Amplitude Pattern Synthesis for Conformal Array Antennas using Mean-Field Neural Networks

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Abstract— **In this paper, we deal with the synthesis problem of conformal array antennas using a mean-field neural network. We applied a discrete version of mean-field neural network proposed by Vidyasagar [1]. This technique is used to find the global minimum of the objective function, which represents the square of the distance between the required and actual radiation pattern. We compared the results with those obtained by an iterative least square synthesis.**

I. Introduction

Since a few decades, the synthesis problem for a given complex radiation pattern has been extensively studied and investigated [2]. For its linear characteristic, the creation of shaped patterns is well-known in technical literature.

In recent years, conformal arrays have attracted great attention. They can offer some advantages in comparison with linear or planar arrays, like: fulfillment of structural and/or aerodynamic requirements without appreciable degradation of the antenna performances, reduction of the interaction between the radome and the antenna, wider scan angles without rotating/moving antennas and reduced radar cross section of the platform. Unfortunately, the corresponding mathematical formulation of the far field is more complicated. This is due to the fact that the effective height of the array is no longer the product of element pattern and array factor. Besides, the polarization properties depend on the single radiator and on the geometry of the array.

Moreover, many engineering applications require a more and more sophisticated design of the radiation pattern, given in terms of the amplitude of the co-polar and cross-polar components. The phase pattern is not constrained and is available as a further degree of freedom to improve the design. However, the corresponding problem is not linear, and the least square method leads to the minimization of a functional, which can have several local minima. For this kind of problems, algorithms like the iterative least square [3], generalized projection [4] and adaptive beamforming [5], based on Newton method, can be trapped in spurious solutions.

In last years, stochastic algorithms, like simulated annealing [6] and genetic algorithm [7], and deterministic algorithms, like mean-field neural network [6] have been extensively developed. They are more reliable and robust to the spurious solutions, although more time consuming.

The matching points technique and the least square method lead to the minimization of an objective functional, which represents the square of the distance between the required and actual radiation pattern. Due to the complexity of this functional, we have used a discrete version [8] of a reliable algorithm, based on Vidyasagar mean-field neural network [1].

In section II, we apply the least square method and matching point technique to the synthesis problem. A brief description of Vidyasagar mean-field neural network is given in section III. Finally, in section IV, some numerical simulations show the performances of the proposed algorithm.

II. Synthesis Problem Formulation

We consider a conformal array antenna of N elements. $\vec{h}_n$ is the effective height (radiation vector) of the array when only the n-th antenna is excited by a unitary current. In a linear system, the effective height $\vec{h}$ of the overall array can be written as a linear combination of vectors $\vec{h}_n$, i.e.:

$$\vec{h} = (h^{(co)}, h^{(cr)}) = \sum_{n=1}^{N} I_n (h_n^{(co)}, h_n^{(cr)}) = \sum_{n=1}^{N} I_n \vec{h}_n \quad (1)$$

where the symbols $^{(co)}$ $^{(cr)}$ indicate the copolar and crosspolar components, respectively, and I_n are the unknown excitation coefficients normalized to a unitary current. Given the design requirement on the amplitude $\vec{f}_d$ of the radiation vector $\vec{h}_d$, the synthesis problem is to find the N-dimensional complex vector $\vec{I}$ and/or geometrical parameters that make the array radiation pattern as close as possible to the required one. In order to solve this problem, we have considered the following functional equation:

$$\sum_{n=1}^{N} I_n (h_n^{(co)}, h_n^{(cr)}) = (h_d^{(co)}, h_d^{(cr)}). \quad (2)$$

By applying the matching point method in the M directions $\{(\theta_m, \phi_m)\}_{m=1}^{M}$, (2) can be rewritten as:

$$\sum_{n=1}^{N}(Z_{m,n}^{(co)}, Z_{m,n}^{(cr)})I_n = (h_{d,m}^{(co)}, h_{d,m}^{(cr)}) \tag{3}$$

where:

$$Z_{m,n}^{(co)} = h_n^{(co)}(\theta_m, \phi_m) \qquad Z_{m,n}^{(cr)} = h_n^{(cr)}(\theta_m, \phi_m)$$

$$h_{d,m}^{(co)} = h_d^{(co)}(\theta_m, \phi_m) \quad h_{d,m}^{(cr)} = h_d^{(cr)}(\theta_m, \phi_m)$$

Least squares formulation leads to the minimization of the functional f:

$$f(\vec{I}, \psi_d^{(co)}, \psi_d^{(cr)}) = \sum_{m=1}^{M} \alpha_m^{(co)} | \sum_{n=1}^{N} Z_{mn}^{(co)} I_n - f_d^{(co)} e^{j\psi_d^{(co)}} |^2 + \\ + \sum_{m=1}^{M} \alpha_m^{(cr)} | \sum_{n=1}^{N} Z_{mn}^{(cr)} I_n - f_d^{(cr)} e^{j\psi_d^{(cr)}} |^2 \tag{4}$$

where α are the weights of the functional, $(f_d^{(co)}, f_d^{(cr)})$ and $(\psi_d^{(co)}, \psi_d^{(cr)})$ are amplitude and phase of the radiation pattern, respectively.

