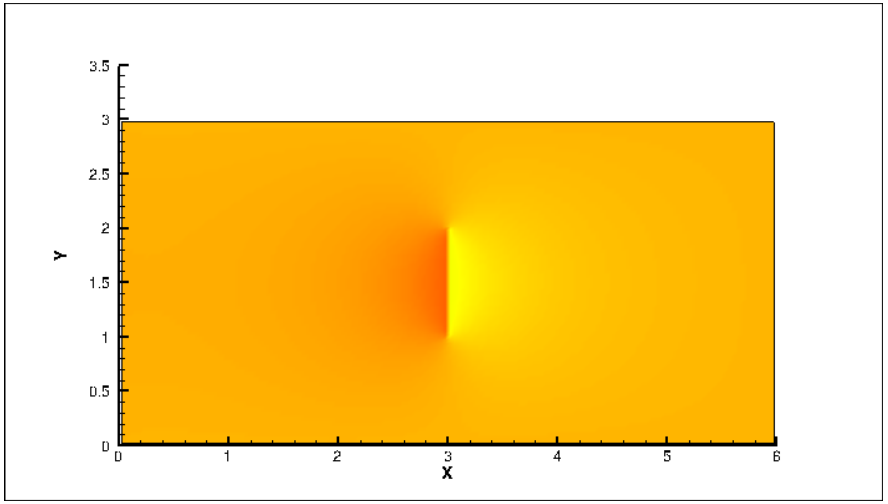

pressure

# Moving actuators

- A **rotating** actuator disk with prescribed motion and uniform loading



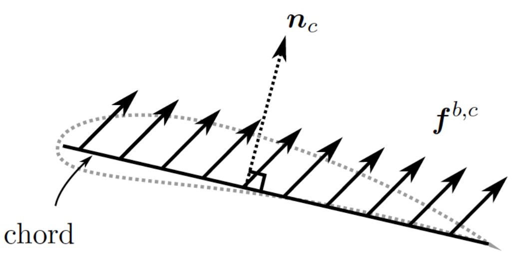

velocity



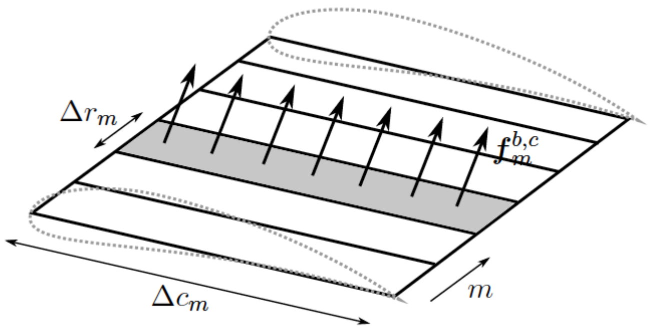

pressure

# A general framework

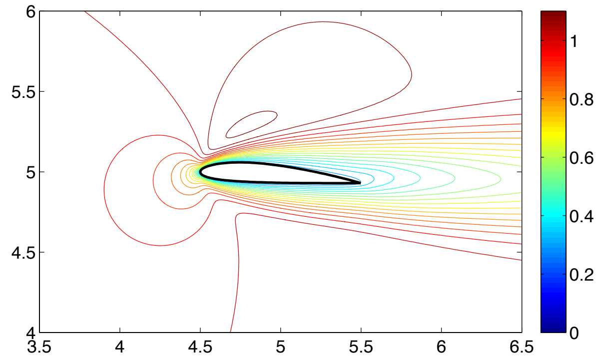
- Current actuator line: ‘point’ force in 2D, ‘line’ force in 3D
- A new actuator line model: distribute lift and drag over chord line:
  - Forces are associated with surface
  - Actuator line model is a special case of actuator surface on coarse meshes



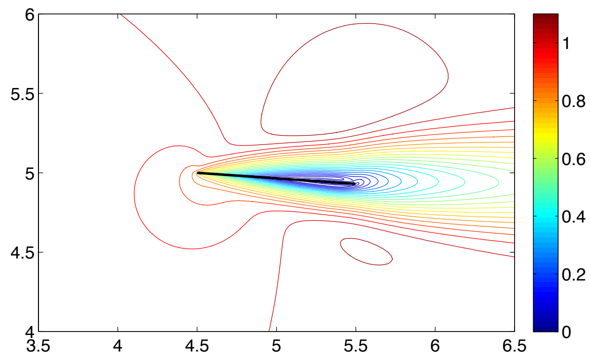

2D




3D



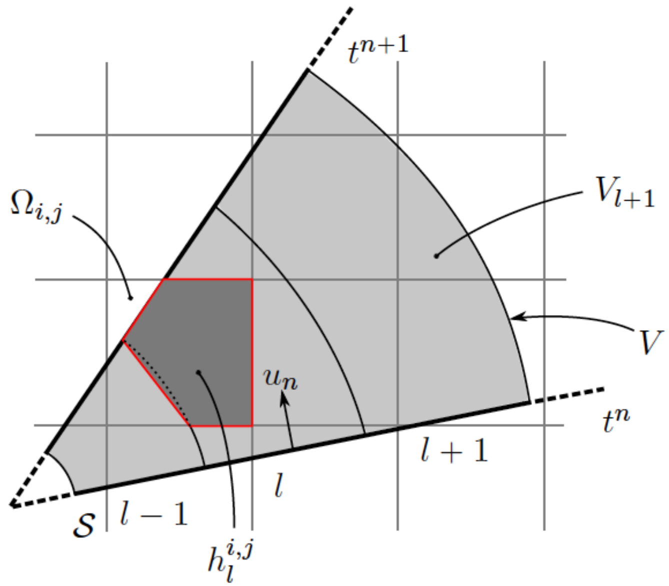
Actuator surface: Cp and Cf



New actuator line: CL and CD

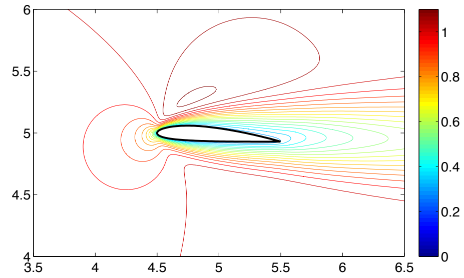
# A general framework

- Actuator disk follows from actuator line for large time steps



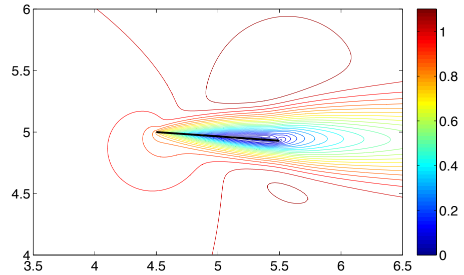
# A general framework

- Actuator surfaces



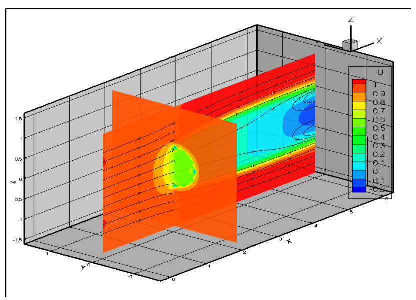

coarser model
OR
coarser mesh

- Actuator 'line'
  (actuator surface with uniform loading)




coarser model
OR
larger time step

- Actuator disk



# Conclusions

- Immersed actuator method (IAM):
  - Sharp interface treatment
  - Jump discontinuities in discretization
  - No discrete Dirac functions
- Moving actuators
  - Discretization based on swept area
  - Large time steps allowed
- General framework
  - Actuator surface -> line -> disk

# Thank you for your attention

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