LMI-based Robust LPV Control of Wind Turbines

Literature Review and Problem Statement

Valentin Pascu

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Abstract

Wind power has proven to be one of the most versatile forms of renewable energy. In a similar fashion, offshore wind is emerging as the most promising method of addressing the increased power production requirements, given the massive potential of wind at sea. The Dutch consortium for innovation in offshore wind energy TKI Wind op Zee performs active research with the main goal of allowing higher penetration of offshore wind power. D4REL (Design for Reliable Power Performance) is one such research project which, within the consortium, is coordinated by the Energy Research Centre of the Netherlands (ECN). Within D4REL, ECN works in close cooperation with TU Delft and have established a definite goal of developing novel control tools for the reduction of uncertainty in the design and operation of offshore wind farms.

Offshore wind turbines are subject to large variations in terms of their physical parameters. Aerodynamic performance can change dramatically based on weather conditions and the passage of time. Modal characteristics for bottom-supported structures experience considerable variations due to environmental conditions such as rise and fall of sea levels, formation of ice around the turbine tower and scour. Control systems designed only based on an ideal fixed-parameter model will, therefore, perform suboptimally in such situations. In this literature review attention is first given to introducing the practicalities associated with the mentioned problem into a model structure. A framework for the robust and linear parameter-varying control theory is then established. Up-to-date efforts in wind turbine control are described in strict relation to the goals of the D4REL project. Subsequently, a new control problem is formulated and a research direction towards addressing it is proposed.

Contents

List of Abbreviations

D4REL	Design for Reliable Power Performance R&D Project
ECN	Energy Research Centre of the Netherlands
LFT	Linear Fractional Transformation
LMI	Linear Matrix Inequality
LPV	Linear Parameter-Varying System
LTI	Linear Time-Invariant System
LQR	Linear Quadratic Regulator
TU Delft	Technische Universiteit Delft
VS-VP	Variable-Speed Variable-Pitch Wind Turbine

1

Introduction and Conceptual Requirements

Many aspects of the modern times are highly debatable upon but there is no doubt about the fact that during the past 40 years the renewable energy sector has seen an impressive growth. Such a development has brought an enormous improvement in both the quality of our lives and the way in which we interact with our fragile environment. At European level, the use of renewable energy is gradually approaching a definite level of maturity, with the European Union being one of the leaders in its development and use [28]. Equally certain is the rapid increase in installed wind power capacity, both onshore and offshore, with a total estimated rate of growth of approximately 30% per year [7]. Although in use since ancient times and reaching a reasonable degree of sophistication with the wind mills of the late Middle Ages, wind energy had been partially left aside during the 19^{th} and early 20^{th} centuries. Driven by a strong desire to make them a sought-after power production alternative, a large group of scientists and inventors such as *Charles Francis Brush*, *Poul la Cour* and *Palmer Cosslett Putnam* to name only a few, have continued to improve the technology despite the general lack of market interest. Due to such efforts there has been no consistent gap in the progress of wind energy technology development and the transition from fossil fuels to wind power has been found to be incredibly smooth compared to other forms of renewable energy.

Despite its intrinsic variable nature and non-constant availability, wind has long been considered an important resource. Ancient uses of wind included water pumping, grain grinding and sailing. Masterful wind mill designs have been performed in Denmark, Scotland and The Netherlands in the 17^{th} and 18^{th} centuries, albeit the idea of a wind mill allegedly originated in England in the 12^{th} century [48]. The main challenges of the time included improving the power capture and increasing reliability. In the late 19^{th} century the first wind mill with electricity-producing capabilities has been constructed by using the rotation of the mill's shaft to drive an electric generator. The idea brought awareness of new potential uses of wind energy. Thus, the concept of *wind turbine* emerged. Through various experimental designs and theoretical research, the technology has considerably progressed now offering very flexible solutions which can suit the needs associated with all types of environments that possess wind power potential [23]. One of the focal points in wind energy technology development is the creation of suitable *automatic wind turbine control* strategies and algorithms in order to address the various operation objectives. Current challenges include optimal wind turbine operation (increasing power capture while simultaneously decreasing loads), creating the possibility for reasonable turbine functioning in exceptional situations (both external - *environmental* and internal - *turbine-related*) and optimal wind farm operation [12].

With a long history in the development of wind power, The Netherlands possesses an impressively large amount of *know-how* in this field in its academia and research institutes, even though the few national wind turbine manufacturers do not currently hold a big share in the global market. The Dutch consortium for innovation in offshore wind energy, **TKI Wind op Zee**, performs active research with the main goal of allowing higher penetration of offshore wind power. One such project is **D4REL** which aims at developing novel tools for the reduction of uncertainty in the design and operation of offshore wind farms. Such goals can be achieved both by creating the possibility of better understanding of wind turbine and wind farm behaviour using advanced modelling methods and by improving upon the manner in which a high-level control strategy translates into a practical control system design methodology.

As opposed to their land-based counterparts, offshore wind turbines experience a wider range of factors that can potentially take a toll on their productivity and lifespan. Changes in relative water depth due to tides and scour can affect the modal characteristics of the turbines, making such a lightly-damped structure subject to abnormal operation, especially if not accounted-for during in the design phase. Hence, any *fixed-parameter* control system is likely to underperform and perhaps even be problematic [12]. Variations in weather conditions together with the passage of time undoubtedly change the aerodynamic properties of the blades due to such factors as ice formation, dirt formation and blade erosion; any control system designed with only the perfect wind turbine model in mind is bound to be *sub-optimal* in such circumstances. Therefore, a synthesis procedure which can lead to appropriate control system designs in the face of such uncertain and time-varying models needs to be employed and, as part of the **D4REL** project, **ECN** and **TU Delft** aim at applying state-of-the-art academic research in control theory in order to address these stringent issues. The current thesis work is an integral part of this project and is focused on designing robust and adaptive wind turbine controllers that optimize operation under different environmental conditions. The associated design task shall be performed for the ART 5 MW wind turbine, which is an offshore monopile bottom-supported VS-VP horizontal-axis wind turbine. This literature survey has been carried out in the framework of the final thesis assignment "**LMI-based Robust LPV Control of Wind Turbines**", supervised by **Stoyan Kanev** (**ECN**) and **Jan-Willem van Wingerden** (**TU Delft**); its goal is defined as follows.

> This literature report aims at providing an overview of the current state-of-the-art in **LMI formulations for robust and parameter-varying systems analysis and control synthesis**, together with the associated challenges in **wind turbine control**.

The previous statements regarding modern offshore wind turbine operation and motivation behind the **D4REL** project are now explicitly shown in the following problem formulation which represents the purpose of the thesis work.

> Given the complexity of offshore wind turbines behaviour, owed to both the **uncertain** aerodynamic characteristics of the blades (by means of the associated *power* and *torque coefficients*) and **time-varying** environmental conditions (more explicitly, *wind speed* and *tower frequency*, both assumed to be online-available), the goal of this thesis is to propose a **robust** and **linear parameter-varying** control system design, **optimal** in the sense of fully exploiting this given nature of offshore wind turbines.

In **Chapter 2**, the survey establishes a framework for control-oriented wind turbine modelling that shall be used in all further developments. **Chapter 3** is dedicated to providing all essential information regarding uncertain and parameter-varying systems theory. Essential properties are summarized and methods by which synthesis procedures for such systems can be posed are shown. **Chapter 4** deals with the application of the presented theory to the control of wind turbines. Several existing approaches are described in relation to the presented aims and clear statements are made regarding how the approach to be used will need to differ from current procedures. In **Chapter 5** a formulation of the control problem is given together with all essential links to the physical process. **Chapter 6** concludes the review with some remarks on how the formulated problem might be tackled and gives important remarks regarding the advantages and disadvantages of each available option. ■

2

Control-Oriented Wind Turbine Modelling

Whether based on physical laws or on collected input-output plant data, *modelling* is an essential step that helps in the design of controllers that use and drive a real-life system to its full potential. However, an important and oftentimes neglected aspect of this task is remembering that any dynamical model is always derived in order to suit several particular needs, therefore a good balance needs to be stricken between its *complexity* and its *adequacy*. Simplifications are, thus, unavoidable. With this idea in mind, the approach of this section is practical. First, several comments are made about the nature of *wind*, which acts both as a *raison d'être* and as a potential source of problems for wind turbines. Secondly, control-related remarks are made on what a suitable wind turbine model should consist of. All such comments are made for VS-VP three-bladed horizontal axis wind turbines. Building towards the peak of the chapter, a block structure is presented in order to show how a wind turbine and its subsystems may be looked at from the perspective of dynamic modelling. In doing so, every subsystem is properly described. Eventually, a full model is presented and several remarks are made about how such a model may fit into the theoretical framework shown in **Chapter 3**. All presented information is necessary when looking at recent developments in the field of wind turbine control in **Chapter 4** and when formulating a new practical problem in **Chapter 5**.

2.1 The Wind and Reasons for Wind Turbine Control

The wind is an inexhaustible source of energy of *local* nature, with availability depending on the geographical positioning, on the vertical positioning (height with respect to the Earth's surface) and on the weather conditions (owed to the season of the year, the time of day or any other factors of meteorological essence). Though the nature of wind is highly variable, it can be described statistically with sufficient accuracy on any given site. This can be achieved by combining knowledge from series of measurements of nearby weather stations taken over long periods of time (which show clear trends regarding the mean values of the speed of the local winds) with mathematical models that describe the turbulent components (e.g. *von Kármán* or *Kolmogorov*) [48]. The characteristics of the local wind are considered during the development of any turbine (including the control system design) but prove to be of extreme importance especially for the simulation and validation process [23]. Therefore, for the needs of the current work, a simplification can be made and we shall further assume that the wind is in the form of a steady (stationary) inflow that can be completely described by its speed $u_{col}(t)$, considered to be the same at all points in the rotor plane and containing all definitory deterministic and stochastic components. Then, the available power in the wind that passes an area of turbine rotor size with radius R is given by:

$$P_r(t) = \frac{1}{2}\rho\pi R^2 u_{col}^3(t) \tag{2.1}$$

where ρ is the air density, considered to be constant in both time and space.

Generally speaking, a wind turbine is a device whose main goal is to capture as much wind energy as possible and to convert it into useful work [23]. The aerodynamic efficiency of a wind turbine may be given in terms of the *tip-speed-ratio* $\lambda(t)$:

$$\lambda(t) = \frac{\Omega_r(t)R}{u_{col}(t)} \tag{2.2}$$

which is the ratio between the tangential speed of the tip of the blade (as a product between the rotor speed $\Omega_r(t)$ and the rotor radius) and the wind speed $u_{col}(t)$. Here, these are given as functions of time. At constant wind speed, maximum power production is then achieved for some given values of the tip-speed-ratio and of the blade pitch angle, as is explained in the section on **Aerodynamic Subsystem** modelling, later on in this chapter.

However, in recent years challenges related to the need of decreasing the cost of wind energy have been phrased [6, 63] and, as a result, energy maximization has been shown to no longer be suitable as the *only* objective associated with wind turbine operation. Current trends point towards the formulation of the overall objective in terms of several goals (such as *energy capture maximization*, *mechanical loads reduction* and *power quality conditioning*) that need to be satisfied by means of feedback control, thus ensuring overall optimal wind turbine operation [20, 12]. A control-oriented model of a wind turbine should therefore reflect some of the intrinsic physical processes occurring in relation to the mentioned goals. Any further information is, in fact, of little use. What this model needs to contain, on the other hand, can be hinted at by looking more specifically at *how* a wind turbine is controlled, from the point of view of the adopted strategy [12], as will be further explained.

