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CALCULATIONS ON LOCATION AND DIMENSIONS OF HOLES IN A CLARINET

by

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Summary

For a cylindrical woodwind and in particular for a clarinet, methods are derived to calculate location and diameter of the holes. In a simple way the calculations are carried out for each hole separately, and independently of the other holes.

Corrections are introduced for the influence on the resonance frequency of the overblowing hole and of the row of closed holes in the main tube.

The usefulness of the formulae is proved by calculating all holes of a B-flat-Boehm-clarinet.

It appears that if the clarinet is assumed to be cylindrical from the top downwards, the pressure antinode is located at 7 mm below the top.

Some phenomena accompanying tuning of the clarinet are clarified.

Zusammenfassung

Für zylindrische Holzblasinstrumente, und zwar speziell für Klarinetten, wurde eine Methode zur Berechnung der Lage und des Durchmessers der Löcher entwickelt. Die Berechnungen werden auf einfache Weise für jedes Loch einzeln und unabhängig von den anderen durchgeführt.

Für den Einfluß des Überblasloches und der geschlossenen Löcher im Hauptrohr werden entsprechende Korrekturen angegeben.

Die Brauchbarkeit der Formeln wird durch die Berechnung aller Löcher einer Böhm-Klarinette bestätigt.

Es scheint, daß der Druckbauch etwa 7 mm vom oberen Ende entfernt ist, wenn man annimmt, daß die Klarinette zylindrisch geformt ist.

Einige mit der Stimmung zusammenhängende Erscheinungen werden ebenfalls aufgeklärt.

Sommaire

On expose des méthodes de calcul de la disposition et du diamètre des trous d'un instrument à vent cylindrique et particulièrement d'une clarinette. Des calculs simples sont effectués pour chaque trou séparément indépendamment des autres.

On introduit des corrections pour l'influence des trous trop résonnants et du souffle bruyant de trous fermés dans le tube principal, sur la fréquence de résonance.

On vérifie l'utilité des formules en les appliquant au calcul de tous les trous d'une clarinette en Si Bémol. Il apparaît que si la clarinette est cylindrique du sommet jusqu'en bas, le ventre de pression se trouve à 7 mm au-dessous du sommet.

On explique quelques phénomènes relatifs à l'accord de la clarinette.

1. Introduction

Knowledge about the acoustic behaviour of musical instruments is not very wide. Their construction and use is merely based on experience. Highly empirical are the position and sizes of the holes in woodwind instruments. These holes act as an effective shortening of the main tube which allows the playing of all tones situated between the natural tones. Calculations with respect to these holes have been carried out earlier [1], [2]; in this paper an extension to these is given by deriving formulae for the holes, which can be used in practice. As an example the clarinet is chosen, although the calculations are not exclusively for clarinet or even for cylindrical instruments.

A clarinet is a long cylindrical tube with practically constant cylindrical cross-sectional area [3]. At the lower end the tube has a horn-like expansion into the free air. At the top its diameter diminishes conically and ends in a flat chink which is formed between the tube and a flat wooden tongue, the reed. The top of the instrument is placed between the lips of the player, who presses air through the chink. The pressure fluctuations in the instrument cause the reed to close and open the chink successively, so that at the frequency of this oscillation air is supplied into the instrument, by which the oscillation is maintained. Acoustically it appears that the instrument is closed at its top: it overblows in uneven harmonics.

The cylindrical part of the tube is provided with some twenty holes. These holes can be closed by fingers or by keys. A calculation of the resonance frequency could involve a calculation of the impedance of a very intricate pipe-combination. Carefully neglecting unimportant quantities, it appears to be possible to simplify these expressions.

2. List of symbols

a = radius of a tube,
c = velocity of sound,
d = diameter of a tube,
f = frequency,
F = cross-sectional area of a side-tube,
g = relative frequency shift,
h = length of a side-tube,
H = effective length of a side-tube (including end-corrections),
j = $\sqrt{-1}$,
k = wave number,
l = geometrical length,
L = effective length (including end-corrections),
m = vibrational mode,
S = cross-sectional area of main tube,
t = hole function,
V = frequency shift expressed in semitones,
z = hole function,
Z = acoustical impedance,
ϱ = density.

