STEP RESPONSE AND FREQUENCY RESPONSE OF AN AIR CONDITIONING SYSTEM

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ABSTRACT

A system of induction units of an existing air conditioning system has been analyzed with respect to its dynamic properties. Time constants were calcalucalculated and measured by analogue models. Comparison with measurements at the installation itself showed a reasonable agreement. Frequency responses were studied in the same way. Lumping of the thermal capacity of the water eliminates the frequency peaks predicted by calculations. The frequency responses at high frequencies in the analogue were charaterized by system of order 0, 1, 2 or 3 which is explained by the calculations if the frequency peaks are neglected.

NOMENCLATURE

- A = Area Surface
- C = Thermal capacity, J/K
- c = Specific heat, J/kgK
- L = Length of an iduction unit or pipe m
- s = Subsidiary domain of t, sec^{-1}
- T = Temperature, K
- t = Time, sec.
- V = Velocity, m/sec
- x = Coordinate in the direction of flow, m
- α = Heat transfer coefficient, W/m^2 K
- θ = Amplitude of the temperature at sinusoidal input disturbance, K
- ϑ = Laplace transformed temperature
- τ = Time constant of the system or a part of the system, sec.
- τ_V = Running time through a part of the system, sec
- $\emptyset_m$ = Mass flow rate, kg/sec
- ω = circular frequency, rad/sec

SUBSCRIPTS

- a = air
- m = metal of the induction units
- mp = metal of the piping system behind the induction units
- mr = metal of the return piping system before the induction units
- p = water in the piping system behind the induction units
- r = water in the return piping system before the induction units
- w = water in the induction units

INTRODUCTION

When considering the system of the piping with the induction units of an air conditioning system the scheme of figure 1 may be useful.
In this scheme it has been assumed that the piping system is perfectly isolated. Heat transfer between the water and the air is only possible at the induction units. In the following it will further be assumed that the thermal resistances in the metal of the induction units are negligibly small [1] but that no heat conduction occurs in the piping system in the direction of flow.
The equations governing this system are:

water in the induction units:

$$\frac{\partial T_w}{\partial t} + V_w \frac{\partial T_w}{\partial x} + \left(\frac{\alpha A}{\emptyset_m c}\right)_w \frac{V_w}{L_w} T_w = \left(\frac{\alpha A}{\emptyset_m c}\right)_w \frac{V_w}{L_w} T_m \qquad (1)$$

air induced:

$$\frac{\partial T_a}{\partial t} + V_a \frac{\partial T_a}{\partial x} + \left(\frac{\alpha A}{\emptyset_m c}\right)_a \frac{V_a}{L_a} T_a = \left(\frac{\alpha A}{\emptyset_m c}\right)_a \frac{V_a}{L_a} T_m \qquad (2)$$

Paper presented at the Sixth International Heat Transfer Conference, Toronto, Canada, August 7-11, 1978.
Energy Utilization (EU-4)

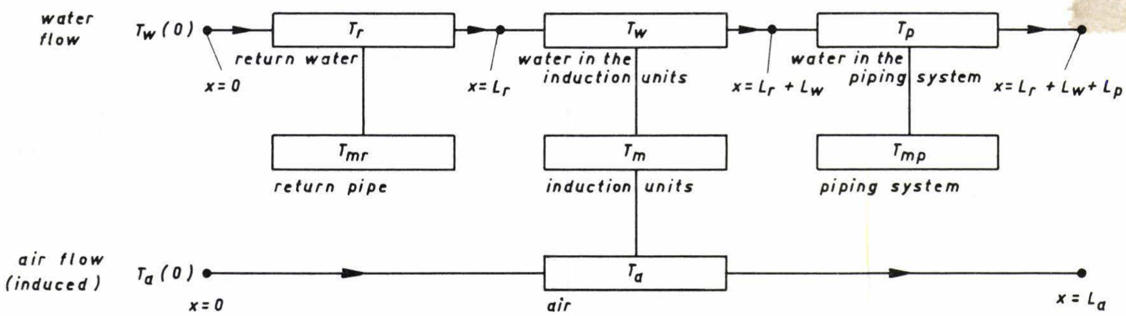

FIG. 1 SCHEME OF THE HEAT TRANSFER PROCESSES IN THE SYSTEM

metal of the induction units:

$$C_m \frac{dT_m}{dt} = \left(\frac{\alpha A}{L}\right)_w \int_{L_r}^{L_r+L_w} (T_w - T_m)\,dx + \left(\frac{\alpha A}{L}\right)_a \int_0^{L_a} (T_a - T_m)\,dx \quad (3)$$

water in the piping system

$$\frac{\partial T_p}{\partial t} + V_p \frac{\partial T_p}{\partial x} + \left(\frac{\alpha A}{\emptyset_m c}\right)_p \frac{V_p}{L_p} T_p = \left(\frac{\alpha A}{\emptyset_m c}\right)_p \frac{V_p}{L_p} T_{mp} \quad (4)$$

return water

$$\frac{\partial T_r}{\partial t} + V_r \frac{\partial T_r}{\partial x} + \left(\frac{\alpha A}{\emptyset_m c}\right)_r \frac{V_r}{L_r} T_r = \left(\frac{\alpha A}{\emptyset_m c}\right)_r \frac{V_r}{L_r} T_{mr} \quad (5)$$

piping system

$$\frac{\partial T_{mp}}{\partial t} + \frac{(\alpha A)_p}{C_{mp}} T_{mp} = \frac{(\alpha A)_p}{C_{mp}} T_p \quad (6)$$

return piping system

$$\frac{\partial T_{mr}}{\partial t} + \frac{(\alpha A)_r}{C_{mr}} T_{mr} = \frac{(\alpha A)_r}{C_{mr}} T_r \quad (7)$$

For steady state conditions the above equations can easily by solved. The solutions are:

$$T_r = T_{mr} = T_w(0) \quad (8)$$

$$T_m = \frac{N_1 T_w(0) + N_2 T_a(0)}{N_1 + N_2} \quad (9)$$

$$T_w = T_m + \frac{T_w(0) - T_a(0)}{N_1 + N_2} N_2 e^{-\left(\frac{\alpha A}{\emptyset_m c}\right)_w \frac{x - L_r}{L_w}} \quad (10)$$

$$T_a = T_m + \frac{T_a(0) - T_w(0)}{N_1 + N_2} N_1 e^{-\left(\frac{\alpha A}{\emptyset_m c}\right)_a \cdot \frac{x}{L_a}} \quad (11)$$

$$T_p = T_{mp} = T_m + \frac{T_w(0) - T_a(0)}{N_1 + N_2} N_2 e^{-\left(\frac{\alpha A}{\emptyset_m c}\right)_w} \quad (12)$$

$$\left.\begin{aligned} N_1 &= (\emptyset_m c)_w \left(1 - e^{-\left(\frac{A}{\emptyset_m c}\right)_w}\right), \\ N_2 &= (\emptyset_m c)_a \left(1 - e^{-\left(\frac{\alpha A}{\emptyset_m c}\right)_a}\right) \end{aligned}\right\} \quad (13)$$

2. <u>STEP INPUT DISTURBANCE</u>

If the supply of hot water is stopped and the water flow becomes a closed circuit all temperatures will finally approach to the air temperature $T_a(0)$. The temperature can then be written as:

$T_w(x, t) = T_{w1}(x, t) + T_w(0)$, etc.

The first terms on the right hand side will approach to zero and satisfy the equations (1)-(7). It will be convenient to introduce into the calculations the running times of the water and air and the time constants of the blocks in fig. 1

$$\left.\begin{aligned} &\tau_w = \left(\frac{C}{\alpha A}\right)_w, \quad \tau_a = \left(\frac{C}{\alpha A}\right)_a, \quad \tau_{vw} = \frac{L_w}{V_w}, \quad \tau_{va} = \frac{L_a}{V_a} \\ &\tau_p = \left(\frac{C}{\alpha A}\right)_p, \quad \tau_r = \left(\frac{C}{\alpha A}\right)_r, \quad \tau_{vp} = \frac{L_p}{V_p}, \quad \tau_{vr} = \frac{L_r}{V_r} \\ &\tau_{mp} = \frac{C_{mp}}{(\alpha A)_p}, \quad \tau_{mr} = \frac{C_{mr}}{(\alpha A)_r} \end{aligned}\right\} \quad (14)$$