(4) can be rewritten as:

$$f(\vec{I}, \psi_d^{(co)}, \psi_d^{(cr)}) = \sum_{i=1,N} \sum_{j=1,N} I_i^* I_j A_{i,j} - \sum_{i=1,N} \sum_{r=1,M} \\ \alpha_m^{(co)} (Z_{mi}^{(co)} I_i)^* f_d^{(co)} e^{j\psi_d^{(co)}} + \alpha_m^{(cr)} (Z_{mi}^{(cr)} I_i)^* f_d^{(cr)} e^{j\psi_d^{(cr)}} \tag{5}$$

where

$$A_{i,j} = \sum_{m=1}^{M} \alpha_m^{(co)} (Z_{mi}^{(co)})^* (Z_{mj}^{(co)}) + \alpha_m^{(cr)} (Z_{mi}^{(cr)})^* (Z_{mj}^{(cr)}) \tag{6}$$

and * is the conjugate operation. In (5), the constant term is neglected, since it is irrelevant in the minimization procedure. Note that the functional f is not quadratic, since it depends exponentially on $2M$ phases of the required radiation pattern.

III. Mean-Field Neural Networks

The non-linear mean-field neural network, introduced by M. Vidyasagar [1], is described by the following system of differential equations:

$$\begin{cases} \dfrac{du_i}{dt} + u_i = -\dfrac{\partial H}{\partial s_i} \\ s_i = g(\beta u_i), \quad i = 1, \cdots, Nb \end{cases} \tag{7}$$

where $\vec{u}$ and $\vec{s}$ denote the potential and the state of the net, respectively, $H(\cdot)$ is a multilinear polynomial, $g(\cdot)$ is a strictly increasing C^∞ threshold function such that:

$$\lim_{x \to -\infty} g(x) = 0, \quad \lim_{x \to \infty} g(x) = 1, \\ g(0) = g'(0) = 1/2, \quad g'(x) \geq 0 \tag{8}$$

β is a positive real parameter, which is analogous to the inverse of the temperature in the annealing physical phenomenon [9], and Nb is the number of neurons. An important property of mean-field neural networks is that the stationary solution of (7) converges to the local minima of Λ [6]:

$$\Lambda(\vec{s}) = H(\vec{s}) + \sum_{i=1}^{Nb} \frac{1}{\beta} \int^{s_i} g^{-1}(s) ds \tag{9}$$

which is a Lyapounov function for (7). It has been showed in [1] that for $\beta \to \infty$ the state $\vec{s}$ of the net converges to a local minimum[1] of $H(\vec{s})$. In [8], it has been proposed a discrete version of Vidyasagar mean-field neural net:

$$\begin{cases} u_i^{(n+1)} = (1-\Delta) u_i^{(n)} + \Delta \dfrac{\partial H}{\partial s_i} \\ s_i^{(n+1)} = g(\beta u_i^{(n+1)}) \end{cases} \tag{10}$$

which has the same properties of the continuous one, if $0 \leq \Delta \leq 1$.

IV. Numerical Simulations

In order to apply the neural net, we must turn the functional f into a multilinear polynomial, with the same global minimum. This can be accomplished by expanding the feeding currents and the phases of the required radiation pattern in terms of boolean variables, i.e.:

$$I_i = |I_0| \sum_{b=0}^{b_1 - 1} 2^{-b} [(2s_{i,b} - 1) + j(2\tilde{s}_{i,b} - 1)] \\ e^{j\psi_r} = e^{j2\pi \sum_{b=0}^{b_2-1} 2^{-b} p_{r,b}} \tag{11}$$

where $|I_0|$ is a constant current, j is the imaginary unit and $s, \tilde{s}, p$ are boolean variables. For these unknowns, it is banal to verify that:

$$s_{i,b}^2 = s_{i,b} \qquad \tilde{s}_{i,b}^2 = \tilde{s}_{i,b} \\ e^{j\psi_r} = \Pi_{b=1}^{bit} \left[1 + p_{r,b}(e^{j2\pi 2^{-b}} - 1)\right] \tag{12}$$