3. Condition for position and size of a hole

To start with, we consider the length-correction for a cylindrical tube with an open end. In a loss-free cylindrical tube with area S and length l, terminated by an acoustical impedance Z_1 the input impedance Z_0 is [4]:

$$Z_0 = \frac{\varrho c}{S} \frac{Z_1 S/\varrho c + \mathrm{j} \tan k l}{1 + \mathrm{j}(Z_1 S/\varrho c) \tan k l}. \tag{1}$$

Here k denotes the wave number and c the sound velocity in free air of density ϱ. The acoustical impedance of an open end, when losses are neglected and if $k \xi a \ll 1$, is equal to $Z_1 = (\varrho c/S)\, \mathrm{j}\, k \xi a$, where a is the radius of the tube and ξ depends on the surroundings of the end: $0.6 < \zeta < 0.85$ [4]. Because $k \xi a$ is small it may be replaced by $\tan k \xi a$ to a good approximation. Substitution into eq. (1) gives $Z_0 = (\varrho c/S)\, \mathrm{j} \tan k L$, where $\mathrm{L} = l + \xi a$. In this way we have obtained a tube with effective length L terminated by zero impedance, by correcting its geometrical length l with an end-correction ξa.

In the following we shall denote geometrical (measured) lengths in small print and effective (corrected) lengths in capitals.

Consider a tube with a single hole of effective length H and cross-sectional area F at an effective distance L_2 from the open end (see Fig. 1). The

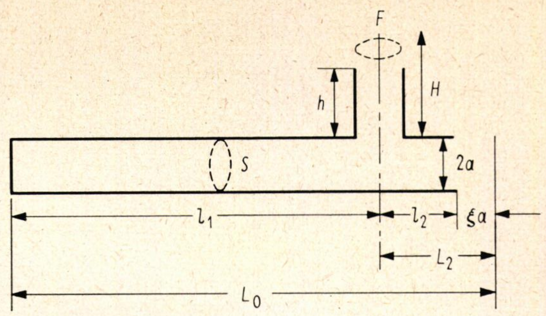

Fig. 1. Explanation of dimensions and locations of a tube with a single side-hole.

terminating impedance Z_1 of the preceding tube-piece l_1 is considered as a network with two parallel branches H and L_2

$$\frac{1}{Z_1} = \frac{S}{\varrho c \,\mathrm{j} \tan k L_2} + \frac{F}{\varrho c\, \mathrm{j} \tan k H}. \tag{2}$$

For a clarinet the input impedance is infinite: $Z_0 = \infty$. Insertion of this condition and of $l = l_1$ in eq. (1) gives:

$$1 + \mathrm{j} \frac{Z_1 S}{\varrho c} \tan k l_1 = 0. \tag{3}$$

Elimination of Z_1 from eqs. (2) and (3) gives the condition for the hole:

$$F \cot k H + S \cot k L_2 - S \tan k l_1 = 0. \tag{4}$$

This may be written as

$$\cos k L_1 = 0, \tag{5}$$

where

$$L_1 = l_1 + \frac{1}{k} \operatorname{arc\,tan} \frac{S}{F \cot k H + S \cot k L_2}. \tag{6}$$

Apparently the tube-pieces L_2 and H form an end-correction to the tube l_1. L_2 and H are small with respect to the wave-length. So it is useful to expand the goniometric functions in power series and neglect higher powers. This gives ($\cot k H = 1/k H$):

$$L_1 = l_1 + \frac{1}{k} \frac{S}{\dfrac{F}{k H} + \dfrac{S}{k L_2}\left(1 - \dfrac{1}{3} k_2 L_2{}^2 \ldots\right)} \times$$
$$\times \left[1 + \frac{S_2}{3\left(\dfrac{F}{k H} + \dfrac{S}{k L_2}\right)^2} \ldots\ldots\right]$$

or approximately:

$$L_1 = l_1 + \frac{S H L_2}{F L_2 + S H} \times$$
$$\times \left[1 - \frac{k^2}{3}\left(\frac{S H L_2}{F L_2 + S H}\right)\left(\frac{S H L_2}{F L_2 + S H} - L_2\right)\right]. \tag{7}$$

If we neglect the frequency dependent term (with k^2) we get:

$$L_1 = l_1 + \frac{S H L_2}{F L_2 + S H} \tag{8}$$

and have obtained an expression for the effective length of the tube of Fig. 1. This length is independent of frequency provided the higher power terms of eq. (7) may be neglected. We shall revert to this validity later.