In terms of *energy capture*, wind turbines are expected to follow power curve of the type shown in Figure 1. As can be seen, four distinct regions of operation can be described. In region 0 the wind speed is below the defined cut-in speed and the turbine is idling (not producing electricity). Region I can be described by low wind speeds and here typically the aim is to control the turbine such that as much energy is captured as possible. This is achieved by maximizing the aerodynamic power coefficient C_P. In practice this is achieved by reducing the opposing torque from the electrical generator. In region II, lower-bounded by the wind speed corresponding to nominal rotor speed and upper-bounded by the rated wind speed, optimal energy capture needs to be compromised over so as to reduce noise emissions and to maintain all centrifugal forces below values tolerated by the rotor. Region III corresponds to above-rated wind speeds and power production is limited to the maximum power that can be delivered by safe operation of the electric generator. This is achieved by changing the pitch angle $\theta(t)$ of the blades which has an effect on C_P. As a measure of power capture, C_P falls within a range upper-bounded by a theoretical maximum called the *Betz limit* [23]. Though both *pitch-to-feather* (*i.e.* increasing the angle θ from its optimal value) and *pitch-to-stall* (*i.e.* decreasing the angle θ from its optimal value which results in induced stall) strategies can be used to reduce C_P in high wind speeds as shown in Figure 3), in practice usually *pitch-to-feather* strategies are used because the mechanical loads associated to reduced angles of attack are lower; moreover, stall is an aerodynamic effect which is well-understood but cannot be modelled with sufficient accuracy.

On the other hand, mitigation of *mechanical loads*, either static or dynamic, is an auxiliary task to be achieved in order to reduce the fatigue damage in wind turbines, thus reducing operational costs. Of these, the most severe are considered to be the dynamic loads which might appear as a result of unequal spatial distribution in the wind field (which can potentially be mitigated by *individual pitch control*), unequal temporal distribution (turbulence or wind gusts) or may

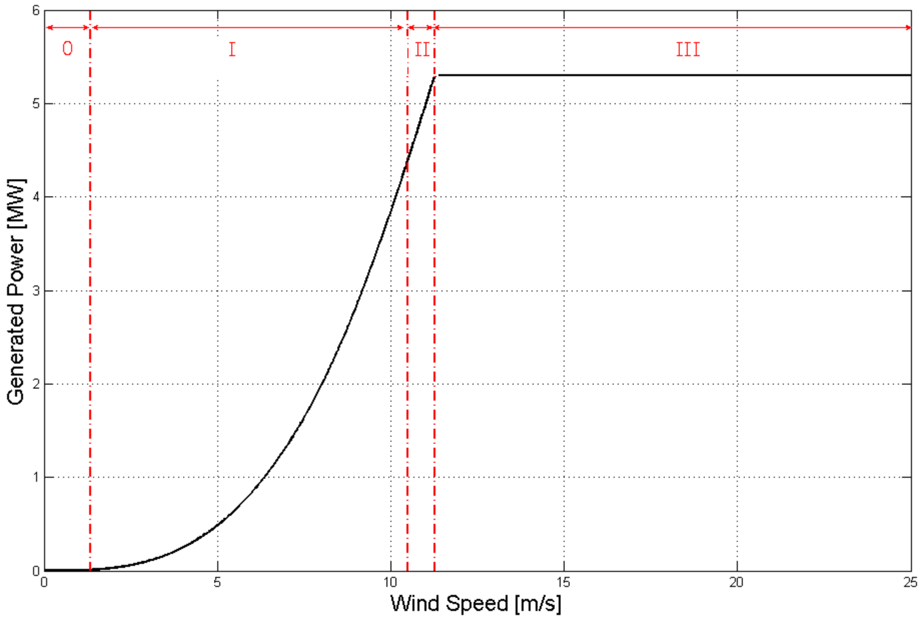

Figure 1: Power Curve for a Generic Wind Turbine

be control-induced (*e.g.* due to tight control for maximum energy capture). Several conflicts between these first two goals become visible in regions I, II and III. The third goal is associated with the *quality of power* that wind turbines can supply.

2.2 Dynamic Modelling

Modern wind turbines are complex electromechanical systems, comprised of many parts. Whilst there exist a handful of ways of looking at each of these from the *physical* point of view [48], we shall only describe the essential parts from a *dynamical systems* perspective and model the *interaction* between these. In Figure 4 we can observe the most typical way of decomposing a wind turbine into its subcomponents based on their functions [12]. Here the angle θ mentioned thus far is denoted $\theta_{lim}(t)$ for reasons which shall be explained in this section. We may consider that the starting point of wind turbine modelling is such a general block diagram representation where one is allowed to set a collective pitch angle of the turbine blades $\theta_{ref}(t)$ and the torque of the electrical generator $T_{gen}^{ref}(t)$. The turbine is operating under the influence of wind (with speed $u_{col}(t)$ as mentioned previously) and of the constant requirements from the electrical grid (*e.g.* the frequency f_s and voltage U_s that the generator is allowed to inject into the power network). Typically, the rotor speed $\Omega_r(t)$, the tower acceleration $\ddot{x}_{fa}(t)$ at hub height and the blade pitch angle $\theta_{lim}(t)$ are measured, alongside other variables of less importance for our study. Figure 4 reveals four subsystems whose dynamic behaviour we are interested in, namely the *aerodynamic subsystem*, the *drivetrain* together with the *tower structure*, the *pitch actuator servomechanism* and the *power generator*.

Aerodynamic Subsystem

The kinetic energy in the wind follows a sequential transformation towards electrical energy. First, by means of its aerodynamics, a wind turbine will be able to convert a generic wind field into specific blade-effective *thrust forces* and *aerodynamic torques*. However, assuming that the wind has the same speed $u_{col}(t)$ at all points in the rotor plane, that the aerodynamic thrust coefficient $C_T(\cdot,\cdot)$ and torque coefficient $C_Q(\cdot,\cdot)$ is the same for all blades and that the term $\dot{x}_{fl+fa}(t)$ which encompasses the axial displacement of the blades caused by flapping and tower bending is negligible

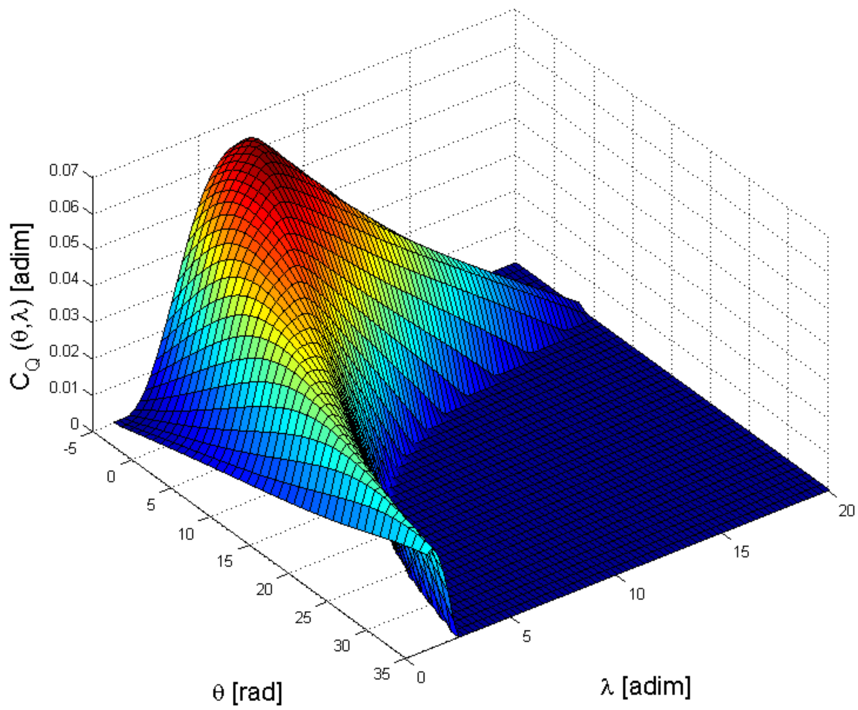

Figure 2: Typical Torque Coefficient $C_Q(\theta, \lambda)$

reduces only to tower fore-aft displacement $\dot{x}_{fa}(t)$, we may instead express the rotor-effective thrust force $F_{ax}(t)$:

$$F_{ax}(t) = \frac{1}{2}\rho\pi R^2(u_{col}(t) - \dot{x}_{fa}(t))^2 C_T(\lambda(t), \theta_{lim}(t)) \tag{2.3}$$

and the rotor-effective aerodynamic torque $T_a(t)$:

$$T_a(t) = \frac{1}{2}\rho\pi R^3(u_{col}(t) - \dot{x}_{fa}(t))^2 C_Q(\lambda(t), \theta_{lim}(t)) \tag{2.4}$$

Several remarks are necessary before concluding this section. The aerodynamic behaviour of wind turbines has been mentioned to be uncertain in the problem statement of the previous chapter. This uncertainty arises from different sources. First of all, airfoils are designed such that they allow the achievement of wind turbine specifications over as large an operational envelope as possible. The characteristics of the designed airfoil (in terms of aerodynamic coefficients C_P, C_T, C_Q, etc.) are never the same on the constructed blades. Stall is an effect which can hardly be numerically modelled with sufficient accuracy and should the turbine blades find themselves in the stall region, there is little information as to what the coefficient values would be for any such particular angle of attack. Moreover, blade erosion occurs in time and this also has a significant effect; similarly, due to exposure to open air, contact with insects or birds is also potentially changing these coefficients. For wind turbines placed in rough environments such as humid and cold areas even more severe phenomena can occur, such as ice formation. All these are contributing factors to uncertainty in the aerodynamic behaviour of the blades and an available estimate of the proportion of this uncertainty would indeed help as turbine operation is governed by controllers that can alleviate the consequences of these factors by appropriate operation. Uncertain characteristics in the aerodynamic parameters C_P, C_T and C_Q will further be denoted by C_P^Δ, C_T^Δ and C_Q^Δ, respectively.

