



Energy research Centre of the Netherlands

A quasi-steady wind farm flow model

A.J. Brand

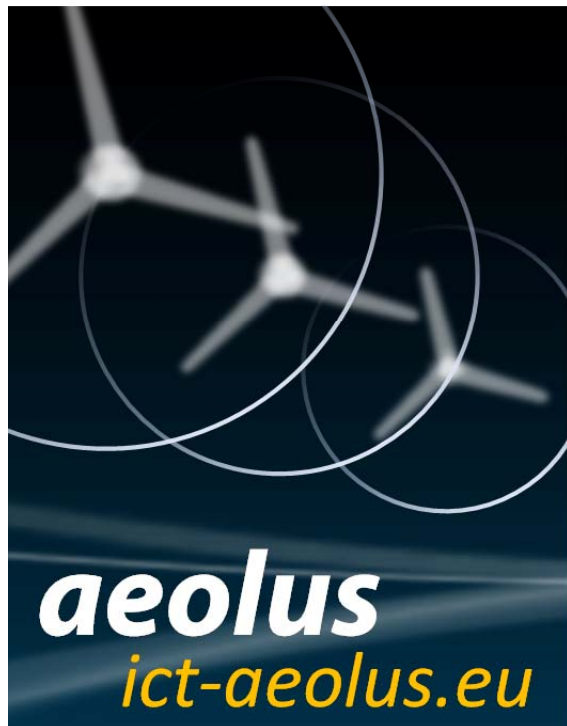
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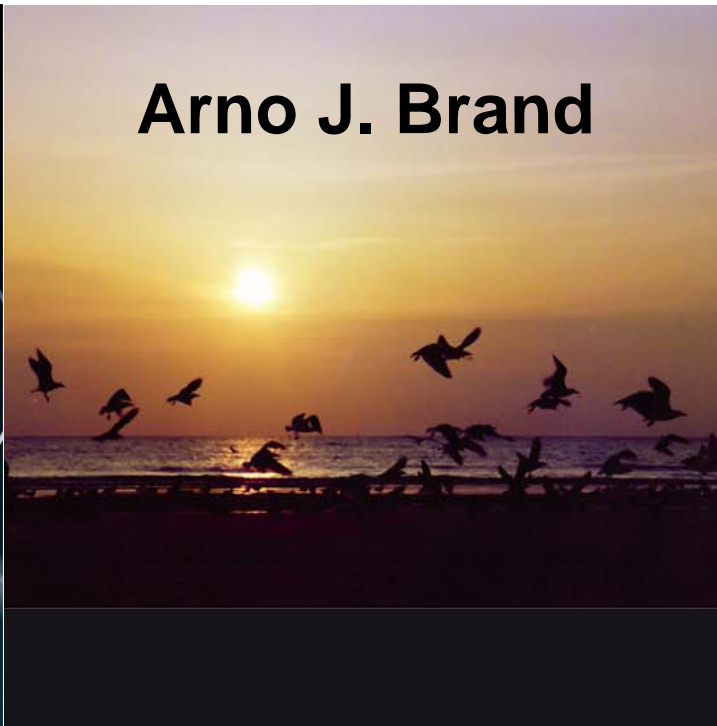


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FP7-ICT STREP 224548 / Aeolus

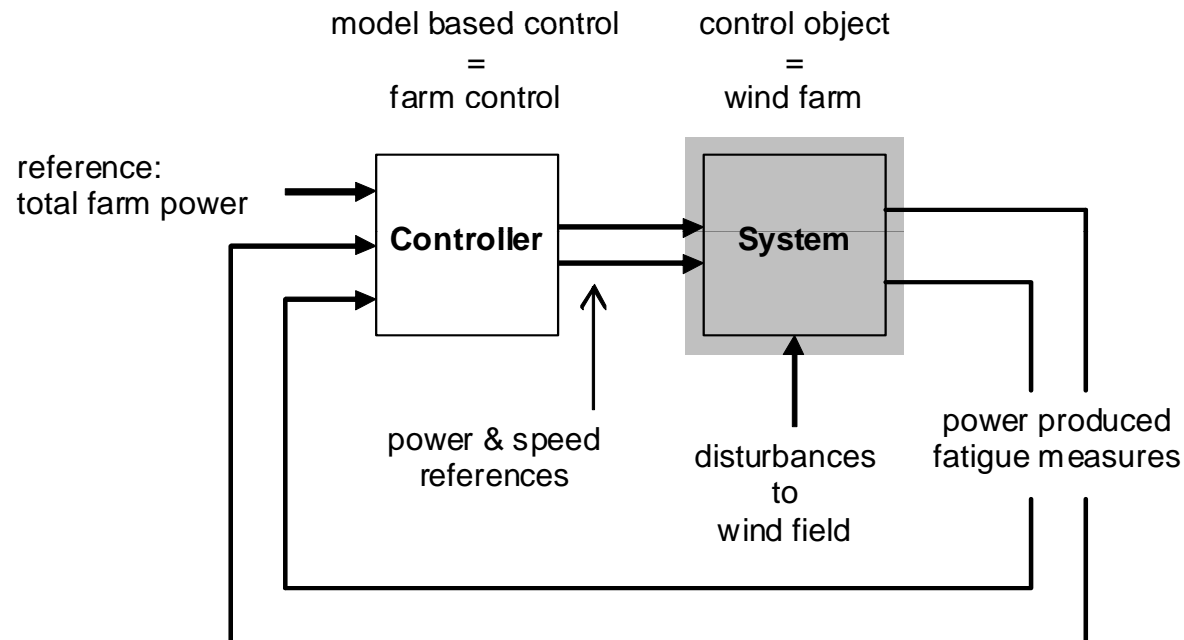
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- ❑ Objectives
- ❑ Method
- ❑ Tuning & Validation
- ❑ Application
- ❑ Summary & Outlook

To develop a wind farm flow model that

- ❑ Relates production and loading to wind
- ❑ Models the control object “wind farm”
- ❑ Is valid over averaging periods of 10 minutes
- ❑ Can be part of a wind farm control algorithm

Quasi-steady flow model basic structure



Assumptions

- Momentum theory for wind turbine states and wakes
- Ten minute wind speed normally distributed

$$U : N(\mu_U, \sigma_U)$$

- Derived quantities normally distributed, e.g.

$$x = f(U)$$

$$x : N(\mu_x, \sigma_x) \quad \mu_x = g_1(\mu_U, \sigma_U) \quad \sigma_x = g_2(\mu_U, \sigma_U)$$

Wind turbine state

- Tip-speed ratio

$$\mu_\lambda = \frac{\Omega R}{\mu_U} \left[1 + \frac{\sigma_U^2}{\mu_U^2} \right]$$

$$\sigma_\lambda^2 = \left(\frac{\Omega R}{\mu_U} \right)^2 \left[1 + 2 \frac{\sigma_U^2}{\mu_U^2} \right] \frac{\sigma_U^2}{\mu_U^2}$$

- Thrust coefficient

$$\mu_{c_T} \approx f(\mu_\lambda) + \frac{1}{2} \sigma_\lambda^2 \left. \frac{d^2 f}{d\lambda^2} \right|_{\mu_\lambda}$$