So far everything is known from the literature [2]. We shall now make a useful change in eq. (8) by introducing the relative frequency deviation g, defined by

$$1+g=2^{v/12} \tag{9}$$

where v is the number of semitones with which the pitch changes when the hole is opened. The frequency then becomes $(1+g)$ times as high, as follows from eq. (9). In Table I the numerical value of g for some values of v is given

Table I.
Relative frequency change g as a function of the number of semitones v.

frequency interval	v	g
semitone	1	0.059463
wholetone	2	0.122462
3 semitones	3	0.189207
fifth	7	0.498307
octave	12	1.000000

From eq. (8) L_2 is eliminated with $L_2=L_0-L_1$ and L_0 with $L_0=(1+g)L_1$. Solving for the hole area F gives:

$$F=\frac{g\,S\,H\,L_1}{(L_1+g\,L_1-l_1)\,(L_1-l_1)}\,. \tag{10}$$

The fact that H is dependent on F, because the end-correction in H depends on the hole diameter, is neglected here.

Instead of solving for the hole area F, the hole position l_1 may be calculated. The quadratic equation of l_1 has only one meaningful solution:

$$l_1=L_1(1-z), \tag{11}$$

where

$$z=\frac{1}{2}\,g(\sqrt{1+4\,t/g}-1) \tag{12}$$

and

$$t=\frac{S\,H}{F\,L_1}\,. \tag{13}$$

It will appear that z and t are quantities with numerical values around those of g. z, as a function of t is calculated for three values of v according to eq. (12) and given in Fig. 2.

Eqs. (10) and (11) gave size in dependence on location and location in dependence on size of a single hole in a tube. When a second hole is made,

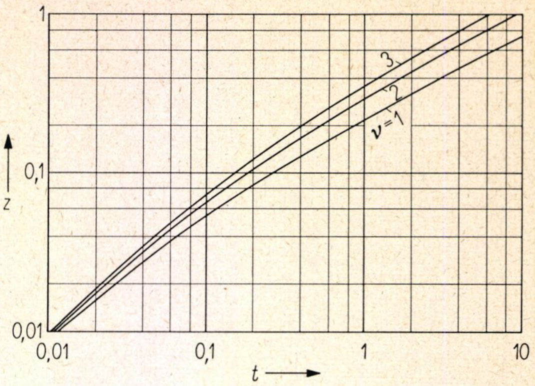

Fig. 2. Correlation between the two hole-functions z and t for various values of the frequency shift v.

these formulas appear to maintain their validity. We then replace the pipe with the first hole by a tube without holes according to eq. (8), and calculate the second hole according to eqs. (10) or (11). This procedure is repeated for every next hole. We remark that it is not necessary to know anything about any hole other than the one in consideration.

4. Accuracy considerations

First we investigate, assuming eqs. (10) and (11) to be exact, which frequency deviations will arise when a hole is slightly displaced or varied in size. We, therefore, calculate the influence on the frequency of a small dimensional change of the hole. After elimination of L_2 and L_1 from eq. (10) the following relation between F, l_1 and g is found

$$F=\frac{g\,S\,H\,L_0}{(L_0-L_1)\,(L_0-l_1-g\,l_1)}\,, \tag{14}$$

where L_0 is a constant.

By partial differentiation we obtain

$$\left(\frac{\partial F}{\partial g}\right)_{l_1=\text{constant}}=\frac{F(L_0-L_1)}{g(L_0-l_1-g\,l_1)}\,;$$

$$\left(\frac{\partial g}{\partial l_1}\right)_{F=\text{constant}}=-g\frac{2(L_0-l_1-g\,l_1)+g\,L_0}{(L_0-l_1)^2}\,.$$

These expressions are only exactly valid for the infinitesimal region. We will use them here assuming they are still valid for finite frequency shifts δg due to finite relative size and location deviations. Therefore, we substitute $L_0=(1+g)L_1$ and insert z by means of eq. (11). In this way we obtain:

$$\delta g=\frac{z\,g(g+1)}{z+g}\,\frac{\delta F}{F}\,; \tag{15}$$

$$\delta g=-\frac{g(1+g)\,(2\,z+g)}{(z+g)^2}\,\frac{\delta l_1}{L_1}\,. \tag{16}$$