If ϑ denotes for the Laplace-transformed temperature then the Laplace transformation of (6) and (7) gives for ϑ_{mp1} and ϑ_{mr1}:

$$\left.\begin{aligned} \vartheta_{mp1} = \frac{\vartheta_{p1}}{S\tau_{mp} + 1} + (T_w(0) - T_a(0)) \cdot \\ \frac{\tau_{mp}}{S\tau_{mp} + 1} \frac{N_1 + N_2 e^{-\frac{\tau_{vw}}{\tau_w}}}{N_1 + N_2} \end{aligned}\right\} \quad (15)$$

$$\vartheta_{mr1} = \frac{\vartheta_{r1}}{s\tau_{mr} + 1} + (T_w(0) - T_a(0))\frac{\tau_{mr}}{s\tau_{mr} + 1} \qquad (16)$$

Laplace transformation of (5) with the condition $(T_{mr1})_{t=0} = (T_{r1})_{t=0}$ gives:

$$\vartheta_{r1} = \frac{T_w(0) - T_a(0)}{s} + B_1 e^{-b_1 \frac{x}{L_r}} \qquad (17)$$

$$\text{with } b_1 = \frac{s\tau_{vr}\left(\tau_r + \frac{\tau_{mr}}{s\tau_{mr} + 1}\right)}{\tau_r}$$

The value for the constant B_1 can be found when all the equations for the water flow have been solved.

The initial value of T_{w1} is

$$(T_{w1})_{t=0} = \frac{T_w(0) - T_a(0)}{N_1 + N_2}\left(N_1 + N_2 e^{-\frac{\tau_{vw}}{\tau_w}\frac{x-L_r}{L_w}}\right)$$

Laplace transformation of (1) then gives, if $\vartheta_{w1} = \vartheta_{r1}$ at $x = L_r$

$$\vartheta_{w1} = \frac{\vartheta_{m1}}{s\tau_w + 1}\left(1 - e^{-b_2\frac{x-L_r}{L_w}}\right) + B_1 e^{-b_1 - b_2\frac{x-L_r}{L_w}} + (T_w(0) - T_a(0)).$$
$$\left\{\frac{1}{s} e^{-b_2\frac{x-L_r}{L_w}} + \frac{N_1\tau_w}{(N_1 + N_2)(s\tau_w + 1)}\left(1 - e^{-b_2\frac{x-L_r}{L_w}}\right) + \frac{N_2 e^{-\frac{\tau_{vw}}{\tau_w}\frac{x-L_r}{L_w}}}{s(N_1 + N_2)}.\left(1 - e^{-s\tau_{vw}\frac{x-L_r}{L_w}}\right)\right\} \qquad (18)$$

$$\text{with } b_2 = \frac{\tau_{vw}(s\tau_w + 1)}{\tau_w}$$

Laplace transformation of equation (4) with theboundary condition $\vartheta_{p1} = \vartheta_{w1}$ at $x = L_r + L_w$ gives

$$\vartheta_{p1} = \frac{\vartheta_{m1}}{s\tau_w + 1}\left(1 - e^{-b_2}\right)e^{-\frac{b_3}{L_p}(x - L_r - L_w)} + B_1 e^{-\left(b_1 + b_2 + \frac{b_3}{L_p}(x - L_r - L_w)\right)} +$$
$$(T_w(0) - T_a(0))\left\{\frac{N_1 + N_2 e^{-\frac{\tau_{vw}}{\tau_w}}}{s(N_1 + N_2)}\left(1 - e^{-\frac{b_3}{L_p}(x - L_r - L_w)}\right) + \frac{1}{s}e^{-\left(b_2 + \frac{b_3}{L_p}(x - L_r - L_w)\right)} + \frac{N_1\tau_w(1 - e^{-b_2})e^{-\frac{b_3}{L_p}(x-L_r-L_w)}}{(N_1 + N_2)(s\tau_w + 1)} + \frac{N_2 e^{-\frac{\tau_{vw}}{\tau_w}}}{s(N_1 + N_2)}\left(1 - e^{-s\tau_{vw}}\right)e^{-\frac{b_3}{L_p}(x - L_r - L_w)}\right\}$$

$$\text{with } b_3 = \frac{s\tau_{vp}\left(\tau_p + \frac{\tau_{mp}}{s\tau_{mp} + 1}\right)}{\tau_p} \qquad (19)$$