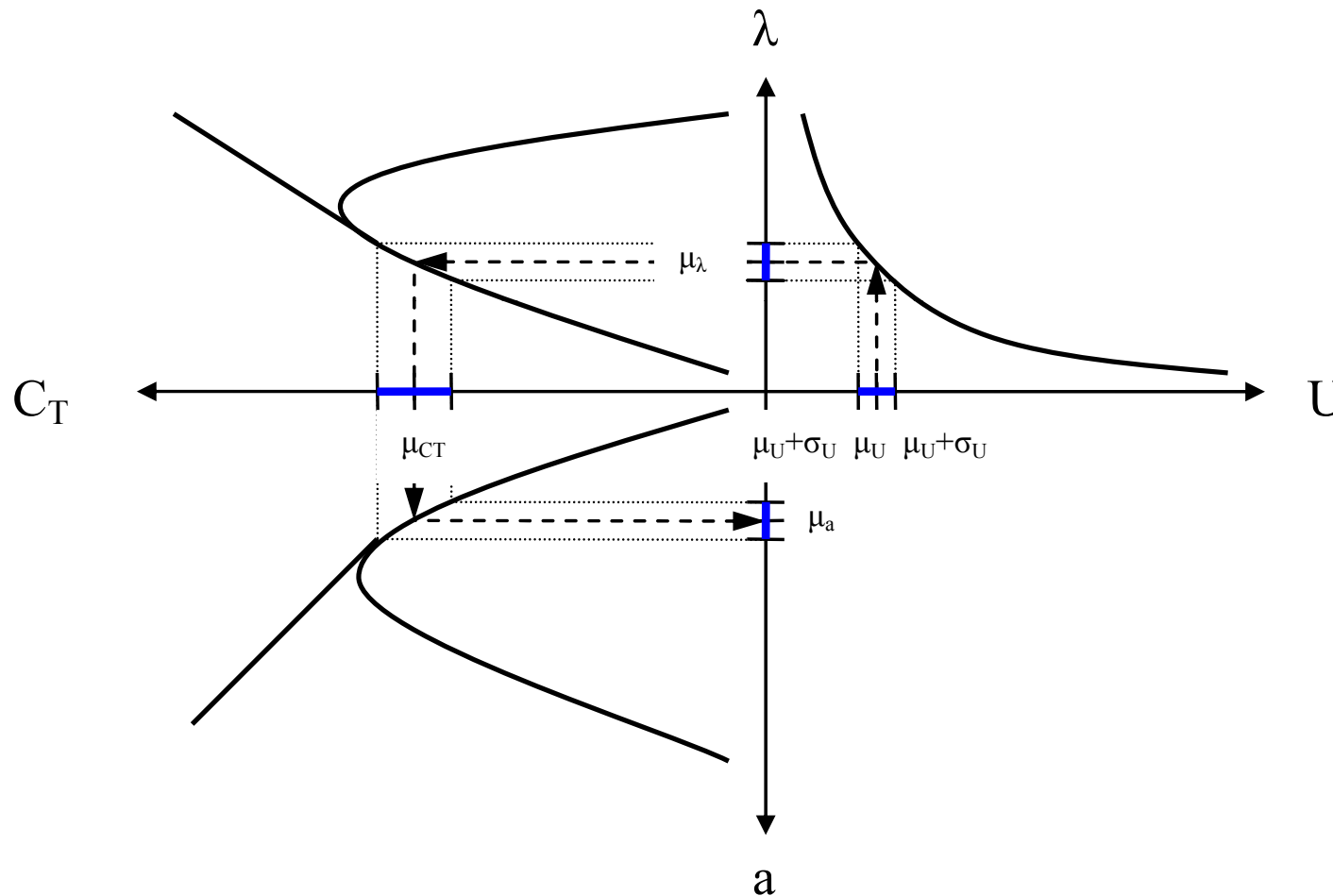
$$\sigma_{c_T}^2 \approx \sigma_\lambda^2 \left[\left(\left. \frac{df}{d\lambda} \right|_{\mu_\lambda} \right)^2 + \frac{1}{2} \sigma_\lambda^2 \left(\left. \frac{d^2 f}{d\lambda^2} \right|_{\mu_\lambda} \right)^2 \right]$$

- Axial induction factor

$$\mu_a = \frac{1}{2} - \frac{1}{2} (1 - \mu_{c_T})^{0.5} + \frac{1}{16} \sigma_{c_T}^2 (1 - \mu_{c_T})^{-1.5}$$

$$\sigma_a^2 = \frac{1}{16} \sigma_{c_T}^2 \left[1 + \frac{1}{8} \sigma_{c_T}^2 (1 - \mu_{c_T})^{-2} \right] (1 - \mu_{c_T})^{-1}$$

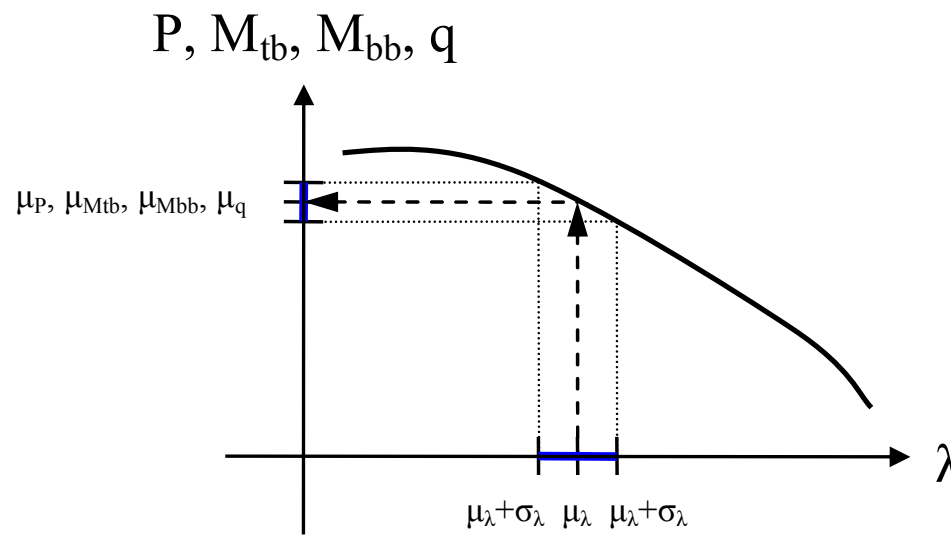
Wind turbine state



Wind turbine production and loading

- ❑ Power
- ❑ Tower bending moment
- ❑ Blade bending moment
- ❑ Rotor shaft torque

Wind turbine production and loading



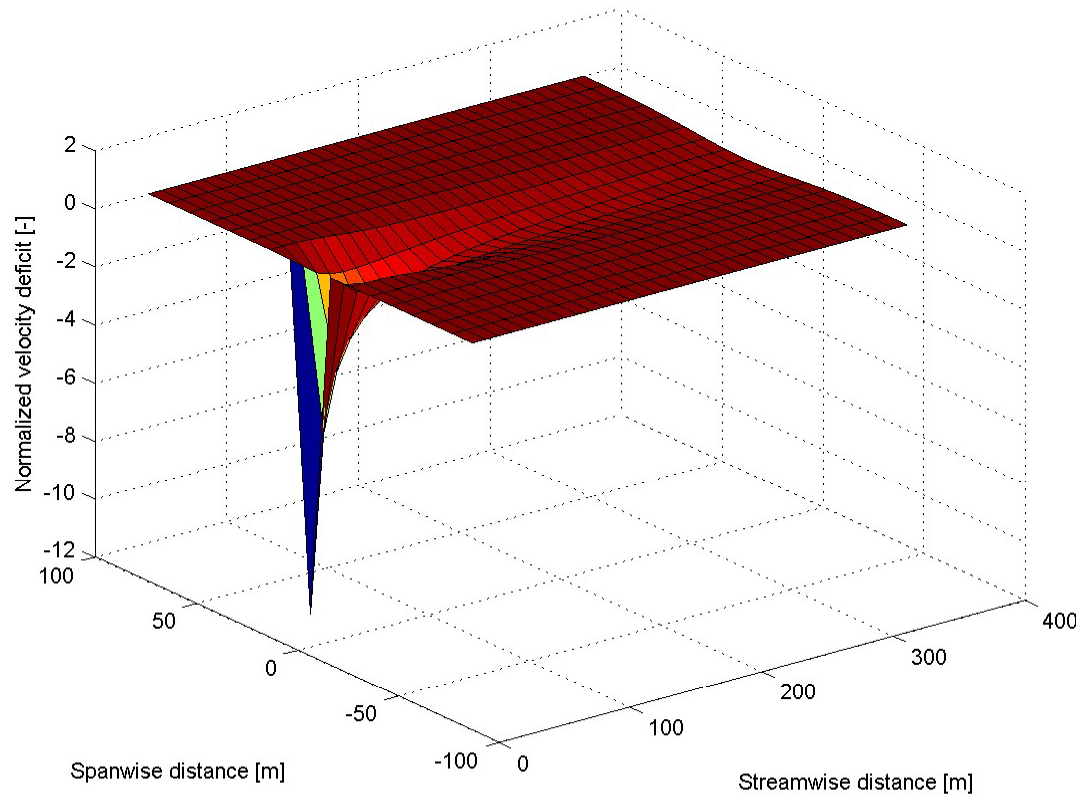
Velocity deficit

$$\mu(r, x) = \mu_0 - \Delta\mu(r, x)$$

$$\frac{\Delta\mu(r, x)}{\Delta\mu_{ini}} = \left(\frac{x}{x_0}\right)^n \exp\left\{-\alpha_1 \frac{r^2}{\beta^2(x)}\right\}$$