In Fig. 3 both functions are plotted against t (t being a function of z) for $\delta F/F = 10\%$ and $\delta l_1/L_1 = 1\%$ for three different values of v (v being a function of g), under the assumption that the relations are still valid. We shall assume that a mistuning of 0.1 semitone is acceptable in practice, as such a value may be compensated by lip pressure of the player. For values of t between 0.03 and 3, which will later appear to be most frequent (c.f. Fig. 5) it is visible from Fig. 3 that 0.1 semitone is caused by a change in area of about 10% or a change in location of about 1% (on clarinet about 2 mm). This can be considered as an accuracy criterion in its way.

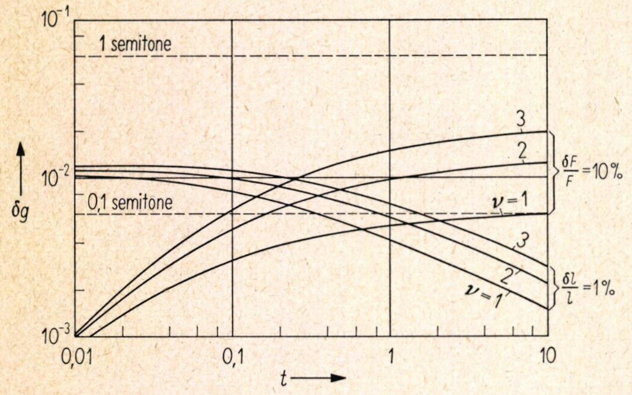

Fig. 3. Frequency shift δg as a function of hole-function t for small alterations of hole dimensions and location.

In the preceding Section we mentioned the general validity of the formulas for the hole calculation. This will be the case only if a tube with one side hole may be replaced by a single, somewhat shorter, tube without a side hole. In other words eq. (8) must be valid, or the higher order term of (7) must be small. In order to get an impression of its magnitude we substitute eqs. (8) and (11), $L_2 - L_1 + l_1 = g\,L_1$ and $k\,L_1 = \frac{1}{2}\,m\pi$ into this higher order term and get

$$L_1 \approx l_1 + z\,L_1 \left(1 + \frac{m^2\,\pi^2}{12}\,g\,z\right). \qquad (17)$$

The higher order term shows frequency dependence because of the presence of the mode-number m. This term is to be found in a frequency-shift δg_m.

$$\delta g_m \approx -\frac{\delta L_1}{L_1} = -\frac{m^2\,\pi^2}{12}\,g\,z^2. \qquad (18)$$

Although the calculations are not exact, they give a good indication to which limit the simple formulas are valid. This is illustrated in Fig. 4, where the relation between δg_m and t is plotted, as calculated with eq. (18) for various values of m and v. The higher order corrections appear to become important for increasing v, m and t. Translating this into normal language it means that we may expect to obtain too low an overtone for long, narrow holes, near the top of the instrument, meant for large frequency intervals. On clarinets with old key mechanisms, where a hole is present for $v = 3$, preceded by one for $v = 2$, this effect is striking. Apart from that these purities do not give much trouble in practice, which is an indication that higher order terms may be neglected and that the methods are useful. Generally speaking t must be smaller than 0.2 to avoid impurities between ground-mode and first overtone.

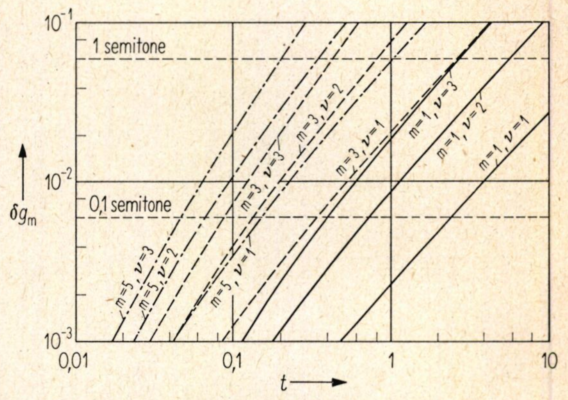

Fig. 4. Higher order frequency shift δg_m as a function of hole-function t.