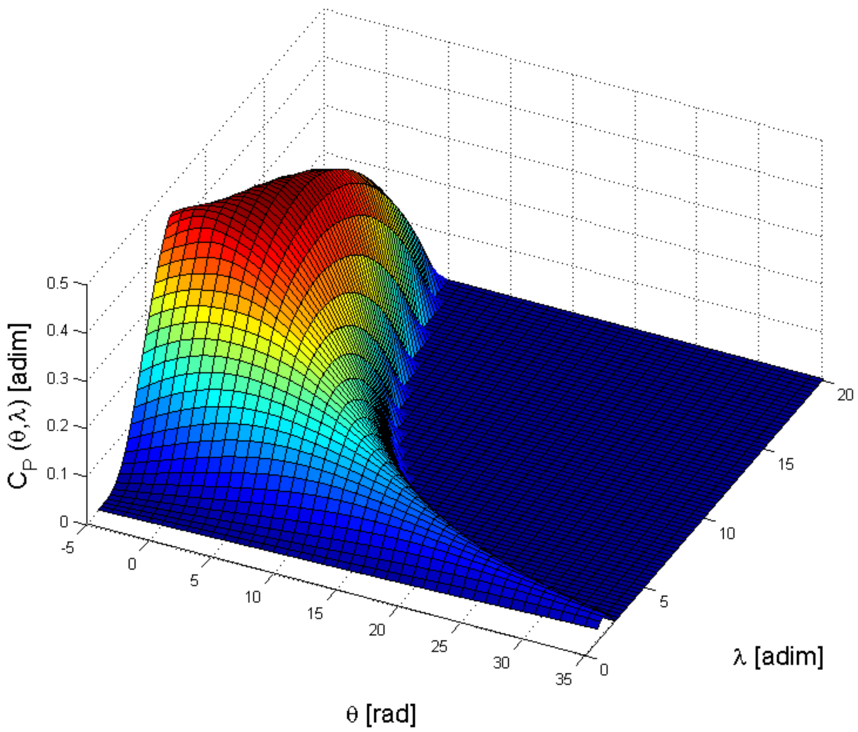

Figure 3: Typical Power Coefficient $C_P(\theta, \lambda)$

Drivetrain and Tower Structure

The drivetrain is composed of two shafts connected through a gearbox with transmission ratio i_{tr}, rigidly connected to the turbine nacelle. The fast shaft is considered rigid, while the slow shaft is flexible with torsional stiffness s_{dt} and damping d_{dt} [36]. The transmission ratio is defined to be positive when the rotation of both shafts is in the same direction and opposite otherwise. Considering no torque losses (*e.g.* due to friction in the gearbox or inelastic shaft torsion), the drivetrain dynamics can be described by the differential equation [36]:

$$\frac{i_{tr}^2 J_r J_g}{J_r + i_{tr}^2 J_g}\ddot{\gamma}(t) + d_{dt}\dot{\gamma}(t) + s_{dt}\gamma(t) = \frac{i_{tr}^2 J_g}{J_r + i_{tr}^2 J_g} T_a(t) + \frac{J_r}{J_r + i_{tr}^2 J_g} |i_{tr}| \, T_{gen}(t) \tag{2.5}$$

where $\gamma(t)$ is the azimuthal difference between the two ends of the drivetrain and J_r and J_g are the rotor and generator inertia, respectively. Similarly, it is sufficient for our needs to model only the first structural tower mode in the fore-aft direction because the higher-order modes usually do not bring an important contribution to the resulting oscillation [36]; the first structural mode considered to be excited by the rotor-effective thrust force $F_{ax}(t)$ alone:

$$m_t(t)\ddot{x}_{fa}(t) + d_t(t)\dot{x}_{fa}(t) + s_t(t)x_{fa}(t) = F_{ax}(t) \tag{2.6}$$

thus disregarding any effect due to blade tilting moments. It is assumed here that $m_t(t)$, $d_t(t)$ and $s_t(t)$, which denote the tower mass, damping and stiffness, respectively vary with time; this especially has a direct consequence on the first fore-aft tower top motion mode, and, as has been previously mentioned in the problem formulation, such effects need to be analysed and taken into account. However, by means of $m_t(t)$, $d_t(t)$ and $s_t(t)$ only a pseudo-natural way of modelling changes in modal behaviour is given. Note that no dynamic effects have been considered in terms of the

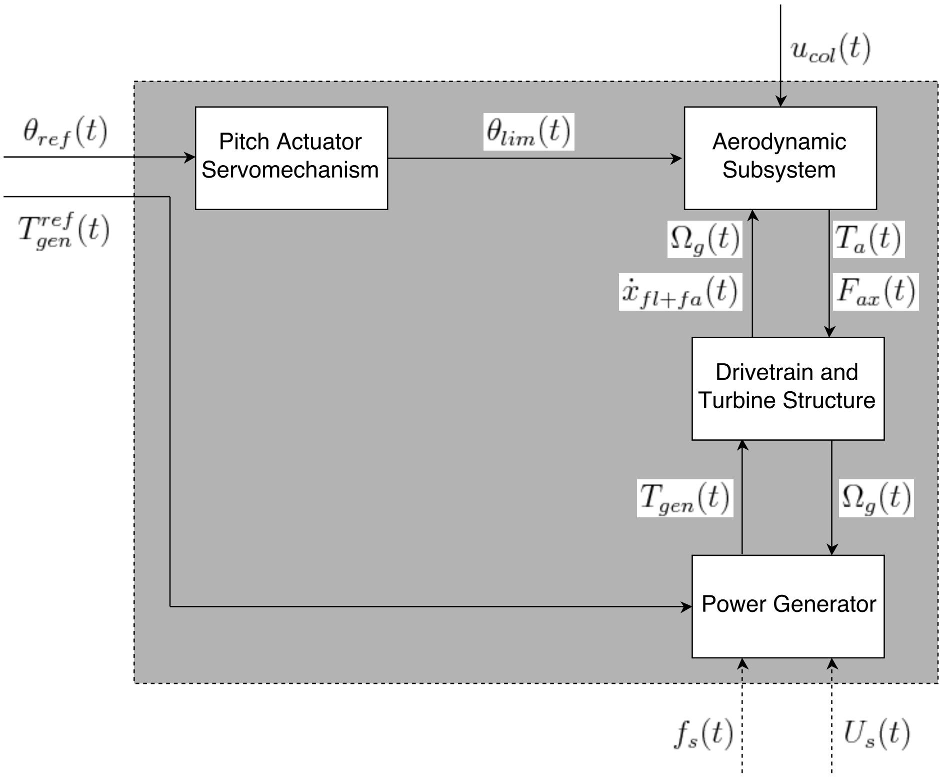

Figure 4: Physical Subsystem Interconnection Inside a Wind Turbine

sidewards movement of the turbine structure also because their contribution to the resulting oscillation is insignificant [36].

Pitch Actuator Servomechanism and Power Generator

The controller outputs setpoints $\theta_{ref}(t)$ and $T_{gen}^{ref}(t)$ for the collective blade pitch angle $\theta_{unlim}(t)$ and for the required generator torque $T_{gen}(t)$, respectively; these are converted to actual physical quantities by the actuation servomechanisms. The pitch pitch actuator servomechanism can be assumed to be partially modelled by the second-order time-delayed linear model [36]:

$$\ddot{\theta}_{unlim}(t) + 2\beta_{pt}\omega_{pt}\dot{\theta}_{unlim}(t) + \omega_{pt}^2\theta_{unlim}(t) = \omega_{pt}^2\theta_{ref}(t - \tau_{pt}) \tag{2.7}$$

with natural frequency ω_{pt}, damping β_{pt} and pure delay τ_{pt}. Though in real life the output of this actuator is subject to physical constraints:

$$\begin{aligned} \theta_{min}(t) \leq \theta_{lim}(t) \leq \theta_{max}(t) \\ \dot{\theta}_{min}(t) \leq \dot{\theta}_{lim}(t) \leq \dot{\theta}_{max}(t) \\ \ddot{\theta}_{min}(t) \leq \ddot{\theta}_{lim}(t) \leq \ddot{\theta}_{max}(t) \end{aligned} \tag{2.8}$$

all these shall be neglected when constructing the necessary model because they cannot be taken explicitly into account with the control approach that is most suitable for the stated needs, subsequently presented in **Chapter 3**. More explicitly, they shall be neglected in the design phase but, of course, in the implementation phase they will be re-introduced. Therefore the actuator may be considered linear and hence $\theta_{lim}(t) \equiv \theta_{unlim}(t)$. On the other hand, the torque generator setpoint $T_{gen}^{ref}(t)$ enters the power generator subsystem directly and is converted to the real torque $T_{gen}(t)$ by means of the unconstrained second-order time-delayed linear servomechanism [36]:

$$\ddot{T}_{gen}(t) + 2\beta_g \omega_g \dot{T}_{gen}(t) + \omega_g^2 T_{gen}(t) = \omega_g^2 T_{gen}^{ref}(t - \tau_{gen}) \tag{2.9}$$

with natural frequency ω_g, damping β_g and pure delay τ_{gen}. The generator terminal voltage $U_s(t)$ and grid frequency $f_s(t)$ are assumed to be fixed and stable variables [12], hence have no dynamic effects on $T_{gen}(t)$. Similarly, we only view the power generator as a subsystem that can supply a torque based on a given demand $T_{gen}^{ref}(t)$ so no torque-related dynamic effect due to the generator speed $\Omega_g(t)$ resulting from the rotational motion of the fast shaft is taken into account [36]. An overall model is shown in Figure 5. Several comments regarding this proposed model are made as a closing of this chapter.

2.3 Conclusions

This chapter has described control-oriented modelling for VS-VP horizontal axis wind turbines. Attention has been given to wind and how its energy is captured by the rotor of the turbine. Further on, the mechanical and electrical subsystems have been described. As is usually the case, simplifications have been made so as to build a model that is not overly complex, while still being sufficiently dynamically accurate and relevant for our needs. Of course, all these simplifying assumptions have been made in strict relation to the aims proposed in **Chapter 1** and appropriate references to the mentioned goals of the thesis work have been given. Our procedure has brought forward a model in the form of the block diagram shown in Figure 5.

We may observe several particularities of the wind turbine, as seen from the current perspective. Though the pitch actuator servomechanism is a nonlinear dynamical system due to equations (2.8), we have chosen to neglect such constraints as they cannot be directly addressed through the proposed control approach of **Chapter 3**. Clearly, the aerodynamic subsystem is *nonlinear* due to the quadratic dependence on its inputs $u_{col}(t)$ and $\dot{x}_{fa}(t)$. Moreover, the coefficients C_T and C_Q *vary based on the same inputs*, but *also on the rotor speed* $\Omega_r(t)$ and *on the pitch angle* $\theta_{unlim}(t)$. In practice, these are given in tabular form. The drivetrain is a linear dynamical system, however it has been proposed that the tower mass $m_t(t)$, the tower damping $d_t(t)$ and the tower stiffness $s_t(t)$ are *time-varying*. Overall, the wind turbine model has become *uncertain*, *time-varying* and *nonlinear*, a fact which will have to be properly dealt with using an appropriate control design framework.