$$\Delta\mu_{ini} = 2a\mu_0$$

Velocity deficit



$D = 80 \text{ m}$

$a = 1/3$

Added turbulence

$$\sigma^2(\mathbf{r}, \mathbf{x}) = \sigma_0^2 + \sigma_{\text{add}}^2(\mathbf{r}, \mathbf{x})$$

$$\sigma_{\text{add}}^2(\mathbf{r}, \mathbf{x}) = \sigma_{\text{add},1}^2(\mathbf{r}, \mathbf{x}) + \sigma_{\text{add},2}^2(\mathbf{r}, \mathbf{x})$$

$$\sigma_{\text{add},1} = f_1(\mathbf{r}, \mathbf{x}) \sigma_{\text{add}}(\mathbf{x})$$

$$f_1(\mathbf{r}, \mathbf{x}) = \exp\left\{-\alpha_2 \left(\frac{\mathbf{r} - \mathbf{R}_1(\mathbf{x})}{R}\right)^2\right\}$$

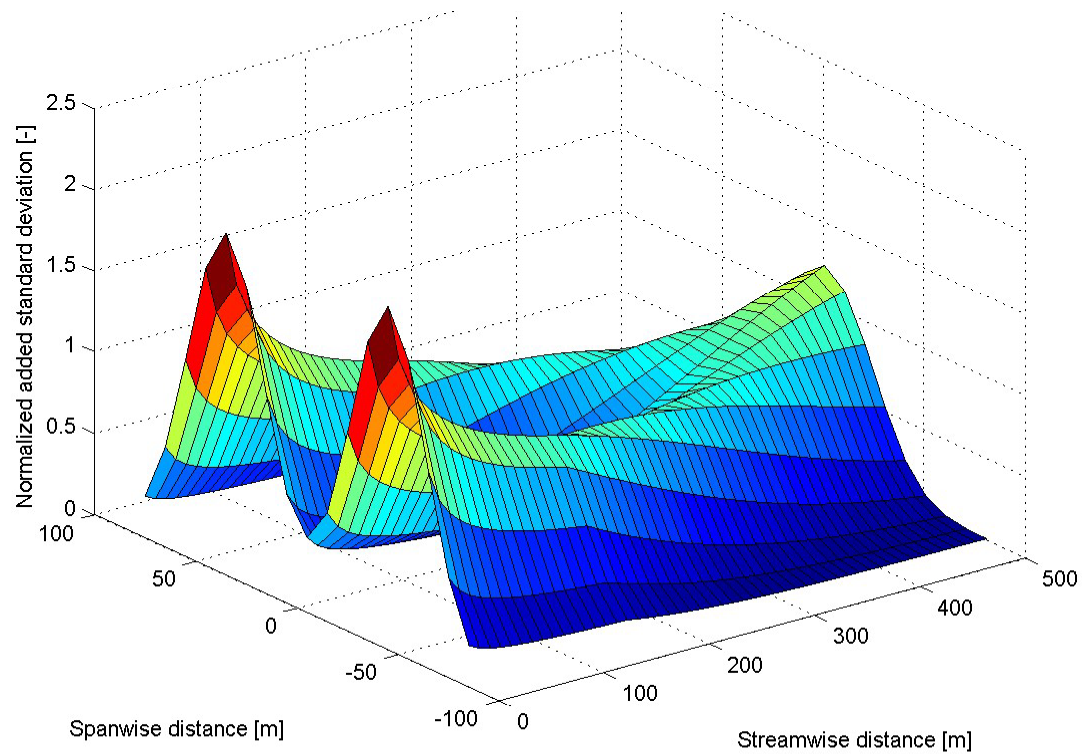
$$\sigma_{\text{add},2} = f_2(\mathbf{r}, \mathbf{x}) \sigma_{\text{add}}(\mathbf{x})$$

$$f_2(\mathbf{r}, \mathbf{x}) = \exp\left\{-\alpha_2 \left(\frac{\mathbf{r} + \mathbf{R}_1(\mathbf{x})}{R}\right)^2\right\}$$

$$\sigma_{\text{add}}^2(\mathbf{x}) = \sigma_{\text{add},\text{ini}}^2 \left(\frac{\mathbf{x}}{\mathbf{x}_0}\right)^m$$