Introduction could have been considered of length corrections according to eq. (18) or an analogous formula. We abandoned this idea, because that would become too intricate: when these corrections are necessary for one hole, they are mostly necessary for preceding holes also and we get an avalanche of corrections, which partly reinforce one another.

5. Comparison with a Boehm-clarinet

A "simple" or "plain" B-flat-Boehm-clarinet, made by Dolnet (France) and bought new at Rotterdam in 1951, was chosen as a specimen to check the formulae. The dimensions of the instrument were measured and are given in Table II. The holes were numbered from top to bottom. The tone sounding with opened hole is given in the second column. The corresponding effective length L_1 is calculated with $340/(4\,f)$, where 340 is the velocity of air in m/s and f denotes the frequency. The walls of the instrument are assumed to be hard. The frequency decrease, when closing the hole, is expressed in the number of semitones, v. The diameter of the main-tube, d_S, is practically constant. As the side holes were more or less conical, for their diameter

Table II.
Survey of measurements and some calculations on location and size of holes of a clarinet.

hole no.	tone with hole opened	L_1 mm	v	d_S mm	d_F min. mm	d_F max. mm	h mm	H mm	l_1 min. mm	l_1 max. mm	l_g mm
1	g^1 #	205	1	14.8	3.0	3.0	12.5	14.5	145	145	155
2	a^1	193	2	14.8	4.5	7	7	10	153	164	169
3	g^1 #	205	1	14.8	6.2	8	7	11	183	186	194
4	g^1	217	1	14.8	5.3	7.5	7	10	191	196	204
5 a	f^1 #	230	1	14.8	5.0	8	7	10	202	207	215
b	f^1	244	2	14.8					206	217	
6	f^1	244	1	14.8	4.6	7	4	7	218	224	231
7	e^1	258	2	14.8	7.8	7.8	10	14	231	231	239
8	e^1	258	1	14.8	5.0	8.5	7	10	228	238	243
9 a	d^1 #	274	1	14.8	5.0	9	10	14	237	249	253
b	d^1	290	2	14.8					240	257	
c	c^1 #	307	2	14.8					256	272	
10	d^1	290	1	14.8	5.0	7.5	7	10	259	266	272
11	c^1 #	307	1	14.8	6.4	12	10	15	276	288	286
12	c^1 #	307	1	14.8	6.0	8.5	7	11	280	286	289
13	c^1 #	307	1	14.8	6.0	7	7	10	282	284	290
14	c^1	325	2	14.8	7.1	11	7	12	296	304	308
15	b	344	1	14.8	5.1	9	7	11	310	321	322
16	a #	365	1	14.8	7.8	10.5	7	12	343	347	348
17	a	386	2	14.8	8.7	11	9	14	360	365	364
18	a	386	1	14.8	8.0	11	7	12	365	370	369
19	g #	410	1	14.8	7.8	11.5	9	14	385	392	388
20	g	434	2	14.8	9.2	11.5	9	14	410	414	412
21	f #	460	1	14.8	10.0	12.5	5	11	444	447	443
22	f	487	2	14.8	12.4	14	4	11	473	475	471
23	e	517	1	15.5	11.0	13	5	11	502	504	503
24	d #	546	1	16.8	12.3	14	4	11	532	534	542
	d	580									

d_F both minimum and maximum values are given. The hole length is h, excluding, and H, including, the end-correction. The magnitude of this end-correction is very uncertain because of this non-cylindrical course and the keys which are hanging above most of the holes. The location of the hole on the instrument is fixed by the length l_g measured from the top of the instrument to the center of the hole.

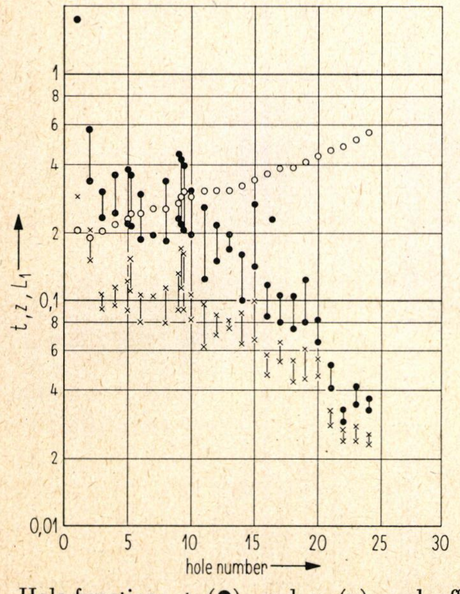

Fig. 5. Hole-functions t (●) and z (×) and effective lengths L_1 (○) (in m) for a clarinet.