The value for B_1 can now be calculated by the boundary conditions for ϑ_{r1} and ϑ_{p1} in a closed system: $\vartheta_{r1}(0) = \vartheta_{p1}(L_r + L_w + L_p)$

Laplace transformation of (3) using the equations from paragraph 1 for the initial value of T_{m1} gives

$$sC_m\vartheta_{m1} - C_m\frac{N_1(T_w(0) - T_a(0))}{N_1 + N_2} = \frac{C_w}{\tau_w L_w}\int_{L_r}^{L_r + L_w}(\vartheta_{w1} - \vartheta_{m1})\,dx + \frac{C_a}{\tau_a L_a}\int_0^{L_a}(\vartheta_{a1} - \vartheta_{m1})\,dx \qquad (20)$$

With the foregoing formulae and the calculated value for B_1, the first integration at the right hand side of (20) can be performed. Before carrying out the second integration of (20) the assumption will again be made that the thermal capacity of the air may be neglected and therefore the first term at the left hand side of (2) becomes zero. Equation (11) is then always valid and the second integral of (20) becomes $- N_2\vartheta_{m1}$

With the calculated integrals equation (20) gives

$$\vartheta_{m1}\left\{sC_m + \frac{sC_w}{s\tau_w + 1} + N_2 + \frac{C_w(1 - e^{-b_2})}{b_2\tau_w(s\tau_w + 1)}\cdot\left(1 - \frac{(1 - e^{-b_2})e^{-(b_1 + b_3)}}{1 - e^{-(b_1 + b_2 + b_3)}}\right)\right\} = (T_w(0) - T_a(0))$$

$$\left[\frac{C_m N_1}{N_1 + N_2} + \frac{C_w}{\tau_w}\left\{\frac{1 - e^{-b_2}}{sb_2} + \frac{N_1\tau_w}{(N_1 + N_2)(s\tau_w + 1)}\cdot\right.\right.$$

$$\left(1 - \frac{1 - e^{-b_2}}{b_2}\right) + \frac{N_2}{s(N_1 + N_2)} \left(\frac{\tau_w}{\tau_{vw}} \left(1 - e^{-\frac{\tau_{vw}}{\tau_w}}\right)\right.$$

$$\left. - \frac{1 - e^{-b_2}}{b_2} \right\} + \frac{(1 - e^{-b_2}) e^{-b_1} \quad C_w}{b_2 (1 - e^{-(b_1 + b_2 + b_3)}) \tau_w} \cdot$$

$$\left\{ \frac{N_1 + N_2 e^{-\frac{\tau_{vw}}{\tau_w}}}{s(N_1 + N_2)} (1 - e^{-b_3}) - \frac{1 - e^{-(b_2 + b_3)}}{s} + \right.$$

$$\left. \frac{N_1 \tau_w (1 - e^{-b2}) e^{-b_3}}{(N_1 + N_2)(s\tau_w + 1)} + \frac{N_2 e^{-\frac{\tau_{vw}}{\tau_w}}}{s(N_1 + N_2)} (1 - e^{-s\tau_{vw}}) e^{-b_3} \right\} \Bigg]$$

(21)

In order to find the time constant of the system the left hand side of (21) should be equated to zero.

The calculations of the numerical value of the time constant will be made for two different assumptions for the piping system

- the thermal capacity of the water and the metal are considered as one capacity
- the thermal capacities of the water and the metal are considered as separated capacities as indicated in fig. 1.

In the analogue experiments the first assumption has been made [1]. With the first assumption τ_{mr} and τ_{mp} are zero and the sum of b_1 and b_3 are determined by the running time of the water through the whole circuit.

From the data reported in [1] it follows that the length of the piping system is 95 m while the average water velocity is about 0,87 m/s yielding a running time of 109.2 sec.

Further the following values are found

$N_1 = 267.925$ W/K, $N_2 = 40.6875$ W/K, $C_w = 2080$ J/K,

$C_m = 2200$ J/K

$\tau_w = 2.7528$ sec., $\tau_{vw} = 7.1944$ sec.