$$\frac{\sigma_{\text{add},\text{ini}}^2}{\sigma_0^2} = c_{\text{GCL}} a$$

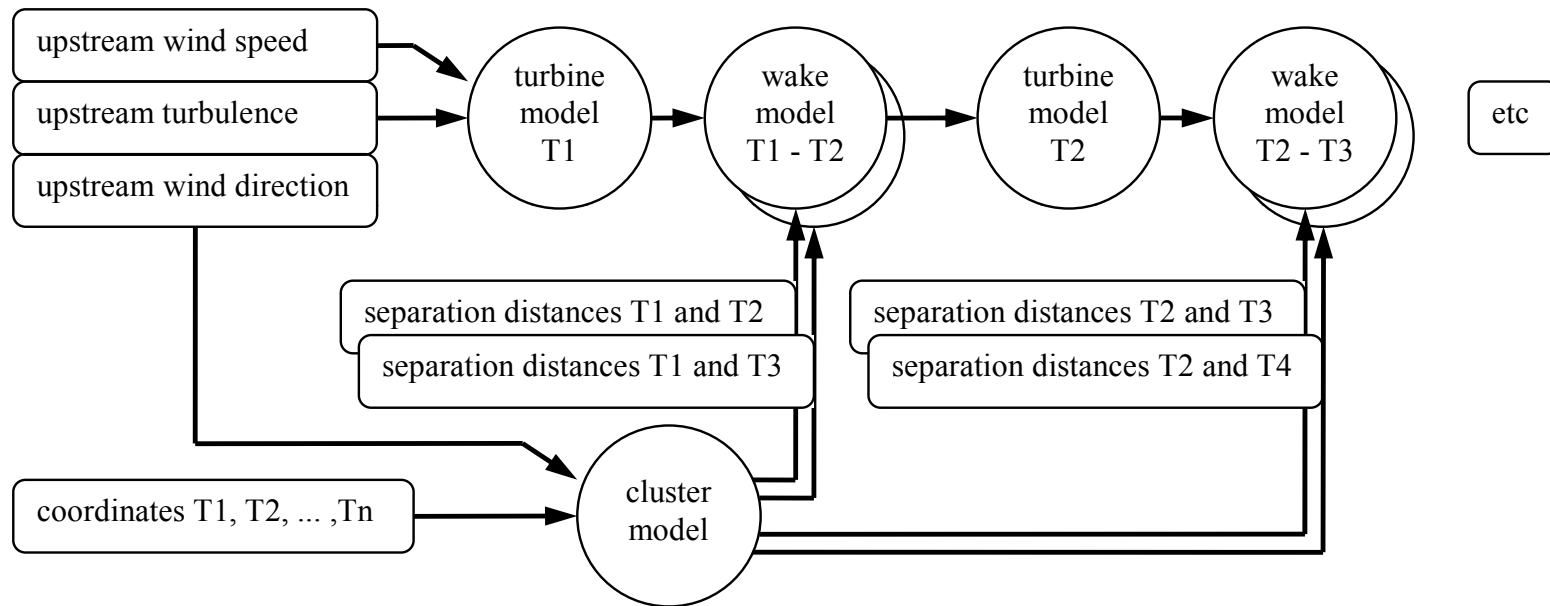
Added turbulence



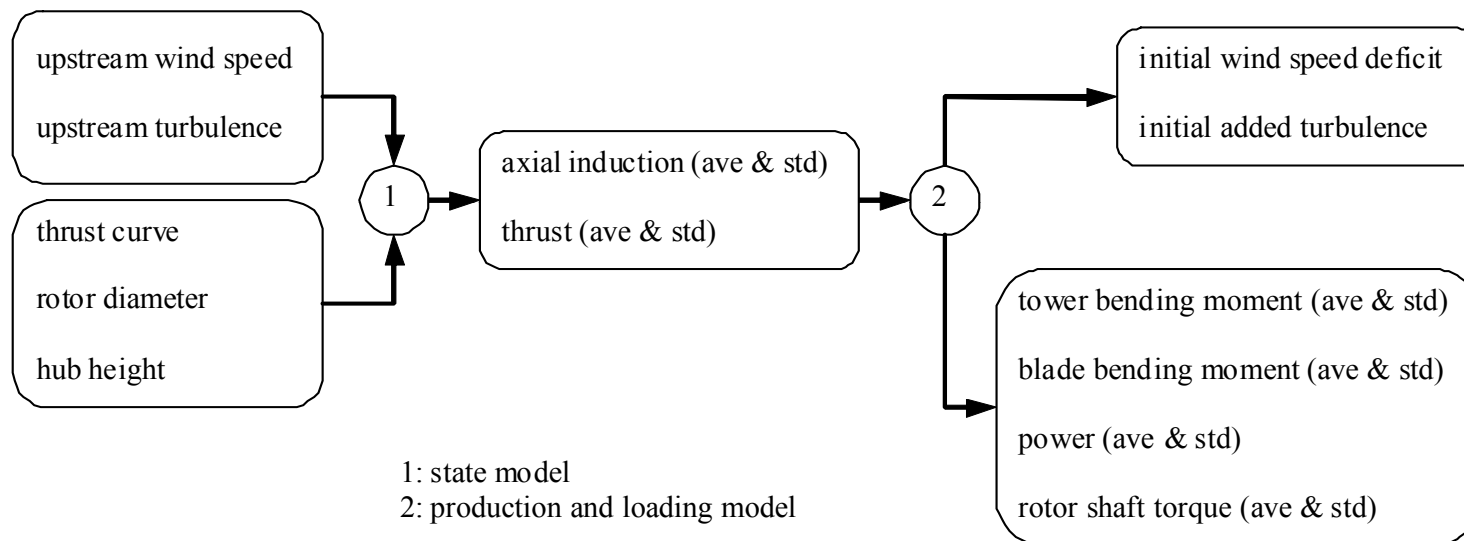
$D = 80 \text{ m}$

$a = 1/3$

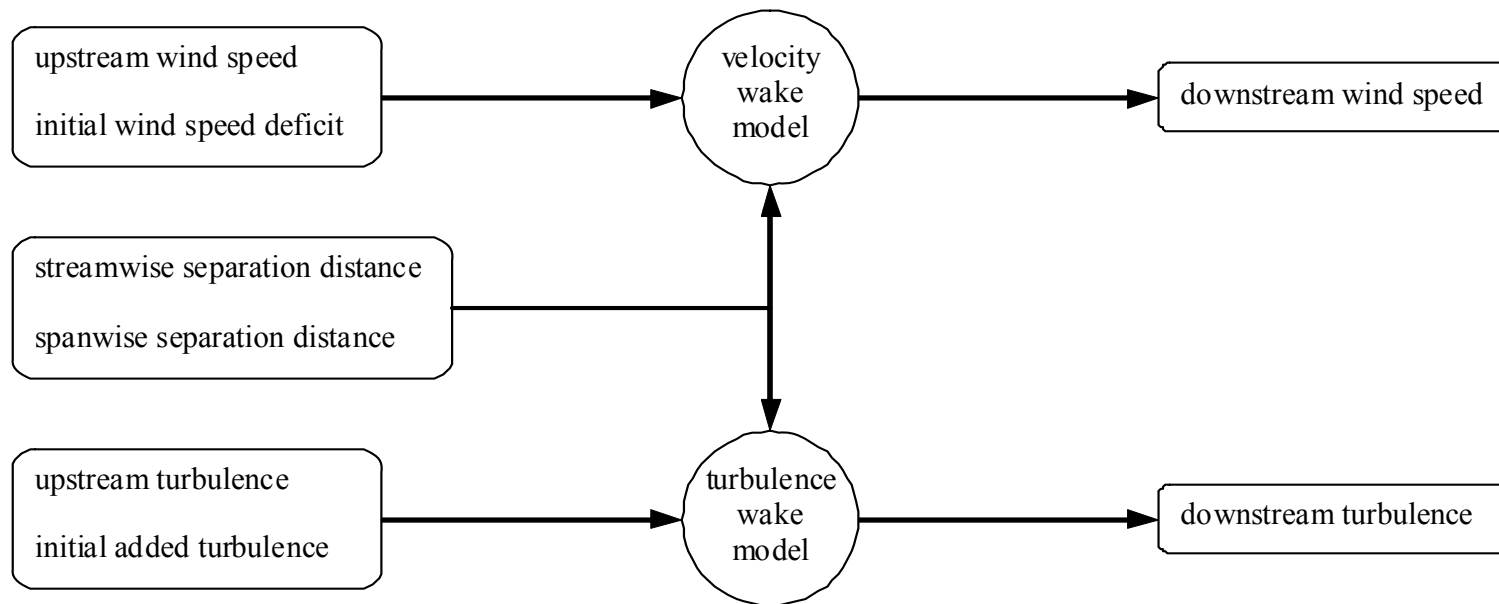
Wind farm flow model



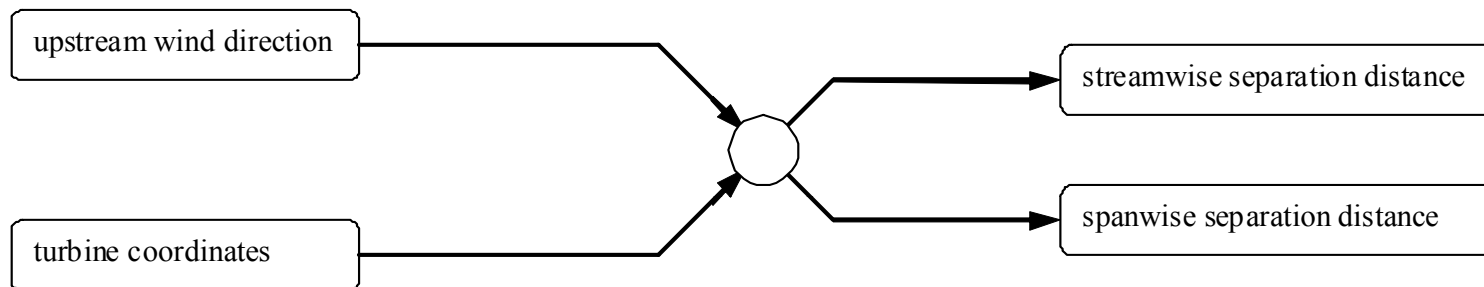
Wind turbine model



Wind turbine wake model



Cluster model



Model output

- ❑ External conditions:
Wind speed, Wind direction, Turbulence intensity
- ❑ State of all turbines:
Hub wind speed, Blade pitch angle, Rotor speed
- ❑ Output of all turbines:
Power, Loads

Model output modes

- Forward

$$(\underbrace{\{WS, BPA, RS\}_{iklm}}_{\text{state}}, \underbrace{\{Power, Loads\}_{iklm}}_{\text{output}}) = f(\underbrace{\{WS_k, WD_l, TI_m\}}_{\text{external conditions}})$$

- Inverse

$$(\underbrace{\{WS, BPA, RS\}_{iklm}}_{\text{state}}, \underbrace{\{Loads\}_{iklm}}_{\text{loads}}) \mid \underbrace{\{WS_k, WD_l, TI_m\}}_{\text{external conditions}} = g(\underbrace{\{Power\}_{iklm}}_{\text{power reference}})$$