The calculation of the hole location was carried out according to eq. (11). First the quantity t had to be calculated. Because of the uncertainty of the diameter of the holes, the two most probable extreme values are calculated by choosing as hole diameter the smallest and the mean value. The results are given in Fig. 5, together with the value L_1 and the value z calculated from t with eq. (12). The two extreme values of l_1, calculated with eq. (11), are

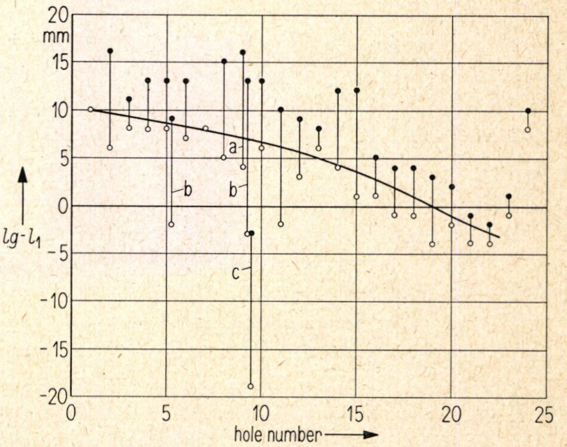

Fig. 6. Difference between geometrical and calculated length, $l_g - l_1$, for the various holes of the clarinet.
● Smallest diameter,
○ mean diameter.

given in Table II. The difference between true location l_g and calculated location l_1 is plotted in Fig. 6 for thet wo extremes of t. A smooth curve is drawn between the separate points.

Some holes perform more than one function. We have marked then a, b and c in Table II and in the figures. A multiple function means that the hole is used for the forming of more tones and intervals. In the opened position two or more frequencies may sound. This is realised by closing one or more holes directly beneath the hole in question. This is called cross-fingering. In general, the frequency shift for opening and closing the hole in question will be different for both functions. A double function fixes the location and size of the hole completely. When we provide the quantities of the cross-fingering with a dash it follows by applying eq. (10) twice that:

$$\frac{F}{H\,S}\,\frac{g\,L_1}{(L_1-l_1+g\,L_1)\,(L_1-l_1)} = \frac{g'\,L_1'}{(L_1'-l_1+g'\,L_1')\,(L_1'-l_1)}\,. \qquad (19)$$

After substitution of $L_1' = (1+G)\,L_1$, where G denotes the difference between both notes, t and z may be solved. For three cases results are given in Table III.

Table III.
Calculated values of t and z for holes with double function (cross-fingering).

	v			Z acoustical ohms	t	examples
	G	g	g'			
1	1	1	2	0.160	0.588	5a, b; 9a, b
2	1	2	3	0.272	0.878	
3	2	1	2	0.276	1.56	9a, c

If we look at the magnitudes of t in this Table, we see that these are larger than those on the instrument, as is apparent from Fig. 5. Besides, they are larger than is permitted with respect to the higher order corrections (see Fig. 4). It seems that cross-fingering in the ideal way according to eq. (19) is impossible because of impurities between the registers. When the cross-fingering on the clarinet is studied by blowing the corresponding notes, it appears that the purity of cross-fingered notes is often bad and especially very bad on the lower register. On the high register, where help of higher order corrections occurs, the purity is sometimes better.

Apart from these elementary hole calculations, some important corrections have been introduced.

For higher order effects no corrections are applied, because these would be doubtful, as was argued earlier. At high frequencies they could become important, however.