In **Chapter 3** a suitable methodology for the analysis of our model and synthesis of a controller is presented. The model that has been derived in the current chapter will need to be slightly altered to fit the presented tools of **Chapter 3**. The notation used in this chapter has been consistent with [36]. ■

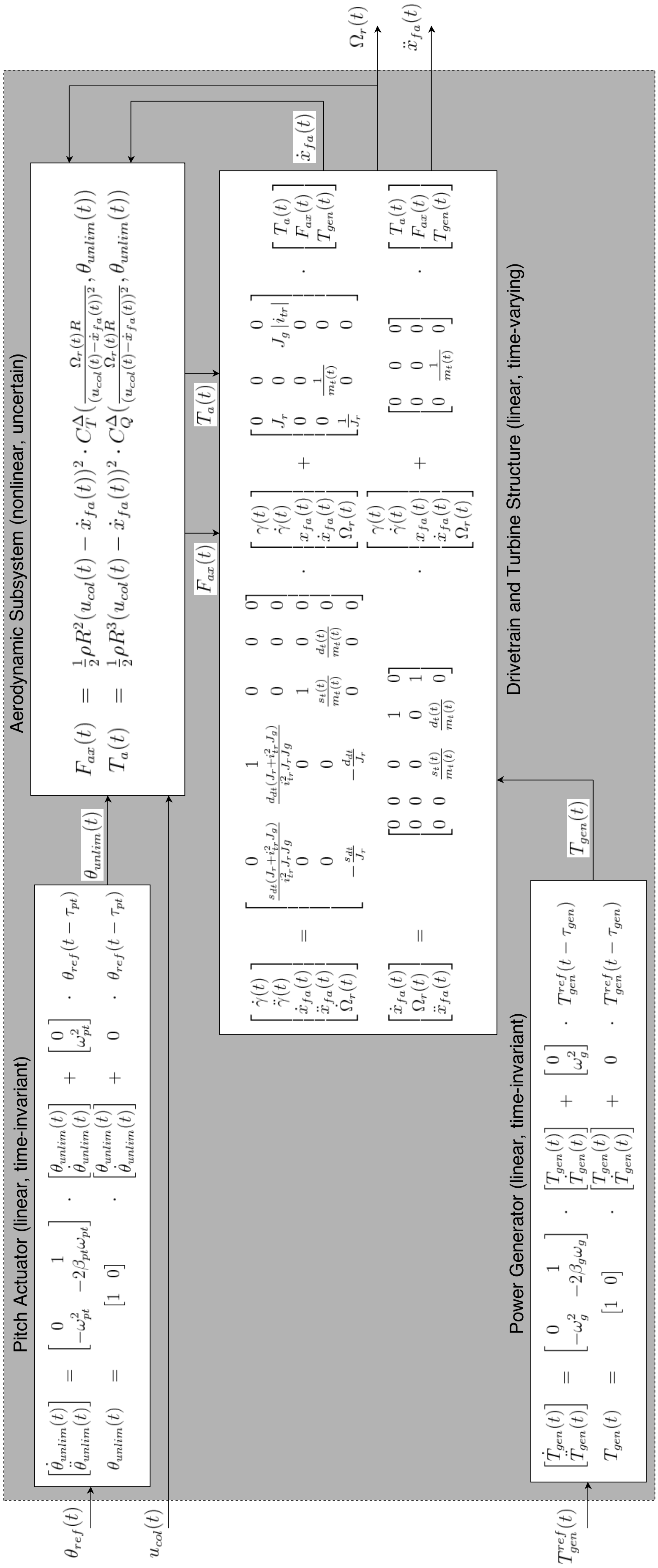

Figure 5: Proposed Model for the Wind Turbine

3

LMI-based Robust and LPV Control: An Overview

In this section of the literature review, the aim is to set up a theoretical framework for the control system design task; this should address the practical needs expressed in **Chapter 1**. The presented framework is further used in **Chapter 4** to relate to some of the recent developments in the field of wind turbine control and, most importantly, in **Chapter 5** when the control problem is formulated. The current state of the art in the analysis of *deterministic uncertain* systems together with the available tools for robust controller synthesis are shown first. Subsequently, *LPV* systems and associated controller design issues are described. The available literature on these topics is vast. Hence only developments that fall within the range of applications of convex optimization are considered; more specifically, we take an approach based on *LMIs* and we introduce them in the opening of the chapter as a necessary background for our discussion.

3.1 Linear Matrix Inequalities in Control Theory

Convex optimization is known to be very attractive from both a theoretical and practical perspective, mainly due to the fact that the troubles associated with *local optimality* can be avoided; it is also extremely useful as tools for large-scale optimization exist for such formulations and have been found to be extremely efficient [22]. Semidefinite programming, as a subfield of convex optimization, deals with optimizing a linear objective function over a set defined by the intersection of a positive semidefinite cone and an affine space [11]. In control engineering, LMIs often appear and, fortunately, they can be cast as semidefinite programs and can be numerically solved by using either *ellipsoid methods* [37] or the more efficient *interior point methods* [51]. In an LMI, the goal is to find a vector $\chi \in \mathbb{R}^N$ such that:

$$F(\chi) \triangleq F_0 + \sum_{i=1}^{N} \chi(i) F_i \succ 0 \tag{3.1}$$

where the matrices $F_0 \in \mathbb{R}^{n_F \times n_F}$ and $F_i \in \mathbb{R}^{n_F \times n_F}$, $i = \overline{1, N}$ are symmetric, often called the *data matrices*. The inequality may be strict or non-strict and may be customized according to the needs due to the practical issue at hand (*e.g.* change of inequality direction, stacking of multiple inequalities). As an example of the usefulness of LMIs for our situation, let us look at stability analysis for systems of the type:

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bw(t) \\ z(t) & = & Cx(t) + Dw(t) \end{array} \right. \tag{3.2}$$

These are called (finite-dimensional) linear time-invariant (LTI) systems. Consider $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times o}$, $C \in \mathbb{R}^{l\times n}$ and $D \in \mathbb{R}^{l\times o}$. Concluding whether equation (3.2) denotes an asymptotically stable system requires finding a quadratic Lyapunov function of the type $V(x) = x^T(t)Xx(t)$ with symmetric positive-definite matrix X such that [21]:

$$\begin{array}{rcl} A^T X + XA & \prec & 0 \\ -X & \prec & 0 \end{array} \tag{3.3}$$

Clearly, the existence of a Lyapunov function is actually an LMI strict feasibility check problem [21]. However, one needs to perform several modifications in order to arrive at an appropriately-posed LMI problem. The first is to look the elements of the unknown matrix $X \in \mathbb{R}^{n\times n}$ as stacked elements in the optimization variable (vector) χ. Since X needs to be symmetric, the dimension of the vector χ turns out to be $N = \frac{n(n+1)}{2}$.

The data matrices for (3.3) can then be retrieved and their structure depends on the structure of A. Let us consider a particular example. If $A \in \mathbb{R}^{2\times 2}$ is of the type:

$$A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \tag{3.4}$$

then the matrices $F_0 \in \mathbb{R}^{n_F\times n_F}$ and $F_i \in \mathbb{R}^{n_F\times n_F}$, $i = \overline{1, N}$ are given by:

$$\begin{array}{rcl} F_0 & = & \left[\begin{array}{cc:cc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \\ F_1 & = & \left[\begin{array}{cc:cc} 2a_1 & a_2 & 0 & 0 \\ a_2 & 0 & 0 & 0 \\ \hdashline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \\ F_2 & = & \left[\begin{array}{cc:cc} 2a_3 & a_1 & 0 & 0 \\ a_1 & 2a_2 & 0 & 0 \\ \hdashline 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{array}\right] \\ F_3 & = & \left[\begin{array}{cc:cc} 0 & a_3 & 0 & 0 \\ a_3 & 2a_4 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right] \end{array} \tag{3.5}$$

Observe that use has been made of the stacking of multiple inequalities (as previously mentioned and hinted by means of the dashed lines) and that all data matrices are symmetric. Indeed, now the problem has been completely cast as an LMI of the type proposed by definition (3.1).

The role of linear matrix inequalities in modern control theory has been introduced in [74]. The same author has later proposed in [75, 76] a specific measure of *dissipativity* for general dynamical systems, which proves to be very useful for our needs, as will be shown next. For the proposed measure, LMIs can be constructed, depending on the class of systems considered. It turns out that two of the fundamental classes for which such theoretical foundations have been laid are the ones that we have proposed at the start of the chapter. For all the developments that will follow, let us consider a feedback structure as shown in Figure 6. By G we denote the *generalized plant* which is comprised of both the model of the system that is to be controlled and all performance specifications; K denotes the *controller* which is feedback-interconnected with G. We further define:

$$\begin{cases} u(t) & \in & \mathbb{R}^m & \text{(input vector)} \\ w(t) & \in & \mathbb{R}^o & \text{(vector of exogenous variables)} \\ y(t) & \in & \mathbb{R}^p & \text{(output/measurements vector)} \\ z(t) & \in & \mathbb{R}^l & \text{(performance outputs vector)} \\ t & \in & \mathbb{R}^+ & \text{(time)} \end{cases} \tag{3.6}$$

3.2 Robust Control of LTI Systems

LTI plants of the type:

$$\begin{cases} \dot{x}_G(t) & = & A_G x_G(t) + B_{Gu} u(t) + B_{Gw} w(t) \\ y(t) & = & C_{Gy} x_G(t) + D_{Guy} u(t) + D_{Gwy} w(t) \\ z(t) & = & C_{Gz} x_G(t) + D_{Guz} u(t) + D_{Gwz} w(t) \end{cases} \tag{3.7}$$

can be controlled using generic controllers given by:

$$\begin{cases} \dot{x}_K(t) & = & A_K x_K(t) + B_K y(t) \\ u(t) & = & C_K x_K(t) + D_K y(t) \end{cases} \tag{3.8}$$

Here $x_G(t) \in \mathbb{R}^{n_G}$ and $x_K(t) \in \mathbb{R}^{n_K}$ denote the state of the plant G and of the controller K, respectively.

The induced $\mathcal{L}_2/\mathcal{L}_2$ norm (often called the $\mathcal{H}_\infty$ norm) for the closed-loop system T is defined by:

$$\|T\|_{\mathcal{L}_2/\mathcal{L}_2} \triangleq \sup_{\substack{\|w\|_{\mathcal{L}_2} \neq 0 \\ w \in \mathcal{L}_2}} \frac{\|z\|_{\mathcal{L}_2}}{\|w\|_{\mathcal{L}_2}} \tag{3.9}$$

The norm defined by (3.9) is generally considered to be the appropriate measure of dissipativity [75, 76] for both system analysis and the design of controllers that achieve closed-loop stability and a specific level of performance, mainly due to the fact that it measures the way in which the system T behaves from an energy viewpoint (even though it may sometimes be more convenient to use other norms *e.g.* for state-feedback control laws usually the $\mathcal{H}_2$ performance measures are used; the extension is trivial in this sense). Here $\mathcal{L}_2$ is the space of *finite energy* signals *i.e.* signals

$s : [0, \infty) \to \mathbb{R}^{n_s}$ that satisfy $\int_0^\infty s^T(t)s(t)dt < \infty$. Given that the interconnection in Figure 6 is formed using LTI systems (3.7) and (3.8), the definition (3.9) simplifies to:

$$\|T\|_{\mathcal{H}_\infty} \triangleq \sup_{\omega \in \mathbb{R}} \bar{\sigma}(T(j\omega)) \tag{3.10}$$

where $\bar{\sigma}(\cdot)$ denotes the maximum singular value of the complex matrix $T(j\omega)$ of this closed-loop system.

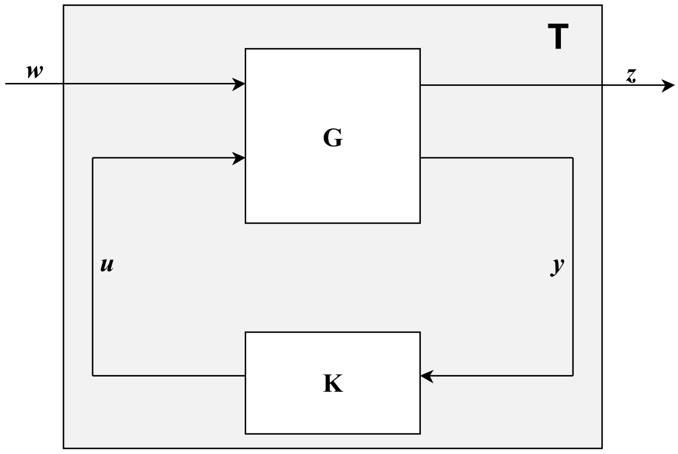

Figure 6: Generalized Feedback Interconnection

In practice, whenever G is constructed it includes an uncertain model of the system that is to be controlled. Uncertainty is introduced because in real life this system is neither linear, nor time-invariant. Moreover, it can never be described in terms of the values of its numerical parameters with arbitrary accuracy. Two approaches may be taken towards modelling the uncertainty. The first, which is rather simplistic, is to suppose that a frequency-dependent norm-bounded perturbation from some nominal model is sufficient to perfectly describe the uncertain model. This is called *nonparametric uncertainty*. Such an uncertain model may be posed either by exploiting some *structure* in the uncertainty associated with the real system or by neglecting such structure. The second, a more realistic one, is to start from the physical process and to model the system while defining the required uncertainty in every parameter appropriately, based on engineering intuition. Because in this case the model will have a fixed structure with uncertainty in the parameters, this is called *parametric uncertainty*. The former method, since it still has to provide realistic results, usually leads to a conservative model, often more conservative than necessary. The latter is thus considered to be more appropriate, especially by practitioners and we will also rely, therefore, on it.