WS = Ambient wind speed

WD = Ambient wind direction

TI = Turbulence intensity

BPA = Blade pitch angle

RS = Rotor speed

Model output modes

- Forward

$$(\underbrace{\{WS, BPA, RS\}_{iklm}}_{\text{state}}, \underbrace{\{Power, Loads\}_{iklm}}_{\text{output}}) = f(\underbrace{\{WS_k, WD_l, TI_m\}}_{\text{external conditions}})$$

- Inverse

$$(\underbrace{\{WS, BPA, RS\}_{iklm}}_{\text{state}}, \underbrace{\{Loads\}_{iklm}}_{\text{loads}}) \mid \underbrace{\{WS_k, WD_l, TI_m\}}_{\text{external conditions}} = g(\underbrace{\{Power\}_{iklm}}_{\text{power reference}})$$

WS = Ambient wind speed

WD = Ambient wind direction

TI = Turbulence intensity

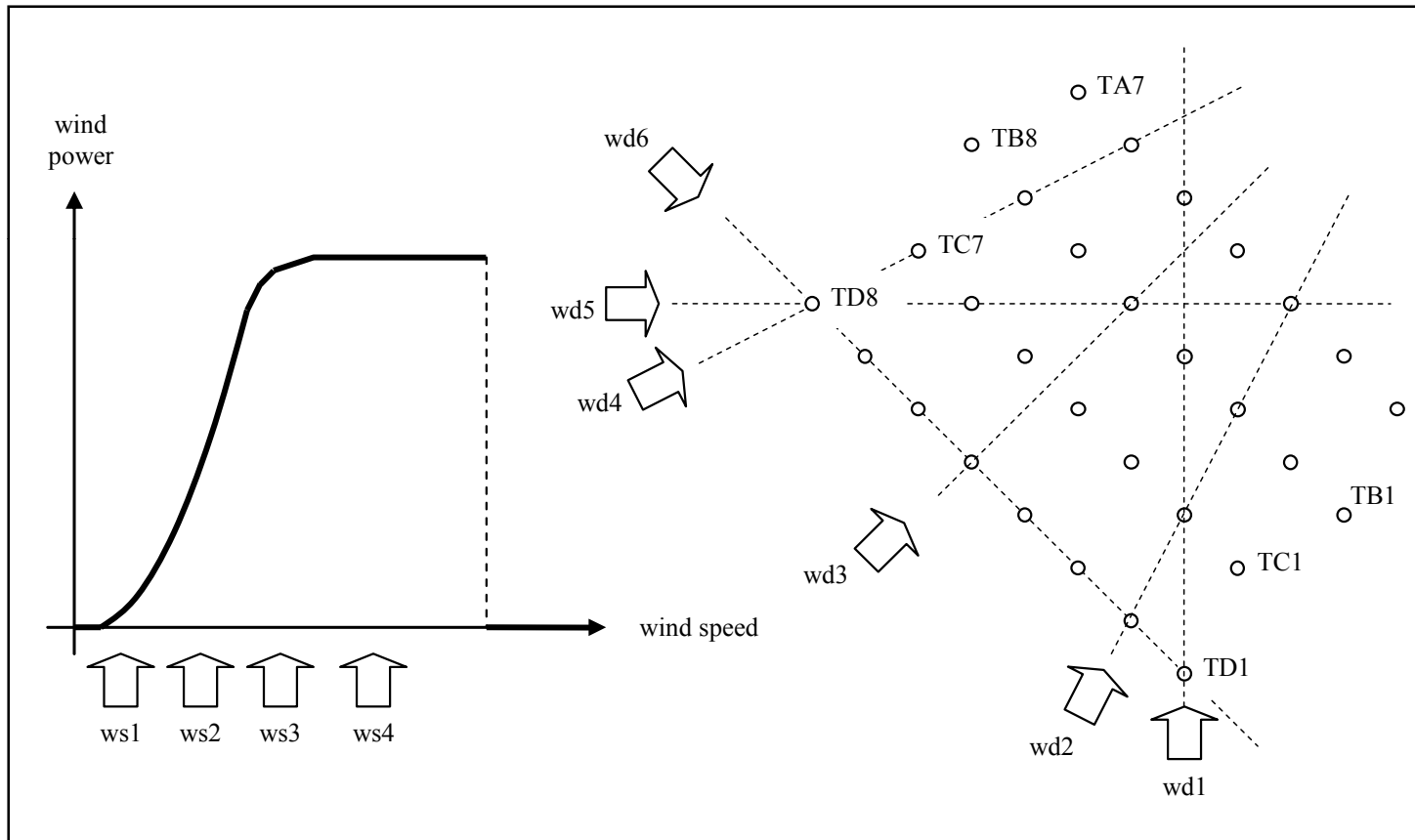
BPA = Blade pitch angle

RS = Rotor speed

Measurements in full-scale wind farm

- ❑ Nominal power: 3 MW
- ❑ Hub height: 80 m
- ❑ Rotor diameter: 90 m
- ❑ Turbine separation: 5.5D
- ❑ Row separation: 8.5D

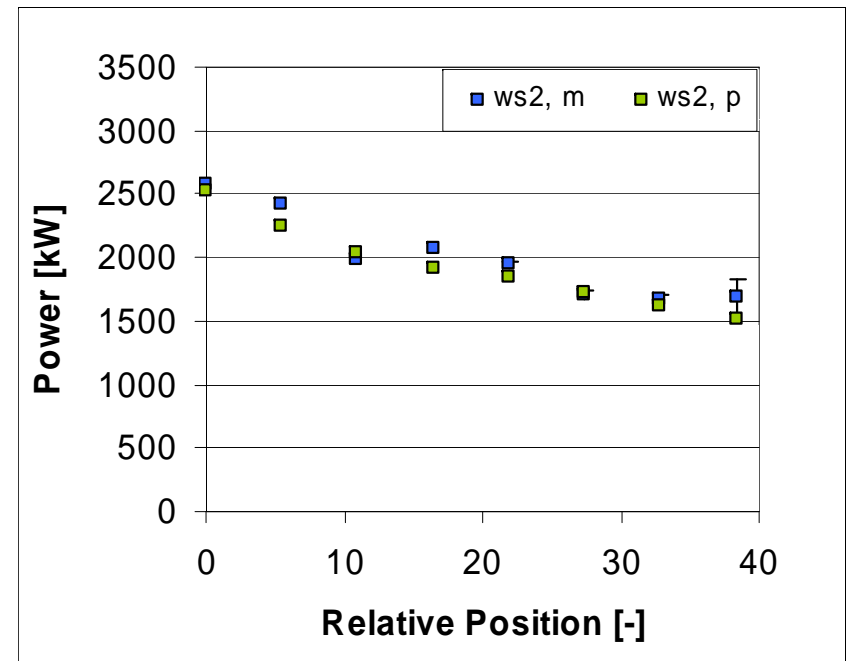
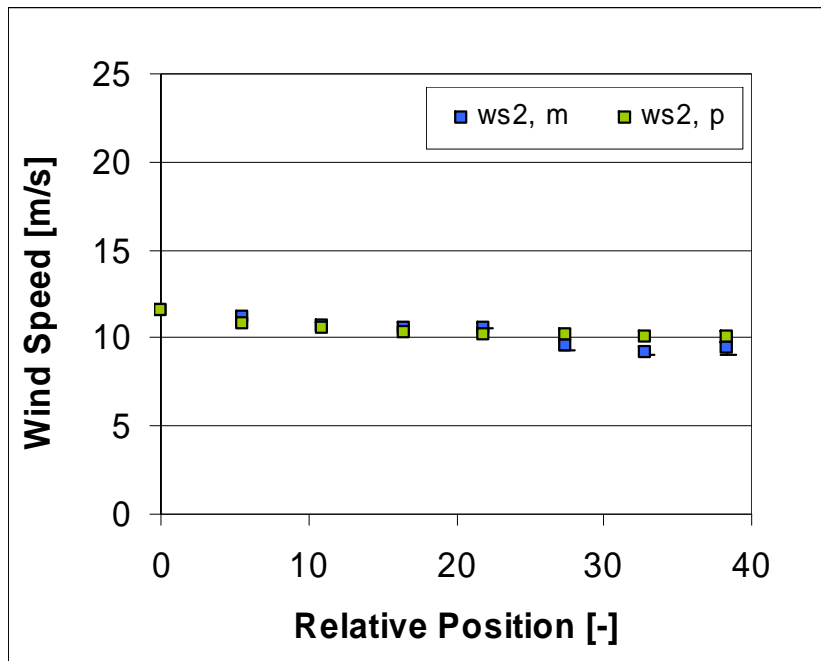
Wind speed and wind direction cases