Then:

$$2.91161\, s^2 + 1.11153\, s + 0.01956 + \frac{1 - 0.07328 e^{-7.1944 s}}{19.8047\, s + 7.1944} \cdot$$

$$\left(1 - \frac{1 - 0.07328 e^{-7.1944\, s}}{e^{109.2\, s} - 0.07328 e^{-7.1944\, s}}\right) = 0 \qquad (22)$$

The root of this equation is obtained by trial and error:

$s = 0.00112\ (\text{sec})^{-1} \rightarrow \tau = 0.00112^{-1}\ \text{sec} = 14.9$ min.

If $s \rightarrow -\frac{1}{\tau_w}$ then both the left hand side and right hand side of (21) approach to zero with order $(s\tau_w + 1)$. So τ_w is not a time constant of the system. The calculated value is the only time constant of the system.

In this calculation it has been assumed that all the induction units are in operation but in fact this is not sure. Therefore the time constant has also been calculated for the case that 70% of the induction units are in operation as was simulated in the analogue experiments. Then the running time through the piping system for an induction unit is increased with a factor $(0.7)^{-1}$ and the first term in the denominator of the last term becomes $e^{156.05} \rightarrow \tau = 20.85$ min.

So the time constant of this system increases nearly proportional with the inverse of the number of induction units in operation.

With the second assumption the piping and return piping systems will be considered as equal so $\tau_{vr} = \tau_{vp}$ etc. The following aditional data can be found which are average values based on a water temperature of 35°C.

$V = 0.87$ m/s, $d = 0.06$ m, $\nu_w = 0{,}73 \times 10^{-6} m^2/s$,

$A = 25.67\ m^2$, $Pr_w = 4.9$, $Nu_w = 340$, $\alpha_w = 3519\ W/m^2K$

$C_p = C_r = 1233 \times 10^3$ J/K, $C_{mp} = 355 \times 10^3$ J/K

$$\tau_p = \tau_r = \frac{1233 \times 10^3}{3519 \times 25.67} = 13.65 \text{ sec.}$$

$$\tau_{mp} = \tau_{mr} = \frac{355 \times 10^3}{3519 \times 25.67} = 3.930 \text{ sec.}$$

$\tau_{vr} = \tau_{vp} = 54.6$ sec.

The first terms of (22) remain unchanged but the first term in the denominator of the last term becomes

$e^{109.2s + 8.00 - \frac{8.00}{3.930s + 1}}$ if all the induction units are in operation

$e^{156.0s + 11.4286 - \frac{11.4286}{3.930s + 1}}$ if 70% of the induction units are in operation

Solving the equation for s gives the time constants:

$\tau = 18.94$ min. if all the induction units are in operation

$\tau = 26.67$ min. if 70% of the induction units are in operation.

Also now the time constant of the system increases nearly proportional with the inverse of the number of induction units in operation.

In the experiment at the installation a time constant 26.6 min. was found [1]. Analogue simulation gave the following results

$\tau = 21.8$ min. if all the induction units are in operation

$\tau = 28.6$ min. if 70% of the induction units are in operation.

From the calculated values it is clear that the second assumption is much better than the first one. The influence of the heat transfer between the water and the pipes in the piping systems appears to be rather strong. If 70% of the induction units are assumed to be in operation the calculated value with heat transfer between the water and the pipes agrees with the experimental value, found previously. Nevertheless it may not be concluded that 70% of the induction units were actually in operation.

3. SINUSOIDAL INPUT DISTURBANCE

The solutions of the equations (1) - (7) will have the general form

$T_w = \Theta_w e^{j\omega t}$, etc., they are:

$$\Theta_{mp} = \frac{\Theta_p}{j\omega\tau_{mp} + 1} \quad (23)$$

$$\Theta_{mr} = \frac{\Theta_r}{j\omega\tau_{mr} + 1} \quad (24)$$

$$\Theta_r = \Theta_w(0)\, e^{-b_1 \frac{x}{L_r}} \quad (25)$$

$$\Theta_w = \frac{\Theta_m}{j\omega\tau_w + 1}\left(1 - e^{-b_2 \frac{x - L_r}{L_w}}\right) + \Theta_w(0) \cdot e^{-\left(b_1 + b_2 \frac{x - L_r}{L_w}\right)} \quad (26)$$

$$\Theta_p = \left\{\frac{\Theta m}{j\omega\tau_w + 1}\left(1 - e^{-b_2}\right) + \Theta_w(0)\, e^{-(b_1 + b_2)}\right\} \cdot e^{-b_3 \frac{x - L_r - L_w}{L_p}} \quad (27)$$