The general robust controller synthesis problem may be described as follows: given a generalized plant G which includes an uncertain model of the real system, find a controller K that ensures stability and performance of the closed-loop system T, for all possible perturbed plants within the uncertainty set. This general definition is, however, of limited use when aiming to solve the problem. As has been previously proposed, when considering interconnected LTI systems, the $\mathcal{H}_\infty$ norm of T is considered to be the appropriate measure of robustness. The synthesis problem may then be customized accordingly, as proposed in [84].

In [27] an analytical solution has been given; it requires solving an associated set of two algebraic Riccati equations and exhibits the entire range of disadvantages related to numerically solving such equations [15]. This first solution is *suboptimal* in the sense that $\|T\|_{\mathcal{H}_\infty}$ is not globally minimized, but brought below a certain level γ. By iteration, an attempt is made at finding a suboptimal controller K that achieves closed-loop stability and a performance level γ, then the problem is scaled and re-iterated upon, until as low a value of γ as possible is achieved or some tolerance level is reached. The solution is convex for some particular classes of controllers (such as, for example, state-feedback controllers) but, as already mentioned, it is suboptimal. One typical practical procedure of translating this solution to the $\mathcal{H}_\infty$ *robust* control problem is the D-K iteration, posed in the μ framework [9]. The μ methodology takes account of both parametric and nonparametric, structured or unstructured uncertainty in the model, therefore it need not be fully conservative, from both the analysis and synthesis point of view. This, however, is non-convex as shown in [85]. This implies that it might not be possible to find a controller that brings the norm below unity (as required by the small-gain theorem [83]) or not even to render it as small as possible; similarly, concluding whether there exists a controller that does bring the norm below unity also cannot be done due to the manner in which the problem is approached.

A more practical and elegant solution, though, is to formulate the problem as a set of matrix inequalities [32, 33] and to find the required controller. This is also a convex problem for some controllers structures (*e.g.* state-feedback) and it is more attractive form a practical point of view because the associated optimization problem is much more easily solved with the currently available computational tools. This formulation, however, is also *suboptimal*.

A strong requirement that has been made so far is that the plant G can be correctly described by an LTI system of the type (3.7). Even if a pragmatic approach has been taken and uncertainty has been considered, in practice some systems may simply not be appropriately described by uncertain LTI models. Such an example is the offshore wind turbine as described in **Chapter 1**. Within the problem formulation it has been mentioned why the system is, in fact, both time-varying and uncertain. Furthermore, in **Chapter 2** more attention has been given to proving these claims. Therefore, a more general class of systems could prove to be more appropriate for this practical problem. This is presented in the next section.

3.3 Control for LPV Systems

One of the most general frameworks available for the description of finite-dimensional continuous-time systems G is given by the set of equations:

$$\begin{cases} \dot{x}_G(t) &= f(x_G(t), u(t), w(t)) \\ y(t) &= g(x_G(t), u(t), w(t)) \\ z(t) &= h(x_G(t), u(t), w(t)) \end{cases} \tag{3.11}$$

where $f : \mathbb{R}^{n+m+o} \to \mathbb{R}^n$, $g : \mathbb{R}^{n+m+o} \to \mathbb{R}^p$ and $f : \mathbb{R}^{n+m+o} \to \mathbb{R}^l$ are smooth nonlinear functions of their variables. This denotes a *state-space nonlinear* system. These, in practice, can often be approximated or reformulated as LPV systems:

$$\begin{cases} \dot{x}_G(t) &= A_G(p(t))x_G(t) + B_{Gu}(p(t))u(t) + B_{Gw}(p(t))w(t) \\ y(t) &= C_{Gy}(p(t))x_G(t) + D_{Guy}(p(t))u(t) + D_{Gwy}(p(t))w(t) \\ z(t) &= C_{Gz}(p(t))x_G(t) + D_{Guz}(p(t))u(t) + D_{Gwz}(p(t))w(t) \end{cases} \tag{3.12}$$

where $p(t) \in \mathbb{R}^{n_p}$ is called the *parameter vector* (or, alternatively, the *scheduling variable*). What can be immediately observed is that systems of the type (3.12) are linear in the mappings $f(\cdot)$, $g(\cdot)$ and $h(\cdot)$ and for each value of the parameter $p(t)$ they are completely defined. The scheduling variable is usually chosen based on knowledge of the real system and because of its time-dependence provides a more flexible framework to model physical systems; for a formal description, knowledge of its possible evolution space needs to be available, therefore we say that it has to lie in some compact set $\mathcal{P} \subset \mathbb{R}^{n_p}$; additionally, its rate of variation is also limited $|\dot{p}_i(t)| < \phi_i$, $i = \overline{1, n_p}$ which corresponds well to what occurs in practice. Let us denote this knowledge of the parameter space as $\mathbb{P}$.

The relationship between LPV systems and LTI systems is easy to see: the latter are simply LPV systems with $\dot{p}_i(t) = 0$, $i = \overline{1, n_p}$ for $\forall t \in \mathbb{R}^+$. In fact, they have been introduced in [68, 69] to properly analyse the guaranteed properties of gain scheduled designs for nonlinear plants approximated by a set of LTI models under the assumption that variation in the model occurs *sufficiently slow*. However, as has been pointed out in [70], the design methodology did not provide sufficient guarantees for closed-loop stability and performance, therefore LPV tools have been developed.

Derivation of LPV Models

There are two fundamental methods of going from the general definition of a smooth nonlinear system (3.11) to the general LPV description (3.12). The *Jacobian-based linearization* [62] relies on first finding all the equilibrium trajectory sets of the system $(x_G^*(t), u^*(t), w^*(t))$ by solving:

$$f(x_G^*(t), u^*(t), w^*(t)) = 0 \tag{3.13}$$

and then denoting $p(t) = \begin{bmatrix} u^*(t) & w^*(t) \end{bmatrix}^T$ only in terms of the equilibrium input/exogeneous variables [62]. The parametrized matrices in (3.12) would then result from computing:

$$\begin{array}{lll} A_G(p(t)) = \frac{\partial f(\cdot)}{\partial x_G(t)}\Big|_{p(t)} & B_{Gu}(p(t)) = \frac{\partial f(\cdot)}{\partial u(t)}\Big|_{p(t)} & B_{Gw}(p(t)) = \frac{\partial f(\cdot)}{\partial w(t)}\Big|_{p(t)} \\ C_{Gy}(p(t)) = \frac{\partial g(\cdot)}{\partial x_G(t)}\Big|_{p(t)} & D_{Guy}(p(t)) = \frac{\partial g(\cdot)}{\partial u(t)}\Big|_{p(t)} & D_{Gwy}(p(t)) = \frac{\partial g(\cdot)}{\partial w(t)}\Big|_{p(t)} \\ C_{Gz}(p(t)) = \frac{\partial h(\cdot)}{\partial x_G(t)}\Big|_{p(t)} & D_{Guz}(p(t)) = \frac{\partial h(\cdot)}{\partial u(t)}\Big|_{p(t)} & D_{Gwz}(p(t)) = \frac{\partial h(\cdot)}{\partial w(t)}\Big|_{p(t)} \end{array} \tag{3.14}$$

and, after first computing the corresponding $y^*(t)$ and $z^*(t)$, which are the associated equilibrium values for the measurement/performance outputs vectors, all signals need to be redefined in terms of deviations away from the equilibrium trajectories:

$$\begin{array}{rcl} x_G(t) & \triangleq & x_G(t) - x^*(t) \\ u(t) & \triangleq & u(t) - u^*(t) \\ w(t) & \triangleq & w(t) - w^*(t) \\ y(t) & \triangleq & y(t) - y^*(t) \\ z(t) & \triangleq & z(t) - z^*(t) \end{array} \tag{3.15}$$

One LPV description of (3.11) will exist for every set of $(x_G^*(t), u^*(t), w^*(t))$. Though very intuitive to work with, Jacobian-based LPV models are not exactly realistic and particularly, accuracy is lost by approximating the nonlinear set (3.12) with a relation of the type (3.12) based on linearizations (3.14) and (3.15) as strongly motivated in [62]. Yet another inconvenience arises from the fact that the scheduling parameter $p(t)$ is only composed of input/exogenous equilibrium trajectories and it is often the case that models of real-life systems also need to be scheduled based on the equilibrium state trajectories (see for example [8]). Such LPV descriptions are called here *quasi-LPV* models [58, 41, 42].

Assume that the state vector can be partitioned as $x(t) = \begin{bmatrix} x_p(t) & \tilde{x}(t) \end{bmatrix}^T$, where $x_p(t)$ denote the state that we wish to be part of the scheduling parameter and $\tilde{x}(t)$ denote the remaining states. By using a suitable choice of the parameter $p(t)$ one can *cover* the nonlinear system (3.11) with (3.12). This parameter will now be composed of both input/exogeneous signals and part of the state vector $x_p(t)$.

LPV Analysis and Synthesis Procedures

Assuming that an overall LPV system T has been formed by interconnecting the plant G given by (3.12) with a full-order controller K parametrized as:

$$\begin{cases} \dot{x}_K(t) &= A_K(p(t))x_K(t) + B_K(p(t))y(t) \\ u(t) &= C_K(p(t))x_K(t) + D_K(p(t))y(t) \end{cases} \tag{3.16}$$

according to Figure 6, the parameter-dependent state-space representation of the closed-loop system is then:

$$\begin{cases} \dot{x}(t) &= A(p(t))x(t) + B(p(t))w(t) \\ z(t) &= C(p(t))x(t) + D(p(t))y(t) \end{cases} \tag{3.17}$$

where the matrices are given by:

$$\begin{aligned} A(\cdot) &= \begin{bmatrix} A_G(\cdot) + B_{Gw}(\cdot)D_K(\cdot)C_{Gz}(\cdot) & B_{Gw}(\cdot)C_K(\cdot) \\ B_K(\cdot)C_{Gz}(\cdot) & A_K(\cdot) \end{bmatrix} \\ B(\cdot) &= \begin{bmatrix} B_{Gu}(\cdot) + B_{Gw}(\cdot)D_K(\cdot)D_{Guz}(\cdot) \\ B_K(\cdot)D_{Guz}(\cdot) \end{bmatrix} \\ C(\cdot) &= \begin{bmatrix} C_{Gy}(\cdot) + D_{Gwy}(\cdot)D_K(\cdot)C_{Gz}(\cdot) & D_{Gwy}(\cdot)C_K(\cdot) \end{bmatrix} \\ D(\cdot) &= \begin{bmatrix} D_{Guy}(\cdot) + D_{Gwy}(\cdot)D_K(\cdot)D_{Guz}(\cdot) \end{bmatrix} \end{aligned} \tag{3.18}$$

Here, the dependence on $p(t)$ has been dropped for brevity. For analysis, we are interested in the stability and the performance of the closed-loop system T. By means of quadratic Lyapunov methods [64], we may directly characterize the system as proposed in [78].