Row with eight 3 MW wind turbines

Wind speed: ws3

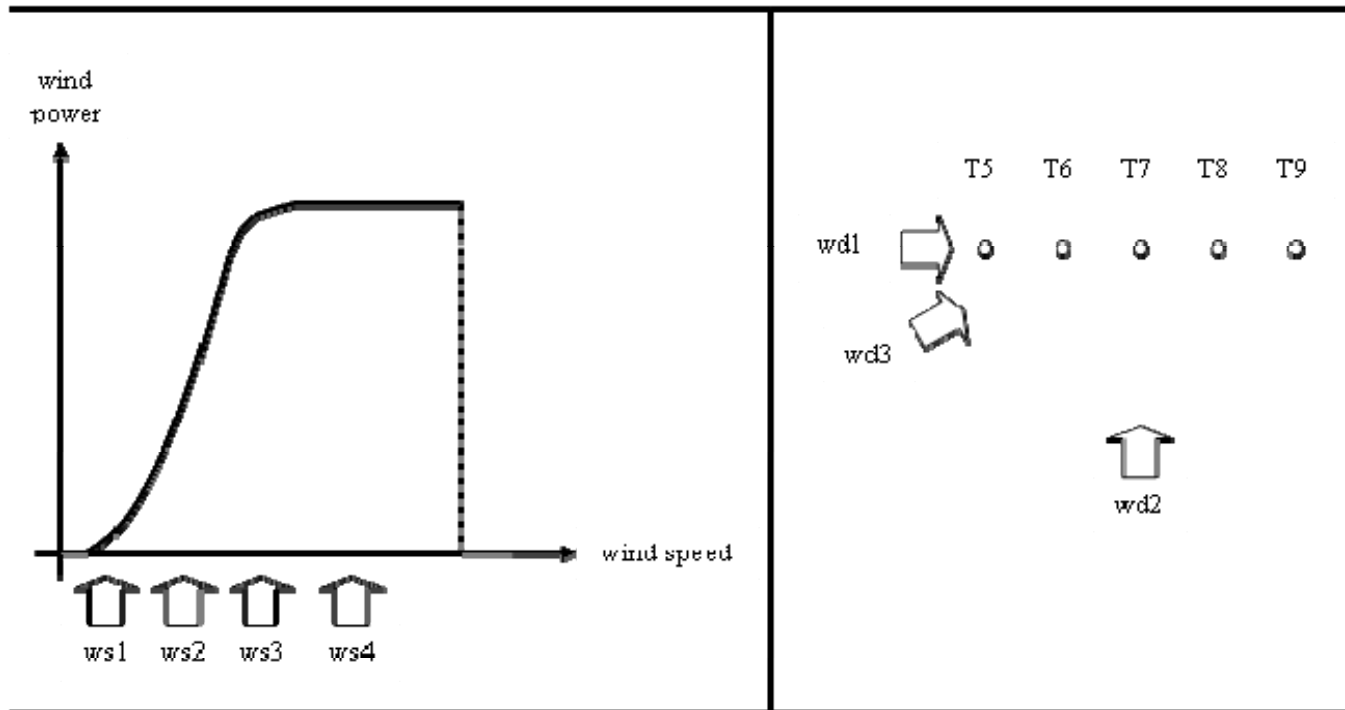
Wind direction: wd6



Measurements in full-scale wind farm

- ❑ Nominal power: 2.5 MW
- ❑ Hub height: 80 m
- ❑ Rotor diameter: 80 m
- ❑ Turbine separation: 3.8D

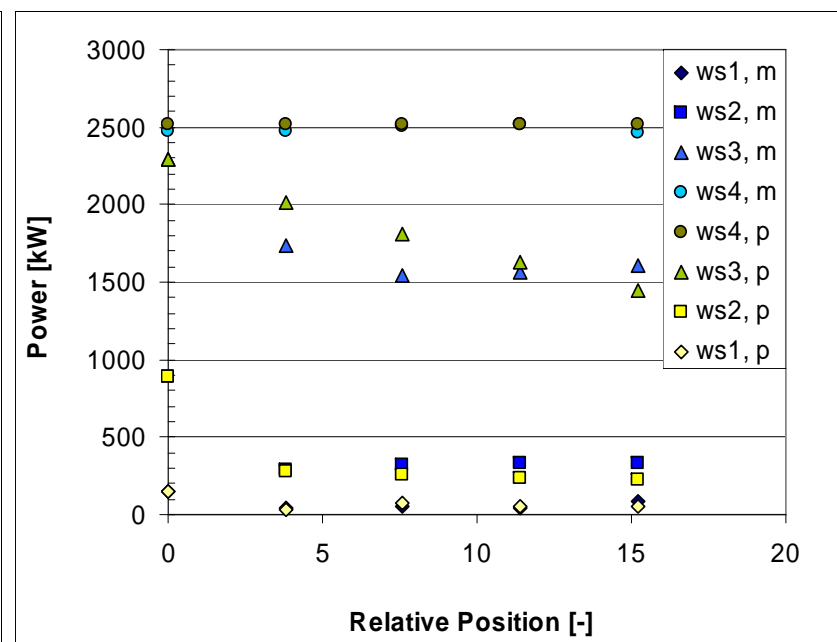
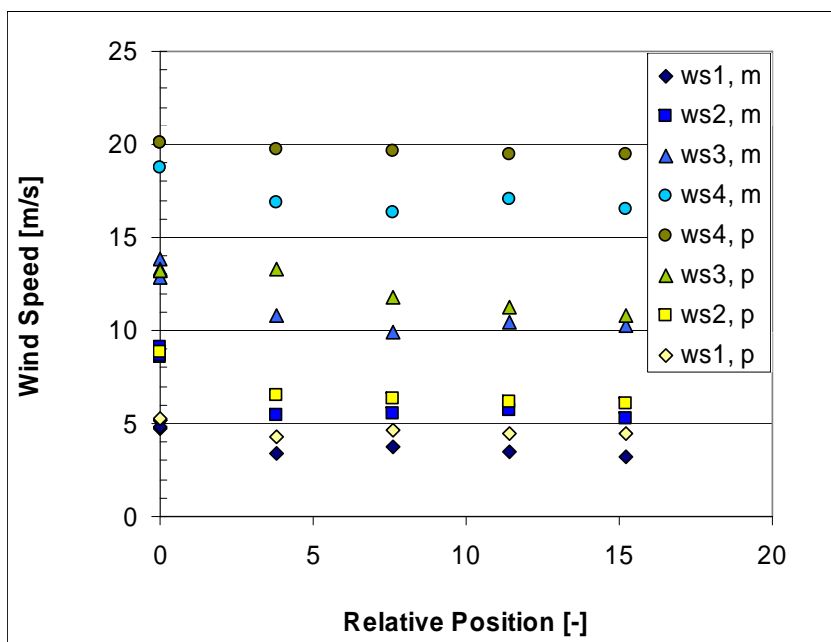
Wind speed and wind direction cases



Row with five 2.5 MW wind turbines

Wind speeds: ws1, ws2, ws3 and ws4

Wind direction: wd1



Conclusion

- ❑ Good fit required tuning of
 - * Velocity deficit decay law
 - * Added turbulence decay law
- ❑ Good fit required variable length of near wake

Model output modes

- Forward

$$(\underbrace{\{WS, BPA, RS\}_{iklm}}_{\text{state}}, \underbrace{\{Power, Loads\}_{iklm}}_{\text{output}}) = f(\underbrace{\{WS_k, WD_l, TI_m\}}_{\text{external conditions}})$$

- Inverse

$$(\underbrace{\{WS, BPA, RS\}_{iklm}}_{\text{state}}, \underbrace{\{Loads\}_{iklm}}_{\text{loads}}) \mid \underbrace{\{WS_k, WD_l, TI_m\}}_{\text{external conditions}} = g(\underbrace{\{Power\}_{iklm}}_{\text{power reference}})$$