The solution for Θ_m expressed in $\Theta_w(0)$ and $\Theta_a(0)$ is:

$$\Theta_m = \frac{\Theta_w(0) C_w (1 - e^{-b_2}) e^{-b_1} + \Theta_a(0) N_2 \tau_w b_2}{j\omega C_m \tau_w b_2 + C_w \tau_{vw} (j\omega\tau_w b_2 + 1 - e^{-b_2}) + N_2 \tau_w b_2} \quad (28)$$

In the formulas for b_1, b_2 and b_3 the s should be replaced by $j\omega$. For high frequencies the following relations are obtained

<u>sinusoidal disturbance on entrance of the return line ($\Theta_a(0) = 0$)</u>

$$\lim_{\omega \to \infty} \frac{\Theta_w}{\Theta_w(0)} = e^{-\left(\frac{\tau_{vr}}{\tau_w} + \frac{\tau_{vw}}{\tau_w}\right) - j\omega\,(\tau_{vr} + \tau_{vw})}$$

for $x = L_r + L_w$ (29)

$$\lim_{\omega \to \infty} \frac{\Theta_m}{\Theta_w(0)} = \frac{C_w\left(1 - e^{-\frac{\tau_{vw}}{\tau_w}(j\omega\tau_w + 1)}\right) e^{-\frac{\tau_{vr}}{\tau_r}(j\omega\tau_r + 1)}}{C_m \tau_{vw} \tau_w (j\omega)^2} \quad (30)$$

$$\lim_{\omega \to \infty} \frac{\Theta a}{\Theta_w(0)} = \frac{C_w\left(1 - e^{-\frac{\tau_{va}}{\tau_a}}\right)\left(1 - e^{-\frac{\tau_{vw}}{\tau_w}(j\omega\tau_w + 1)}\right) e^{-\frac{\tau_{vr}}{\tau_r}(j\omega\tau_r + 1)}}{C_m \tau_{vw} \tau_w (j\omega)^2}$$

for $x = L_a$ (31)

<u>sinusoidal disturbance on air ($\Theta_w(0) = 0$)</u>

$$\lim_{\omega \to \infty} \frac{\Theta_w}{\Theta_a(0)} = \frac{N_2\left(1 - e^{-\frac{\tau_{vw}}{\tau_w}(j\omega\tau_w + 1)}\right)}{C_m \tau_w (j\omega)^2} \quad (32)$$

for $x = L_r + L_w$

$$\lim_{\omega \to \infty} \frac{\Theta_m}{\Theta_a(0)} = \frac{N_2}{j\omega C_m} \quad (33)$$

$$\lim_{\omega \to \infty} \frac{\Theta a}{\Theta_a(0)} = e^{-\frac{\tau_{va}}{\tau_a}} \quad \text{for } x = L_a \quad (34)$$

Equations for Θ_p at $x = L_r + L_w + L_p$ can be obtained from the equations for Θ_w by adding a factor

$$e^{-\frac{\tau_{vp}}{\tau_p}(j\omega\tau_p + 1)}$$

Equations (29)-(34) indicate systems of order 0,1 or 2 as far as the denominators are concerned. But the imaginary parts in the exponents of e indicate periodicity in the amplitude response and phase lag.

The analogue model used for the frequency response was different from the previous model [1] at two points:

- There was one section for the water side instead of four.
- The water side and the piping system is not a closed system anymore. The capacitor representing the piping system (C3) is behind the induction units.

In the analogue model the thermal capacities of the piping system and the water in the induction units are lumped together into one capacitor each. Equations for Θ_w, Θ_m, etc. in this system can easily be derived and they indicate the same order of system as equation (29) - (34). But even without deriving the equations it is clear that no periodicity can occur in such a system. It appears that the equations are obtained from (30) - (34) if $\tau_{vr} = 0$ because the capacitor is behind the induction units and if $\tau_{vw} = 0$ because of lumping together of the thermal capacities. For water at waterinlet fluctuations a first order system is found because there was only one section for the water side.

The temperature at the end of the piping system is obtained from the water temperature in the model by adding a factor $(j\omega R_1 C_3)^{-1}$. As an example the amplitude response at water inlet fluctuation is given in fig. 2. All temperatures show 2nd order system except the water temperature which shows a first order system.
At air inlet temperature fluctuation the water from the induction units shows 2nd order behaviour and the water leaving the piping system shows 3rd order behaviour.