The system (3.17) with possible parameter space $\mathbb{P}$ is parameter-dependent quadratically stable if there exists a continuously differentiable symmetric positive definite matrix $X(\cdot)$ such that:

$$A^T(\cdot)X(\cdot) + X(\cdot)A(\cdot) + \dot{X}(\cdot) \prec 0 \tag{3.19}$$

with the derivative of the Lyapnuov matrix $\dot{X}(\cdot) \triangleq \sum_{i=1}^{n_p} \frac{\partial X(\cdot)}{\partial p_i} p_i$. Note that even though (3.17) bears strong similarities to the given example of Lyapunov stability (3.3), this is actually not an LMI. This is because the inequality depends on the parameter vector p, on one hand, and on the derivative of the Lyapunov matrix $X(\cdot)$, on the other. Notice the similarity between *parameter-dependent quadratic stability* and *exponential stability*.

Suppose that the system (3.17) is stable. Performance may now be measured in terms of the induced $\mathcal{L}_2/\mathcal{L}_2$ norm. Satisfying the dissipation inequality proposed in [75, 76] and measured as mentioned earlier means that a bound $\gamma \in \mathbb{R}^+$ on the norm exists:

$$\|T\|_{\mathcal{L}_2/\mathcal{L}_2} < \gamma \tag{3.20}$$

which, equivalently, implies:

$$\int_0^\infty z^T(\tau)z(\tau)d\tau < \gamma^2 \int_0^\infty w^T(\tau)w(\tau)d\tau \tag{3.21}$$

A condition for both stability and performance in terms of the number γ arrives by extending the *Bounded Real Lemma* for use with LPV systems: the system (3.17) with parameter space $\mathbb{P}$ is parameter-dependent quadratically stable if there exists a continuously differentiable symmetric positive definite matrix $X(\cdot)$ such that:

$$\begin{bmatrix} A^T(\cdot)X(\cdot) + X(\cdot)A(\cdot) + \dot{X}(\cdot) & X(\cdot)B(\cdot) & C^T(\cdot) \\ B^T(\cdot)X(\cdot) & -\gamma I_o & D^T(\cdot) \\ C(\cdot) & D(\cdot) & -\gamma I_l \end{bmatrix} \prec 0 \tag{3.22}$$

which, again, is not an LMI due to the same reasons mentioned earlier. However, the two tools presented for analysis carry through directly for controller synthesis precisely owed to their convenient formulation.

LPV controllers that satisfy (3.22) may be derived by using one of two procedures that have been proposed in the literature. The first, called *quadratic LPV gain scheduling* has been brought forward and developed in [10, 5, 78, 82, 66, 3, 77]. It relies on first writing the corresponding closed-loop system T as in (3.17), by using the unknown controller matrices (3.16) and the generalized plant matrices (3.12), which leads to (3.18). Then, by direct substitution into the relation that defines the objective, the synthesis condition is obtained. However, since this is not an LMI, as previously mentioned, several modifications are necessary to obtain tractable synthesis relations. These are either based on a *change of variables* [66] or on the *matrix projection lemma* [31], used extensively in the context of LMIs but not detailed here for brevity.

The second proposed synthesis technique is called *LFT gain scheduling* [57, 4, 67, 65]. This procedure has aimed to reuse results from LTI robust control theory but it is only applicable to LPV systems in which the parameter dependence can be expressed as structured (diagonal) uncertainty which is interconnected with the generalized plant by a particular type of the mentioned generalized feedback interconnection, namely the LFT. A reformulated version of the small gain theorem is then used to characterize all controllers that satisfy the criteria of the lemma (3.22). In general, though, the former technique is used because it is sufficiently general, provides increased flexibility and is more intuitive due to the clear link to Lyapunov theory and dissipativity of dynamical systems.

State-Feedback versus Output-Feedback

So far, a general framework for the analysis and synthesis of LPV systems has been given. It has been assumed that the generalized plant is given in the form (3.12) and that the associated controller is given by (3.16). These lead to the closed-loop representation (3.17) with parameter-dependent matrices (3.18). This structure is called *output-feedback* control. One of its main advantages is that it is directly applicable to any practical LPV control problem where only the measurement vector $y(t)$ is available for feedback. However, in some cases it may be more convenient to use *state-feedback* control instead. This assumes that the full state vector $x(t)$ is available by some means. State-feedback may be static, in which case the controller becomes:

$$u(t) = D_K(p)x_G(t) \tag{3.23}$$

or dynamic (see, for example [21]), yielding:

$$\begin{cases} \dot{x}_K(t) &= A_K(p(t))x_K(t) + B_K(p(t))x_G(t) \\ u(t) &= C_K(p(t))x_K(t) + D_K(p(t))x_G(t) \end{cases} \tag{3.24}$$

LPV theory is currently considered to be under-developed in terms state-feedback control. Several extensions have been made though (see for example [79, 80, 81]) but the full range of possible developments is far wider.

Practical Remarks

Reduction of Infinite-Dimensionality LPV synthesis procedures typically lead to parametrized matrix inequalities (either linear or not). This implies that in practice several more factors need to be considered before attempting to compute an LPV controller. First, a specific definition (structure) for the parametrization of the generalized plant must be assumed. One possible method is to use an affine description:

$$\begin{bmatrix} A_G(p(t)) & B_{Gu}(p(t)) & B_{Gw}(p(t)) \\ C_{Gy}(p(t)) & D_{Guy}(p(t)) & D_{Gwy}(p(t)) \\ C_{Gz}(p(t)) & D_{Guz}(p(t)) & D_{Gwz}(p(t)) \end{bmatrix} = \begin{bmatrix} A_{G_0} & B_{Gu_0} & B_{Gw_0} \\ C_{Gy_0} & D_{Guy_0} & D_{Gwy_0} \\ C_{Gz_0} & D_{Guz_0} & D_{Gwz_0} \end{bmatrix} + \sum_{i=1}^{n_p} \begin{bmatrix} A_{G_i} & B_{Gu_i} & 0 \\ C_{Gy_i} & D_{Guy_i} & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot p_i \quad (3.25)$$

where it is important to observe that the variables in the parameter vector enter the model linearly and that the matrices $B_{Gw}(\cdot)$, $C_{Gz}(\cdot)$, $D_{Gwy}(\cdot)$, $D_{Guz}(\cdot)$ and $D_{Gwz}(\cdot)$ are actually constant. The latter formulation is very convenient for tractability in the synthesis procedure and, even if not exactly true in practice, appropriate adjustments [4] can be made to allow for it. Assuming that the parameter space is a convex polytope given by the intersection of n_p-dimensional halfspaces now the model is fully defined (recall that the set of parameter variation rates is already given as a convex polytope). Note that this definition is, in practice, more conservative which is not always appropriate [35]. The controller matrices can, however, now be computed. Observe that this may be completely carried out offline.

The polytopic description of the parameter space, as has already been explained, may sometimes be too conservative. Therefore it might be more appropriate to rely on other methods, such as gridding the parameter space. A set of controller matrices is then computed for point in the given grid. The density of the grid and, thus, the accuracy of the controller arrived at may then be adjusted according to the needs.

Scheduling Once a specific definition in terms of the parameter vector is also given for the controller, LPV gain-scheduling can be appropriately performed by only requiring the parameter vector to be online-available to compute the control signal. Its current value is fully determined by the current value of the parameter vector.

From a practical point of view, scheduling cannot be performed unless the parameter $p(t)$ is fully determined (either measured or estimated). Therefore one fundamental condition needs to hold true for quasi-LPV descriptions: it is required that the states $x_p(t)$ that are part of the parameter vector are either fully measured and, therefore, they would also be part of the output $y(t)$ or can be reconstructed by means of an observer. Considering that dynamics that govern the evolution of $y(t)$ are nonlinear as expressed in (3.11), this condition may, in practice, be very difficult to satisfy. More details about addressing such issues may be found in [62].

3.4 Conclusions

As has been mentioned previously in the report, the practical problem that we have formulated is to be addressed by using the tools described in this chapter. First, linear matrix inequalities have been briefly introduced and it has been motivated why they have proven to be very applicable for control engineering purposes. Secondly, an outline of possible approaches for robust controller design has been given. Limitations of the existing procedures have been mentioned from the point of view of our goals and these have been linked to practical considerations regarding assumptions of time-invariance which will not work appropriately for our task. Subsequently, the need for gain-scheduling has been discussed. Linear parameter-varying control has been portrayed and existing synthesis procedures have been listed. Additionally, remarks regarding the computation of the controller matrices have been made together with an evaluation of whether the methodology is conservative.

Linear parameter-varying control is appropriate for our needs and several excellent efforts have already been made towards applying it for wind turbine control, as will be explained in the next chapter. However, a unified theoretical framework for the synthesis of robust LPV controllers would have been preferred and is not yet available. More specifically, scheduling and robustness in controller design would have been useful but are not usually considered simultaneously. Nonetheless, there are a few extensions towards this [73, 16, 61] but they are yet to be applicable due to partial completeness. ■

4

Robust and LPV Wind Turbine Control

As has been previously mentioned in **Chapter 1**, modern wind turbines bring a handful of challenges for control system designers. Power production is highly dependent not only upon wind availability, but also on the aerodynamic characteristics of the turbine. In control-oriented models, these are typically given in terms of power and torque coefficients and, as described in **Chapter 2**, production can be enhanced below rated wind speeds if of these two, C_P is maximized. As explained, this can be actively done by means of a wind turbine controller. On the other hand, above rated wind speeds the turbines need to be controlled such that power production is limited to values maximally allowed by the electrical system. Furthermore, wind turbines tend to be lightly-damped structures. Since safe operation needs to be ensured, control systems need to be designed such that they do not excite the structural modes of the turbine but, on the contrary, damp them, if possible. Even more so, blade-effective mechanical loads need to be reduced, not only to prevent hazards but also for decreasing associated maintenance costs and downtime while increasing the turbine lifespan. Wind turbine control can, thus, be described as *multiobjective*. Typically, each objective is addressed in modern wind turbines as schematically depicted in Figure 7. Based on the proposed theoretical foundations laid in **Chapter 3**, here we shall briefly present relevant approaches taken towards the design of advanced control systems for wind turbines and, in doing so, we shall mainly focus on the controller structures shown in Figure 7. The material is informative and necessary in showing why a new approach is preferred for the goals stated in **Chapter 1** and subsequently formulated in **Chapter 5**.

4.1 Robust Wind Turbine Control

The laws that govern wind turbine operation and power extraction are well understood from a theoretical perspective [23] and there are few physical effects that have not been fully described yet (*e.g.* aerodynamic stall). Control system designers, however, require accurate models and, in practice, these can be rarely supplied, either because the physical parameters cannot be precisely determined or because they are not fixed. Under the assumption that wind turbines are systems whose parameters are constant but can never be completely modelled, several approaches have been taken towards controller design.