A correction which may be calculated to some degree of accuracy, is that due to the row of closed side-holes in the instrument. We follow a method published earlier [5]. If in a cylindrical tube with cross-sectional area S a side-tube is present with volume ΔV at a distance x of the closed end, we must introduce a positive virtual length-correction Δl to the main tube according to

$$\Delta l = \frac{\Delta V}{S}\,(\cos^2 k\,x - \varepsilon \sin^2 k\,x)\,. \qquad (20)$$

where $0<\varepsilon<1$ depends on the shape of the side-tube. Instead of calculating the influence of each side-tube separately we imagine the volume of the side-tube to be spread homogeneously over the main tube, from a point l_3 to the end of the tube. The length to the end of the tube will have some value between l_1 and L_1; we choose L_1. The total correction follows from

$$\Delta l_V = \int_{l_3}^{L_1} \frac{V}{(L_1-l_3)\,S}\,(\cos^2 k\,x - \varepsilon \sin^2 k\,x)\,\mathrm{d}x\,, \quad (21)$$

where V denotes the sum of the volumes of all side-tubes between l_3 and L_1. This integration may be carried out and gives after insertion of $k\,L = \frac{1}{2}\,m\pi$:

$$\Delta l_V = \frac{V}{2\,S}\left[(1-\varepsilon) - (1+\varepsilon)\,\frac{\sin(m\,\pi\,l_3/L_1)}{m\,\pi(1-l_3/L_1)}\right]. \quad (22)$$

In order to decide upon the validity of this procedure for the clarinet, Fig. 7 shows the volume of all

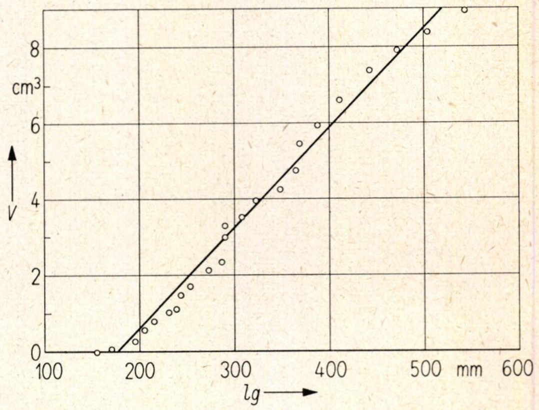

Fig. 7. Volume of closed side holes, V, as a function of location of the highest opened hole, l_g.

holes above a certain hole against the location of the hole. It appears that it is possible to a reasonable approximation to draw a straight line through the points so that integration is permissible. At the same time we find from the intersection point with the horizontal axis the best value for l_3 to be 175 mm, which value was adopted for the calculations. To fix the magnitude of ε, we calculated the quotient of diameter and length for each hole. The mean value was 1.3. From eq. (20) in reference [5] it follows that $\varepsilon = 0.128$. The correction for the side holes was now calculated according to eq. (22) and is given graphically in Fig. 8.

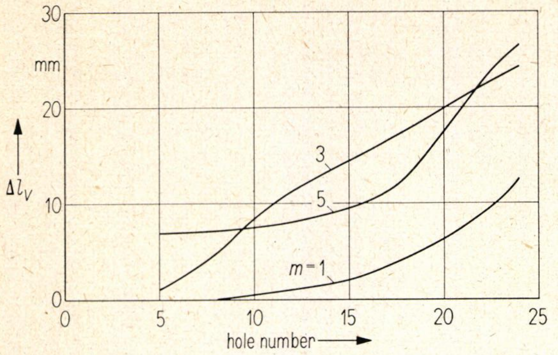

Fig. 8. Length correction Δl_V against hole number due to the closed side hole effect.

Another important correction must introduced for the frequency alteration caused by the opening of a so-called speaker-hole, which is meant to facilitate the overblowing. When in eq. (4) one substitutes $L_2 = L_0 - l_1$, one obtains

$$\cos k(L_0 + \Delta L_0) = 0\,, \tag{23}$$

where

$$\Delta L_0 = \frac{1}{k} \arctan \frac{-F \cos^2 k\, l_1}{S \tan k\, H - F \sin k\, l_1 \cos k\, l_1}\,.$$

When the corrections are small, arc tan and tan may be neglected and we get:

$$\Delta L_0 = \frac{-l_1 \cos^2 k\, l_1}{k\, l_1 [k\, l_1 (S\, H/F\, l_1) - \sin k\, l_1 \cos k\, l_1]} \tag{24}$$

with $k\, l_1$ as variable quantity. When $S\, H/F\, l_1$ is large (long, narrow holes) ΔL_0 is small and negative, so that we have a correction which gives a frequency increase. This correction is calculated for hole 1 at $m = 3$ and $m = 5$ and for hole 9 at $m = 5$. The results are plotted in Fig. 9.