4. REFERENCES

[1] Crommelin, R.D., ASHRAE Journal, p. 64-72, Jan. 1974.

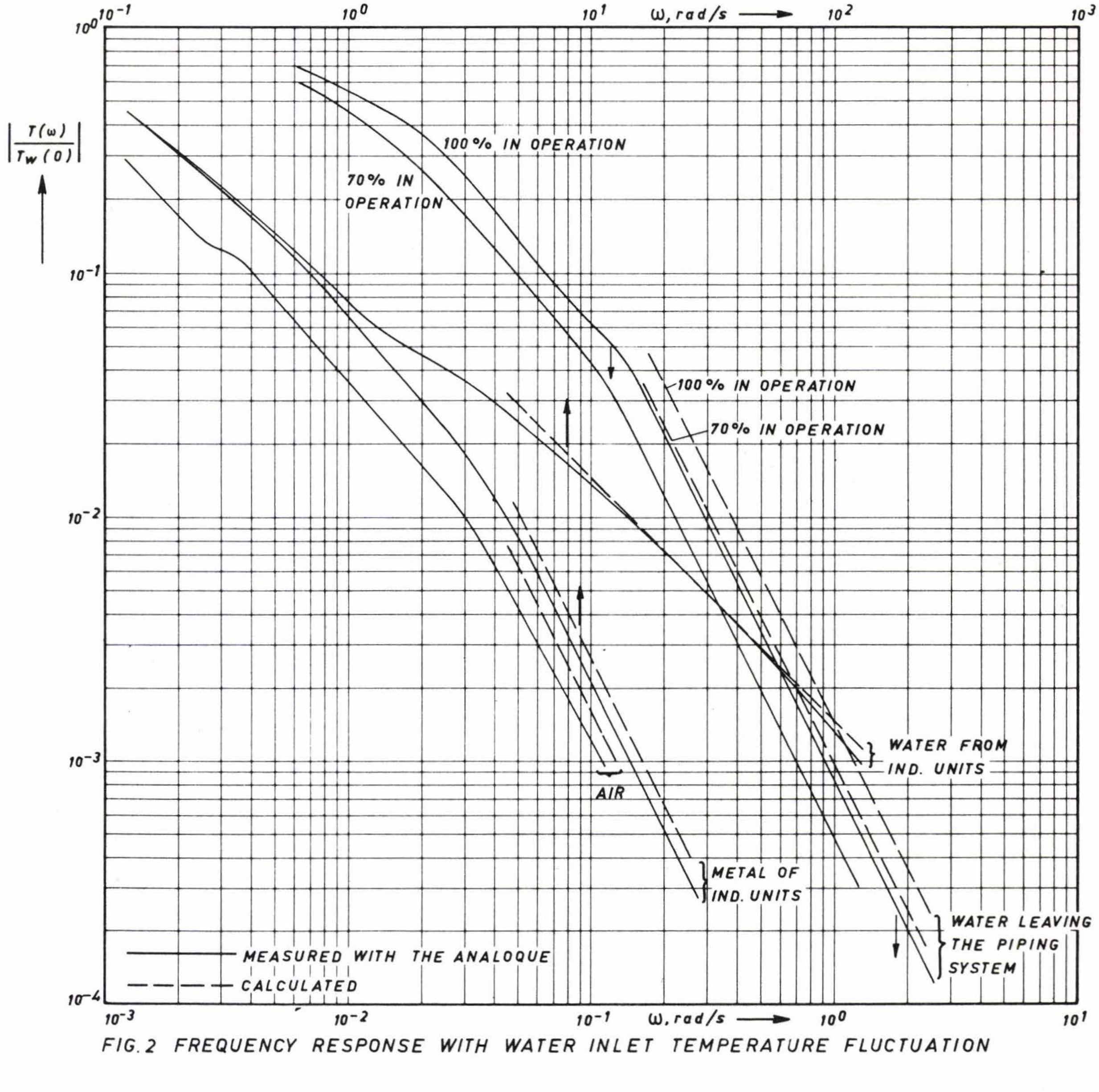

FIG. 2 FREQUENCY RESPONSE WITH WATER INLET TEMPERATURE FLUCTUATION