WS = Ambient wind speed

WD = Ambient wind direction

TI = Turbulence intensity

BPA = Blade pitch angle

RS = Rotor speed

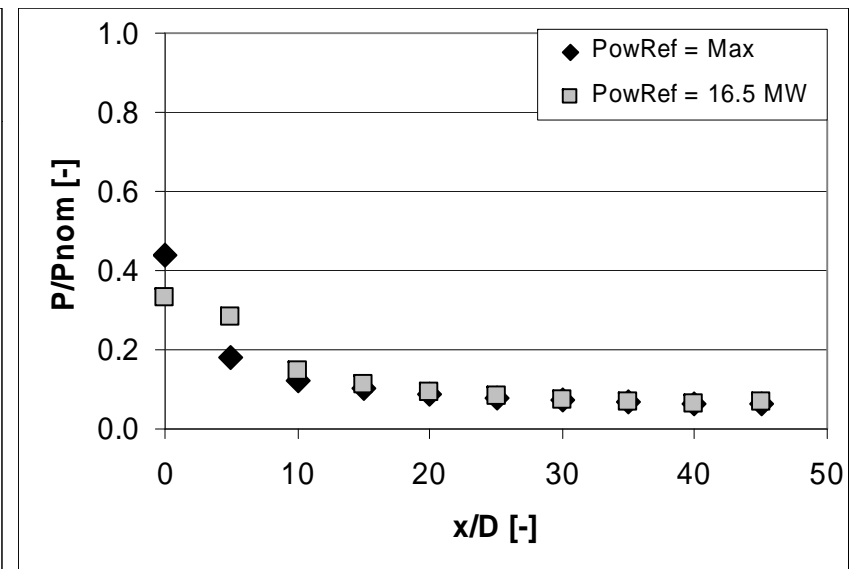
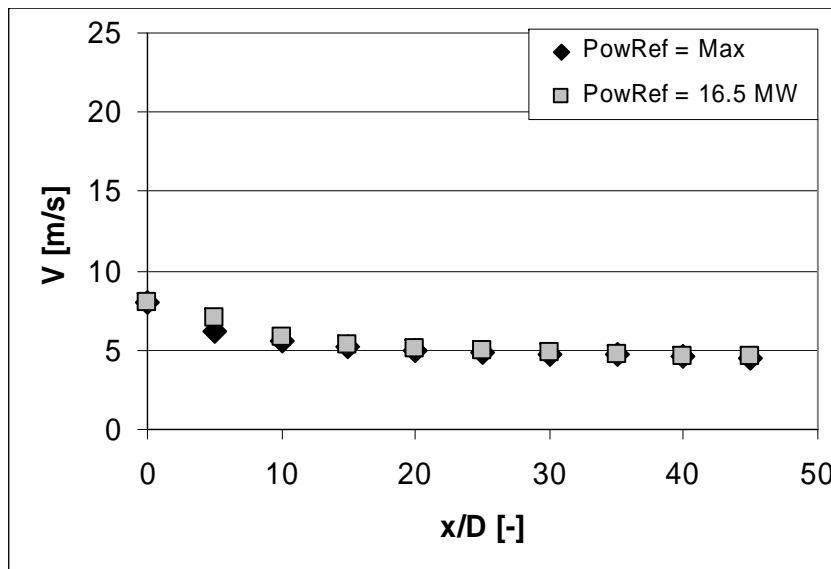
Hypothetical full-scale wind farm

- ❑ Nominal power: 5 MW
- ❑ Hub height: 90 m
- ❑ Rotor diameter: 126 m
- ❑ Turbine separation: 5D

Row with ten 5 MW wind turbines

Wind speed: ws2

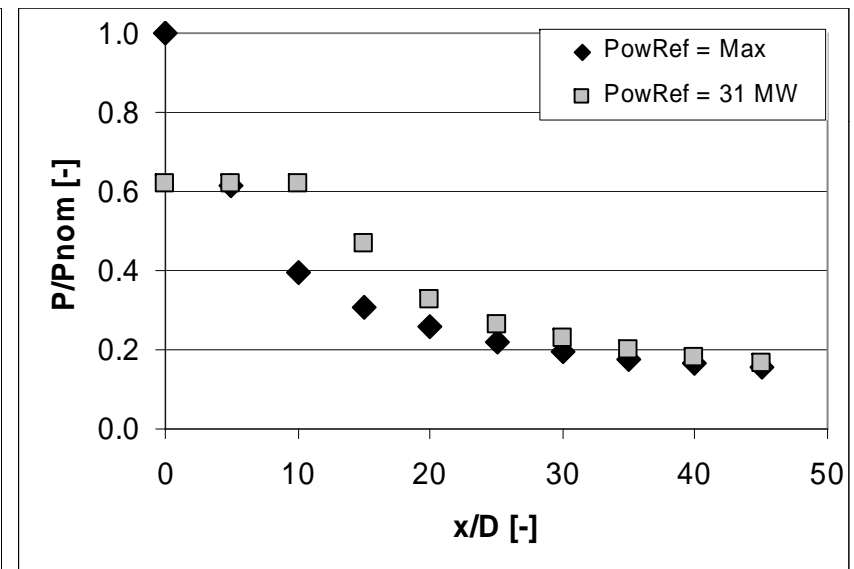
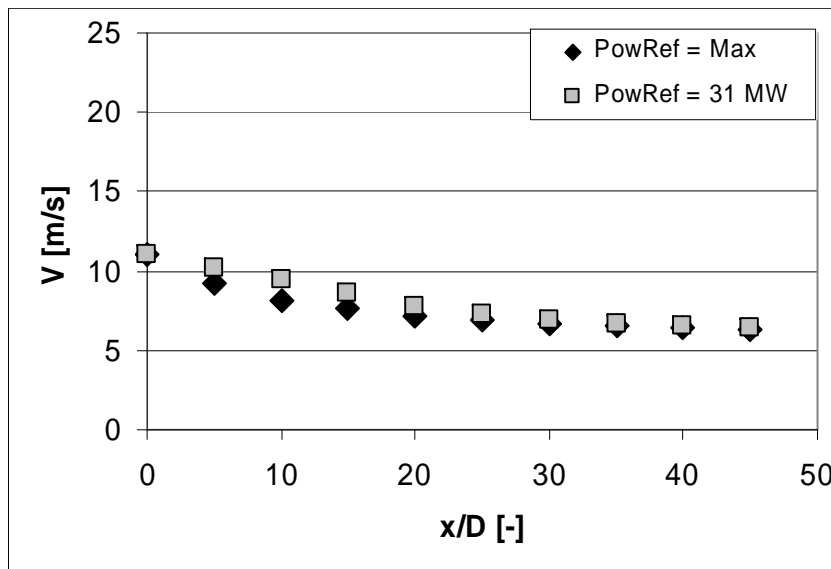
Power references: 10 x 5.0 MW and 10 x 1.65 MW



Row with ten 5 MW wind turbines

Wind speed: ws3

Power references: 10 x 5.0 MW and 10 x 3.1 MW



- ❑ A quasi-steady wind farm flow model has been presented
- ❑ The model can be operated in forward or inverse mode
- ❑ The forward mode has been tuned and validated
- ❑ The inverse mode has been demonstrated

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A quasi-steady wind farm flow model

(slide 1)

My name is Arno Brand from ECN in the Netherlands.

ECN is an independent research institute that performs non-nuclear energy research, among others in wind energy.

The wind energy group of ECN is aimed at the design of large-scale offshore wind turbines and wind farms.

In the wind energy group I am working on wind energy meteorology, with the emphasis on the wind aspects of grid integration, the wind shadow of wind farms, wind power forecasting, and wind resource assessment.

But this morning I will talk about local wind in a wind farm in the context of control of that wind farm.

(slide 2)

The outline of the talk is as follows.

First, I will introduce the objectives of flow modeling in the context of wind farm control.

Next, I will zoom in on the quasi-steady wind farm flow model that was developed by ECN.

Then I will proceed with a comparison of model predictions and discuss the tuning and the validation of the model.