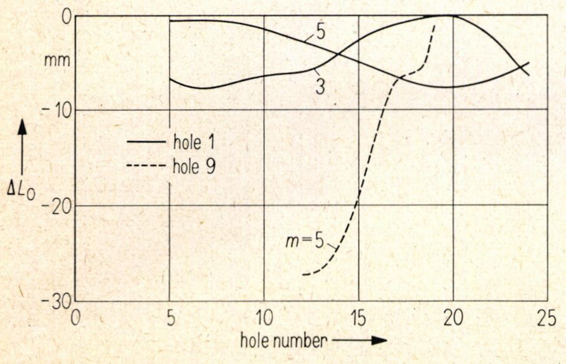

Fig. 9. Length correction ΔL_0 against hole number due to opening of a speaker-hole.

Finally the total result of all calculations is summarized in Fig. 10, where $\Delta l_{\text{tot}} = l_g - l_1 + \Delta l_V + \Delta L_0$ is plotted against the hole number for modes $m = 1$, 3 and 5. for $l_g - l_1$ we started from the smooth curve of Fig. 6.

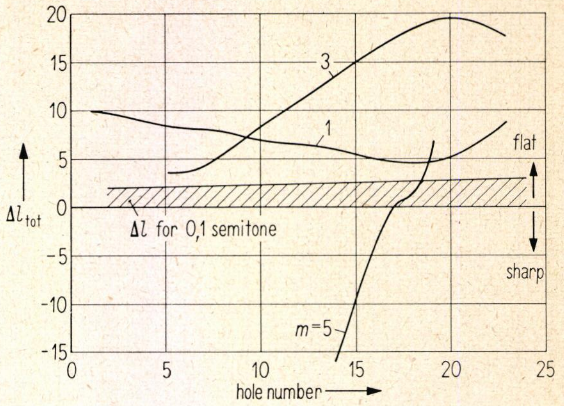

Fig. 10. Difference between geometrical and calculated length, Δl_{tot}, including corrections from Figs. 8 and 9, as a function of hole number.

First we observe from studying Fig. 10 that apparently the clarinet is geometrically 7 mm longer than is expected from calculations. This, however, is not surprising as we completely neglected the conical top of the instrument and assumed that the input impedance at the top was real and infinite. The conical top results according to RAYLEIGH [6] in a frequency increase because one has a decrease in diameter in a pressure antinode.

The impedance at the top is certainly not infinite because the reed leaves at least half of the time a chinkwise opening there [7]. Another striking fact from Fig. 10 is that the notes in the lower part of the lower register ($m = 1$) are too sharp and in the corresponding part of the high register ($m = 3$) are too flat. This phenomenon is well-known among clarinet-players and is found by measurements too [8], [9], [10]. A third fact is the course of the curve of $m = 5$, which seems to involve much too sharp notes. This will, however, be compensated to some extent by higher order effects. Besides, the correction for the conical mouthpiece, which may be assumed constant for low frequencies, will become dependent on the frequency. This follows from the wave length of the vibration being no longer large with respect to the mouthpiece. That this effect is important is proved from the experience that a differently shaped mouthpiece disturbs the tuning completely. Apparently the clarinet-maker has found by trial and error the best shape to compensate all effects.

The difference between the pure fifth, which arises in overblowing, and the tempered fifts, equal to $0.5 - 0.4983 = 0.0017 \approx 0.03$ semitone can easily be compensated by lip pressure. Therefore this effect has been neglected in all calculations.

6. Conclusions

Several investigations of the tuning of a cylindrical woodwind and especially of a clarinet were carried out. It appears to be possible to calculate the location of a hole in a clarinet with an accuracy of some mm. If the instrument is assumed cylindrical from top to bottem, the velocity node is located about 7 mm from the top in the instrument.

When a hole is meant for a large frequency shift and when it is long and narrow, it may be impossible to tune the tone pure on more than one register. As an approximation one may say that the hole function t, defined in eq. (13) must be smaller than 0.2 to avoid impurities. Holes for large frequency shifts are to be avoided!

A long and narrow speaker-hole, when opened, will sometimes increase the frequency somewhat.

The large volume of the closed side-holes causes the notes in the lower register to be too sharp and those in the high register too flat. By choosing shorter holes this effect could be diminshed.

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