As has been presented in the previous chapter, several methods exist for designing controllers for LTI systems by using the $\mathcal{H}_\infty$ norm as a way to measure robustness of stability and performance of the closed-loop system. In [24, 19] the theory has been applied for the first time for wind turbine control, however with the sole purpose of fatigue damage reduction. In [17, 14, 50, 49] the same tools are applied for regulating the power output of a turbine under full load

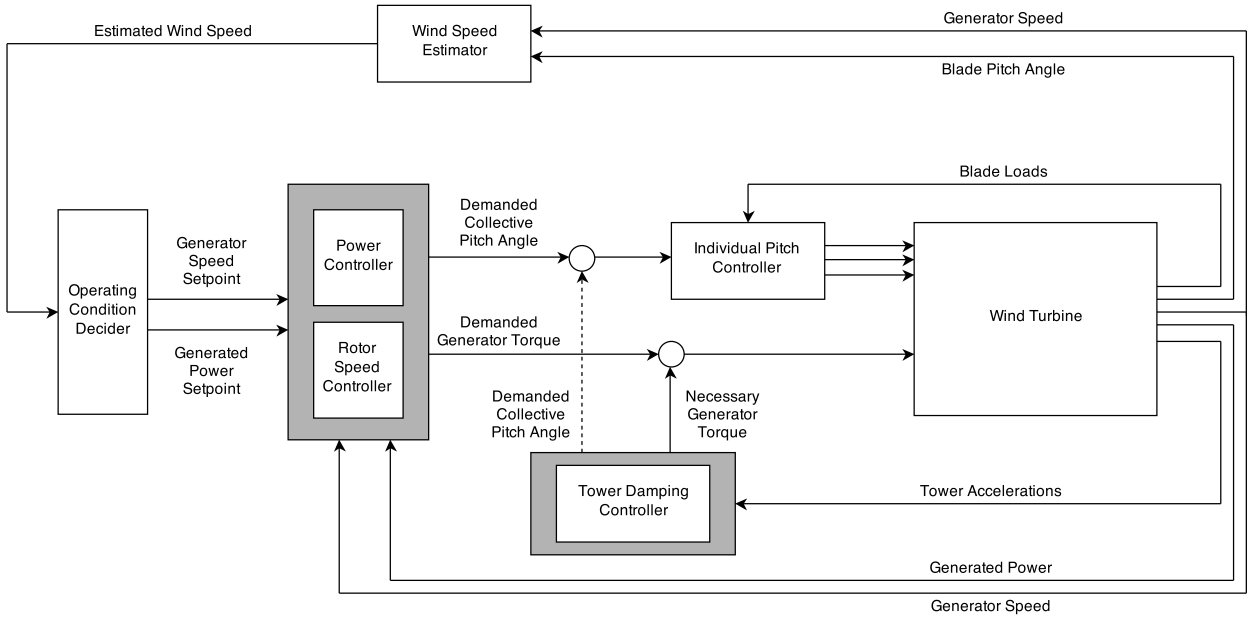

Figure 7: Wind Turbine Control System Structure (based on [53])

conditions. While [17, 14] use simplified models and rely of unstructured uncertainty description, [50] and [49] rely on a more realistic scenario and attempt to model uncertainty in the drivetrain stiffness and drivetrain damping as structured parametric uncertainty; either way, all four works make clear statements regarding the limitation of the methodology, either from the point of view of non-convexity in the controller synthesis problem [50, 49] or from the additional perspective of practically unreasonable model derivation due to the fact that it is not known which physical parameters represent the source of uncertainty [17, 14]. Nonetheless, an extension of [17] is given in [18] when the theory is also applied for below-rated wind speed conditions. Furthermore, an attempt is made in [71] to address the entire operational envelope where a full analysis of the results is given and the same remarks are made about the limitations of the approach. In [59] multivariable robust control techniques are used to address the entire operational envelope, however with severe simplifications (*i.e.* the uncertain linear time-invariant model proposed results from linearising a general wind turbine model around the optimal tip-speed-ratio) and only to conclude that such a design is extremely conservative due to the fact that the uncertainty introduced by such a linearization is considerably high. Refinements are made in [60] where a comparison is shown between $\mathcal{H}_2$ and $\mathcal{H}_\infty$ methods. The latter is indeed concluded to provide more guarantees for reliable designs but at the price of possibly unnecessary pragmatism which, yet again, relates to how the model is proposed to be uncertain. It then becomes gradually clear that unstructured uncertainty is not a suitable option for constructing such wind turbine models for control.

A practical comparison between wind turbine controllers designed using classical loopshaping techniques and using modern robust control theory is given in [38] and further in [30, 29] where designed controllers are tested on experimental turbines. Closed-loop system performance is mainly assessed and it is concluded that indeed safe wind turbine operation can be guaranteed by using the latter procedure, however, with a loss of optimality in the case that the uncertainty in the model is not matched on the real turbine (*i.e.* the uncertainty set is unrealistically large). To summarize all these mentioned works it may be stated that tractability of the synthesis problem is difficult to maintain within the presented methodology because of unnecessary conservativeness - it may be better to have a controller that adjusts its behaviour based on some online-measured parameter rather than a single controller that performs well for all possible values of such a parameter. On the other hand, building an uncertain model to correctly describe the wind turbine can be challenging and severe simplifications need to be made in this linear time-invariant systems framework, which is inconvenient and very non-realistic.

4.2 Gain-Scheduled Control of Wind Turbines

Though the overall behaviour of wind turbines is generally-speaking nonlinear, a more practical way of describing them is as parameter-dependent systems. As an example of the usefulness of such a viewpoint, aerodynamic torque and thrust forces are intrinsically nonlinear functions of the corresponding aerodynamic coefficients and of the relative wind speed, as described previously in the report. Setting up a control problem based on a single LTI model has previously shown to be insufficient, even if uncertainty is accounted for in the general framework of linear robust control. Therefore a controller design approach that relies on a nonlinear model of the plant is considered to be more appropriate, as shown next.

Traditional gain-scheduled designs, as described in the previous chapter, have been attempted for wind turbine control. In [44, 39] the first steps had been taken in this direction with a focus on power production maximization at below-rated wind speeds while allowing for some reduction in fatigue loads. It is later established in [43, 40, 25] that at above-rated wind speeds gain-scheduled controllers also provide a more optimal closed-loop behaviour. However, all these approaches look at rather academical models of wind turbines and, more importantly, only at fixed-speed variable-pitch wind turbines. On the other hand in [72] a proposal is made as to how gain-scheduling could also be performed for variable-speed variable-pitch wind turbines. Attempts have also been made at applying the same methodology for fatigue load reduction in [34]. As also expressed previously in the report, a severe limitation of this approach is that it cannot deal with fast changes in the model of the wind turbine. An alternative approach which does actually do so would be considered more appropriate.

Linear parameter-varying control has first been proposed in the field of wind turbine control in [13]. The paper mainly raises awareness of the suitability of such an approach and does not look into details regarding practical design. In [52] LPV control is applied to wind turbines for rotor speed control in below-rated wind speeds. LPV designs for variable-speed variable-pitch wind turbine load reduction in above-rated wind speeds are presented in [46, 47, 45, 26]. A proposal for a unified approach to address all control objectives over the entire operational envelope is given in [12] and it is proposed that controller scheduling is performed based on wind speed, collective blade pitch angle and generator speed, a fact which corresponds reasonably well with what is expected in practice, as stated in **Chapter 2**. One interesting remark brought forward in [12] is that unified LPV wind turbine control may be carried out for the entire operational envelope by also scheduling the associated weighting functions. This helps in addressing separately the various objectives of each operating region and all works that propose designs for both partial load and full load conditions rely on some implementation of this suggestion.

It may be considered that the most significant contribution has arrived through the work presented in [53]. There, LPV control is applied to variable-speed variable-pitch wind turbines for the entire operational envelope. A gain-scheduled state-feedback controller is first proposed in [54] for full load conditions. Later in [56] a similar approach is used for both partial load and full load conditions and conclusions are drawn from the perspective of fatigue loads reductions. In [55] a less conservative design is proposed for above-rated wind speed power regulation. One of the severe drawbacks of the proposed LPV designs is regarding controller structure. Though more convenient in [54] by means of state feedback, LPV wind turbine control formulated in the traditional output-feedback sense has been found to be problematic due to the the fact that the resulting controllers usually have the same order as the generalized plant that has been set up. In [1] it is investigated how fixed-structure control synthesis may be formulated and eventually applied to wind turbines. Despite the lack of convexity in the formulated control problem, some results are shown in [2] only for full load conditions.

4.3 Conclusions

Though robustness towards uncertainty is a required characteristic of wind turbine controllers, it has been mentioned in this chapter how and why previously-developed methods still need further refinements. A more realistic point of view has been taken and applications of the more advanced gain-scheduling control theory to our field of interest have been shown. Classical methods have first been presented and, as a more reliable alternative, LPV wind turbine control has subsequently been proposed.

The efforts presented in the framework of LPV control rely on scheduling based on the wind speed conditions or blade pitch angle and generator torque as well. However, currently developed wind turbines (such as those placed offshore, for example) would benefit from a more realistic approach towards controller scheduling, therefore all previous proposals are considered insufficient. Moreover, it is considered extremely important from a practical point of view to keep the controller as simple as possible, therefore state feedback, if possible, is preferred. In the light of these facts, in the next chapter we formally state a new theoretical control problem and show how it relates to these mentioned wind turbine-related control challenges. ■

5

Formulation of the Control Problem

In this chapter special attention is given to the formulation of a general LQR control problem for wind turbine control. In **Chapter 1** the general aims of the **D4REL** project have been introduced and a practical control problem has been stated. **Chapter 2** has shown how a wind turbine may be looked at from a dynamical systems point of view and it has further reinforced the motivation for approaching the modelling from the perspective of uncertain time-varying systems. The LQR problem that is stated here is given in the theoretical framework set up in **Chapter 3** but all notions of practicality follow directly from **Chapter 1** and **Chapter 2**. The final two sections give special attention to the considered sub-problems while the first provides a unified description of the overall control design task.

5.1 Mathematical Control Problem Formulation

Given two practical issues (*i.e.* **Rotor Speed Control** and **Tower Damping**, detailed below) and a generalized parameter vector:

$$p^*(t) = \begin{bmatrix} \omega_0^*(t) \\ u_{col}^*(t) \\ \theta_{col}^*(t) \\ \Omega_r^*(t) \end{bmatrix}$$

with the previously-introduced notations and $\omega_0(t)$ denoting the first fore-aft structural tower mode (which provides a unified method of describing variations in $m_t(t)$, $s_t(t)$ and $d_t(t)$) we desire to compute robust LPV state-feedback control laws:

$$u_i(t) = F_i(p^*(t)) \cdot x_i(t)$$

that optimize the cost function minimization problem:

$$J_i^{opt} = \min_{u_i(t)} \sup_{\Delta \in \mathcal{D}} \int_0^\infty (x_i^T(t) Q_i(p^*(t)) x_i(t) + u_i^T(t) R_i(p^*(t)) u_i(t)) dt$$

in the general framework:

$$\dot{x}_i(t) = A_i(p^*(t), \Delta)x_i(t) + B_i(p^*(t), \Delta)u_i(t) + E_i(p^*(t), \Delta)w_i(t).$$

with $i = 1, 2$, depending on the considered problem (**Rotor Speed Control**: $i = 1$ and **Tower Damping**: $i = 2$). It has been chosen here to use a generalized $\mathcal{H}_2$ objective function because state-feedback laws are directly related to such functions, on one hand and performance specifications may be easily given in terms of the system state and/or input, on the other; nonetheless, the previously-presented framework of **Chapter 3** may be easily extended towards this, as has been already mentioned.