Furthermore, I will address an application of the model,

and finally, I will summarize what has been achieved so far and give an outlook into the future.

(slide 3)

The motivation for wind farm control is that wind farms are expected to operate similar to conventional power plants.

During operation this is achieved by addressing control objectives at the farm level, for example tracking an externally issued power set point while at the same time minimizing mechanical loads.

To facilitate this, control algorithms are developed,

as well as wind farm flow models which feed information to the control algorithm.

In wind farm control it is recognized that the individual wind turbines are connected through their wakes.

(slide 4)

The objective of the quasi-steady flow model is to relate the turbine production and loading to the wind speed in a time-averaged sense and,

by doing so, to model the control object "wind farm".

The model is steady in the sense that it is valid over averaging periods of several minutes.

It is quasi-steady as it takes into account the effect of variations on the averages.

This is in contrast to a dynamical model which takes care of the instantaneous deviations from the averages.

Since the model is to be part of a control algorithm, it is computationally fast and cheap.

(slide 5)

The quasi-steady model is based on a number of assumptions:

First, the flow in the wake of a wind turbine is governed by momentum theory.

Second, in a period of several minutes the wind speed is normally distributed.

Third, the quantities that depend on the wind speed are normally distributed too.

(slide 6)

The model expresses the wind turbine state in terms of the tip-speed ratio, the thrust coefficient and the axial induction factor,

while employing the thrust curve as the basic input.

Apart from the averages also the standard deviations are specified.

(slide 7)

The procedure is shown in the figure.

Given the average wind speed and the turbulence intensity,

the average and the standard deviation for the tip-speed ratio,

and subsequently for the thrust coefficient and the axial induction factor are determined.

(slide 8)

As to the electricity production and the mechanical loading of a wind turbine,

four quantities are considered:

the aerodynamic power, the tower bending moment, the blade bending moment, and the rotor-shaft torque.

(slide 9)

The procedure for these quantities is the same:

given the average and the standard deviation of the tip-speed ratio,

the corresponding measures for the production and the loading are determined.

(slide 10)

The sub-model for the velocity deficit in the wake of a wind turbine is based on momentum theory and a large amount of experimental evidence.

Starting from an initial value,

which depends on the axial induction factor and the ambient wind speed average,

in the model the velocity deficit has a power-law decay in downstream direction and a Gaussian-law decay in spanwise direction.

(slide 11)

The structure of the velocity-deficit field is shown here:

a peak in the centre closely behind the rotor,

which peak decreases in both streamwise and spanwise directions.

(slide 12)

The sub-model for the added turbulence in the wake of a wind turbine is based on experimental evidence and the assumption that the wind speed is a stochastic process.

In the model the wind speed variance due to a wind turbine is the sum of two components, where each component originates from the creation of turbulence at the edge of the rotor disk and both components merge at a given downstream position.

Starting from an initial value,

which depends on the axial induction factor and the ambient wind speed variance,

in the model the wind speed variance has a power-law decay in downstream direction and a Gaussian-law decay in spanwise direction.

(slide 13)

The structure of the added wind speed variance field is shown here:

two peaks near the rotor disk edge closely behind the rotor,

which peaks gradually merge into one peak and ultimately disappear.

(slide 14)

The wind farm flow model allows the wind farm controller to take decisions on basis of prediction of the wind, the production and the loading for the next period of several minutes.

The model consists of a sub-model of the wind turbine, a sub-model of the wind turbine wake, and a sub-model of the cluster of wind turbines.

(slide 15)

The wind turbine sub-model has separate sub-models of the turbine state and the turbine production and loading.

(slide 16)

And the wind turbine wake sub-model has separate sub-model of the velocity wake and the turbulence wake.

(slide 17)

The cluster sub-model combines all information on the velocity deficit and the added turbulence of the individual wind turbines into information on the wind, the production and the loading in a wind farm.

(slide 18)

The model output consists of a look-up table which relates the external conditions of a wind farm to the state and the output of all the wind turbines in that wind farm.

(slide 19)

The model can be operated in two modes: forward or inverse.

In the forward mode the state and the output of the wind turbines in a wind farm are the output variables whereas the external conditions are the input variables.

The typical application of the forward mode is to predict the power output for a given wind speed.

In the inverse mode, on the other hand, the power is the input variable whereas the state and the loads are the output variables for given external conditions.

The typical application of the inverse mode is to calculate the distribution of power references over the individual wind turbines if an external power reference is issued.

(slide 20)

The forward mode was used in order to tune and validate the model.

To this end measured data from two different wind farms with two different types of wind turbine were employed.

(slide 21)

The model tuning was performed by using the wind speed and the wind power measured in a row of eight three megawatt wind turbines separated over distances of five point five rotor diameters.

(slide 22)

The external wind speed was near the nominal wind speed of the turbines, and the external wind direction was parallel to the row of wind turbines.

The averaging time was ten minutes.

(slide 23)

The figures show the measured and the predicted wind speed (on the left), and the measured and the predicted wind power (on the right), as a function of the position in the row of turbines.

The blueish symbols display the measurements.

The predicted values were obtained with the optimal values of the parameters in the wake decay laws,

where "optimal" refers to the smallest value of the sum of the squared difference between the measured and the predicted wind powers of the individual wind turbines.

(slide 24)

The model validation was performed by predicting the wind speed and the wind power in a row of five two point five megawatt wind turbines separated over distances of three point eight rotor diameters when using the optimal set of model parameters.

(slide 25)

In this case four wind speeds were considered in combination with a wind direction which is parallel to the row of wind turbines:

near the cut-in wind speed, halfway the cut-in wind speed and the nominal wind speed, near the nominal wind speed, and halfway the nominal and the cut-out wind speed.

(slide 26)

The figures show the measured and the predicted wind speed (on the left), and the measured and the predicted wind power (on the right) as a function of the position in the row.

Again the blueish symbols display the measurements.

For the near-nominal wind speed,

the predicted wind speeds are too high which results in predicted wind powers which are too high.

For the high wind speed,

the predicted wind speeds are too high as well but the predicted wind powers are correct.

For the other two wind speed cases,

the predicted wind speeds are within one meter per second and the predicted wind powers are within a hundred kilowatt.

May these results be insufficient for design purpose,

they are good enough for control purpose.

(slide 27)

Summarizing,

these fits of the predicted and the measured wind power required tuning of the power in the decay law of the velocity deficit and the power in decay law of the added turbulence.

In addition,

but I have not talked about this,

these fits required a variable rather than fixed length of the near wake.

(slide 28)

As already mentioned,

the inverse mode of the model is used in order to determine the distribution of power references over the individual wind turbines if a power reference for the wind farm is given.

(slide 29)

In the remainder of this talk I will show such a wind farm control application for a wind farm which consists of a row of ten five megawatt wind turbines separated over distances of five rotor diameters.

(slide 30)

The figure on the right shows the distribution of the power references over the turbines for the uncontrolled and the controlled situation if the external wind speed is halfway cut-in and nominal.

The wind direction is parallel to the row.

For the uncontrolled situation (the black diamonds) the power references are equal to the power that is available in the wind.

For the controlled situation the control objective is not to exceed thirty three percent of the nominal power of the wind farm.

The grey squares in the figure on the right show that the action of the wind farm controller results in a small redistribution of the power references.

The figure on the left shows the corresponding wind speeds at the turbine positions for the uncontrolled and the controlled situation.

(slide 31)

These figures show the same kind of information for an external wind speed which is near the nominal wind speed.

The control objective in this case is not to exceed sixty two percent of the wind farm nominal power.

The figure on the right shows that the control action results in a reduction of the power reference of the first turbine in the row,

in combination with an increase of the power references of the other turbines.

The figure on the left shows the impact on the wind speeds.

(slide 32)

Summarizing,

a quasi-steady wind farm flow model for use in wind farm control has been presented.

I have shown that this model can be operated in two modes:

one where the external conditions are the input variables,
and the other where the wind power is the input variable.

The forward mode of the model was tuned and validated by using measured wind speed and wind power from two different wind farms.

The inverse mode has demonstrated the ability to redistribute individual power references if an external power reference is not to be exceeded.

(slide 33)

Thanks for your attention.