5.2 LPV Framework for Rotor Speed Control

Rotor speed control allows the achievement of power regulation along the operational curve of a wind turbine. In partial loads (below rated wind speeds) it is necessary to maximize power capture, therefore the turbine blades are fixed to some pitch angle value and the generator torque $\delta T_{gen}(t)$ is modified accordingly using the electronic converters. In full load (above-rated wind speed) the power capture needs to be limited, therefore the turbine blades will need to be pitched-to-feather by using the collective pitch angle $\delta\theta_{col}(t)$. Rotor speed control is thus considered over the entire operational envelope (note that around rated wind speed a conflict between the two methodologies might occur which requires careful design of the transition between the two). Here operating trajectories $p^*(t)$ are considered; these are equilibrium values for the parameter $p(t)$ as described in **Chapter 3** and given in terms of the physical variables in the section **Mathematical Control Problem Formulation**. As a result of the linearisation of the aerodynamic torque $T_a(p(t))$ described in **Chapter 2** around this operating trajectory by Taylor series expansion we may express the deviations away from the equilibrium as:

$$\underbrace{\begin{bmatrix} \delta\dot{\Omega}_r(t) \\ \delta\dot{\Omega}_r^{int}(t) \end{bmatrix}}_{\dot{x}_1(t)} = A_1(p^*(t)) \cdot \underbrace{\begin{bmatrix} \delta\Omega_r(t) \\ \delta\Omega_r^{int}(t) \end{bmatrix}}_{x_1(t)} + B_1(p^*(t)) \cdot \underbrace{\begin{bmatrix} \delta\theta_{col}(t) \\ \delta T_{gen}(t) \end{bmatrix}}_{u_1(t)} + E_1(p^*(t)) \cdot \underbrace{(\delta u_{col}(t) - \dot{x}_{fa}(t))}_{w_1(t)} \tag{5.1}$$

with parameter-dependent state-space matrices:

$$\begin{aligned} A_1(p^*(t)) &= \begin{bmatrix} \frac{(1-T_t)B\nabla_\Omega T_a(p^*(t))}{J_{eff}} & 0 \\ 1 & 0 \end{bmatrix} \\ B_1(p^*(t)) &= \begin{bmatrix} \frac{(1-T_t)B\nabla_\theta T_a(p^*(t))}{J_{eff}} & -\frac{|i_{tr}|}{J_{eff}} \\ 0 & 0 \end{bmatrix} \\ E_1(p^*(t)) &= \begin{bmatrix} \frac{(1-T_t)B\nabla_u T_a(p^*(t))}{J_{eff}} \\ 0 \end{bmatrix} \end{aligned} \tag{5.2}$$

expressed in terms of the partial derivatives:

$$\begin{aligned} \nabla_u T_a(p^*(t)) &= \tfrac{1}{2B}\rho\pi R^3 u^*_{col}(t) \left(2C_Q(\lambda(t), \theta_{col}(t)) - \lambda(t)\frac{\partial C_Q(p(t))}{\partial\lambda(t)}\right)\Bigg|_{p(t)=p^*(t)} \\ \nabla_\theta T_a(p^*(t)) &= \tfrac{1}{2B}\rho\pi R^3 {u^*_{col}}^2(t)\frac{\partial C_Q(p(t))}{\partial\theta_{col}(t)}\Bigg|_{p(t)=p^*(t)} \\ \nabla_\Omega T_a(p^*(t)) &= \tfrac{1}{2B}\rho\pi R^4 u^*_{col}(t)\frac{\partial C_Q(p(t))}{\partial\lambda(t)}\Bigg|_{p(t)=p^*(t)} \end{aligned} \tag{5.3}$$

through which it can be observed that such a formulation describes a *quasi-LPV* model due to the fact that the scheduling variable $p^*(t)$ contains also states of the system (see section **Control Problem Formulation**). As mentioned in **Chapter 1** and **Chapter 2**, the aerodynamic torque coefficient C_Q is considered to be *statically* uncertain and will be described as part of a defined set $\mathcal{D}$.

5.3 LPV Framework for Tower Damping

In this setup it is necessary to dampen the tower oscillations by active control, using the collective pitch angle $\delta\theta_{col}(t)$. It is necessary to achieve this over the entire operational envelope (*i.e.* on both partial and full load). Note here that $m_t(t)$ and $s_t(t)$ denote the tower mass and tower stiffness, respectively, which are considered to be time-varying due to environmental conditions as expressed earlier in the report. Here operating trajectories $p^*(t)$ are considered; these are equilibrium values for the parameter $p(t)$ as described in **Chapter 3** and given in terms of the physical variables in the section **Mathematical Control Problem Formulation**. As a result of the linearisation of the rotor thrust force $F_{ax}(p(t))$ described in **Chapter 2** around this operating trajectory by Taylor series expansion we may express the deviations away from the equilibrium as:

$$\underbrace{\begin{bmatrix}\delta\dot{x}_{fa}(t)\\ \delta\ddot{x}_{fa}(t)\end{bmatrix}}_{\dot{x}_2(t)} = A_2(p^*(t))\cdot\underbrace{\begin{bmatrix}\delta x_{fa}(t)\\ \delta\dot{x}_{fa}(t)\end{bmatrix}}_{x_2(t)} + B_2(p^*(t))\cdot\underbrace{\delta\theta_{col}(t)}_{u_2(t)} + E_2(p^*(t))\cdot\underbrace{\delta u_{col}(t)}_{w_2(t)} \tag{5.4}$$

with parameter-dependent state-space matrices:

$$\begin{aligned}A_2(p^*(t)) &= \begin{bmatrix}0 & 1\\ -\frac{s_t^*(t)}{m_t^*(t)} & -\frac{d_t^*(t)+B\nabla_u F_{ax}(p^*(t))(1+\frac{R^2}{H^2})}{m_t^*(t)}\end{bmatrix}\\ B_2(p^*(t)) &= \begin{bmatrix}0\\ \frac{B\nabla_\theta F_{ax}(p^*(t))}{m_t^*(t)}\end{bmatrix}\\ E_2(p^*(t)) &= \begin{bmatrix}0\\ \frac{B\nabla_u F_{ax}(p^*(t))}{m_t^*(t)}\end{bmatrix}\end{aligned} \tag{5.5}$$

expressed in terms of the partial derivatives:

$$\begin{aligned}\nabla_\Omega F_{ax}(p^*(t)) &= \frac{1}{2B}\rho\pi R^3 u_{col}^*(t)\frac{\partial C_T(p(t))}{\partial\lambda(t)}\bigg|_{p(t)=p^*(t)}\\ \nabla_\theta F_{ax}(p^*(t)) &= \frac{1}{2B}\rho\pi R^2 u_{col}^*(t)\frac{\partial C_T(p(t))}{\partial\theta_{col}(t)}\bigg|_{p(t)=p^*(t)}\\ \nabla_u F_{ax}(p^*(t)) &= \frac{1}{2B}\rho\pi R^2 u_{col}^*(t)\left(2C_T(\lambda(t),\theta_{col}(t)) - \lambda(t)\frac{\partial C_T(p(t))}{\partial\lambda(t)}\right)\bigg|_{p(t)=p^*(t)}\end{aligned} \tag{5.6}$$

through which it can be observed that such a formulation describes a *quasi-LPV* model due to the fact that the scheduling variable $p^*(t)$ contains also states of the system (see section **Mathematical Control Problem Formulation**). As mentioned in **Chapter 1** and **Chapter 2**, the aerodynamic thrust coefficient C_T is considered to be *statically* uncertain and will be described as part of a defined set $\mathcal{D}$. ■

6

Conclusions and Further Developments

In **Chapter 5** a new control problem has been formulated. This has taken into account the objectives of **D4REL** as given in **Chapter 1** as well as information presented in **Chapter 2** regarding wind turbine modelling for control; the control problem has been developed such that it can fit within the framework described in **Chapter 3**. Though similar to other works discussed in **Chapter 4**, our problem can only be partially addressed by using the current theoretical tools. Moreover, the available control system design tool of **ECN** also requires several modifications for accommodating these practical matters.

Several steps need to be taken for solving the problem in an appropriate manner. These are listed below and, as such as shown next, addressing them will require us to follow a specific thread. This may be given as such:

Formulation of an Uncertain LPV Model

In order to be in accordance with the proposed theory, the practical aspects regarding wind turbine modelling given in **Chapter 2** and presented in block diagram form in Figure 5 need to be well integrated in an uncertain LPV model. To this end, we propose that a *quasi-LPV model* for a more accurate description. Nonetheless, simplifications may be necessary therefore the discrepancy between this model and the realistic one started from needs to be investigated.

Robust Controller Synthesis

A conservative controller that ensures robustness of closed-loop stability and optimal performance characteristics considering the mentioned sources of uncertainty needs to be designed.

- **Determination of Uncertainty Set**: the uncertainty profile first needs to be defined as a set;
- **Analysis of Uncertainty Effect and Formulation of Control Objectives**: given the uncertain set, several simulations need to be performed to interpret how turbine behaviour changes within it; relevant physical variables (either measured or estimated) will be taken into account when control objectives are derived;
- **Formulation of Necessary LMIs**: in order to fit our state-feedback robust control problem within the framework of convex optimization, a linear matrix inequality approach will be used;
- **Controller Computation and Closed-Loop Simulations**: the controller will here be computed using the implemented tools for solving semidefinite programs; closed-loop simulations need to be performed and the results will be analysed and interpreted.

LPV Controller Synthesis

A robust gain-scheduled controller that guarantees closed-loop stability and performance considering the mentioned variations in wind turbine behaviour as a function of the operating condition needs to be designed.

- **Determination of Parameter Set**: for the proposed parameter vector, the corresponding set defined by the possible parameter values and associated rates of variation needs to be defined;
- **Formulation of Cost Function and Control Objectives**: given the parameter set, several simulations need to be performed to interpret how turbine behaviour changes within it; relevant physical variables (either measured or estimated) will be taken into account when control objectives are derived;
- **Formulation of Necessary LMIs**: in order to fit our robust state-feedback LPV control problem within the framework of convex optimization, a linear matrix inequality approach will be used;
- **Controller Computation and Closed-Loop Simulations**: the controller will here be computed using the implemented tools for solving semidefinite programs; several more practical issues regarding controller implementation will also be discussed; closed-loop simulations need to be performed and the results will be analysed and interpreted.

Analysis of Results

The performance achieved with the new control structure will be investigated, both with respect to theoretical indicators (*e.g.* performance levels) and, most importantly, in terms of physical variables. A contrast is offered between the results with the proposed approach and those due to the one implemented within the control system design tool of **ECN**. Relevant conclusions are drawn following the comparative study.

A detailed report of the entire work will be provided. This will include both aspects mentioned thus far in the literature review and design results in the exact sequential manner as proposed previously. Conclusions and possible extensions will be offered. ■

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ECN

Westerduinweg 3
1755 LE Petten
The Netherlands

P.O. Box 1
1755 ZG Petten
The Netherlands

T: +31 88 5154949
info@ecn.nl
www.ecn.nl