Finally, substituting (11) in (5) and resorting to (12), we obtain the unique multilinear functional H, which verifies the following equality:

$$H(\vec{s}, \vec{\tilde{s}}, \vec{p}) = f(\vec{s}, \vec{\tilde{s}}, \vec{p}) \\ \forall\, \vec{s} = \{s_{1,1}, \cdots, s_{1,b_1}, \cdots, s_{N,b_1}\} \in \{0,1\}^{b_1 N} \\ \forall\, \vec{\tilde{s}} = \{\tilde{s}_{1,1}, \cdots, \tilde{s}_{1,b_1}, \cdots, \tilde{s}_{N,b_1}\} \in \{0,1\}^{b_1 N} \\ \forall\, \vec{p} = \{p_{1,1}, \cdots, p_{1,b_2}, \cdots, p_{2M,b_2}\} \in \{0,1\}^{2b_2 M}. \tag{13}$$

[1] Note that local minima belong to the set $\{0,1\}^{Nb}$

The design requirement on the radiation vector is given by means of the lower and upper masks for the amplitudes of the copolar and crosspolar components. Since the allowable masks are, in fact, a set of functions, it is always possible to find one that is closer to the pattern, which can be radiated by the given array. Of course, the best approximation is obtained projecting the pattern, radiated by the array, on the set of masks.

Now, we focus the attention on the choice of the parameters $\{\alpha_r\}_{r=1}^{M}$ and β. We assume that the vector $\{\alpha_r\}_{r=1}^{M}$ is equal to one, in order to use the same weights for each point of the required radiation pattern. It is known, from annealing physical phenomenon and from convergence theory of the simulated annealing [9], that the parameter β must increase slowly like the logarithmic function. However, in practical applications the more efficient strategy, from the computational point of view, is obtained with a geometrical progression [6]. In accordance with [8], we have chosen this initial value for β:

$$\beta_{min} = \left\{ max_i \sum_{j=1}^{N} \left| \frac{\partial^2 H}{\partial s_i \partial s_j} \right|_{s=1/2} \right\}^{-1} .$$

The proposed procedure can be summarized as follows:
- Step 1) Given the element patterns, calculate the A matrix.
- Step 2) Evaluate the multilinear polynomial H.
- Step 3) Apply the discrete version of Vidyasagar mean-field neural network, in order to minimize H.
- Step 4) Using the projection method, find the mask closer to the radiation pattern, produced by the array.
- Step 5) If the effective height is not accurate enough, return to step 2.

First, we tested the algorithm on a conformal array antenna demonstrator, manufactured at the TNO-Physics and Electronics Laboratory. The demonstrator consists of an array of 65 open ended waveguides (WR90), mounted on a circular cylindrical structure. The radius of the cylinder is 20.97λ (wavelength) at the frequency 9GHz. The array consists of 5 rows of 13 elements, spaced 1.26λ (vertical interelements distance) and $.98\lambda$ (horizontal interelements distance). In our simulation, the upper and lower masks are rectangular windows 6^0 and 3^0 wide, respectively. In fig.s 1-2, we show the amplitude and phase of the radiation pattern obtained with the proposed algorithm. Fig.s 3-4 show the effective height for the iterative least square method [3]. Although the required radiation pattern is constrained to be different from zero only in a very narrow azimuthal window (6^0), the neural network algorithm achieves a good performance.

In order to stress the role of the phase, we considered the design of a uniform radiation pattern (the lower and upper masks are constants) for an array of 135 elements placed on the same cylinder of the previous example. Fig.s 5-6 show a better performance of the proposed algorithm with respect to the iterative least square method (fig.s 7-8), in terms of reduction of the mean square error ($\approx 6dB$).

V. Conclusions

In this paper, we have proposed an algorithm based on the mean-field neural network and we have compared it with the iterative least square method, presented in [3]. Numerical simulations have shown that the proposed method is reliable and robust to spurious solutions, but it is time consuming.

In the first example, the required radiation pattern is significative only in a narrow azimuthal window. Although the phase distribution has few degrees of freedom, the neural network algorithm achieves a good performance. Of course, the reduction of actual unknowns leads to a functional less complicated, and as a result it decreases the number of local minima. In this case, the iterative least square synthesis, based on Newton method, has a good chance of success.

Whereas, in the second instance, the amplitude distribution is uniform in the azimuthal plane and many more degrees of freedom on the phase distribution are therefore available. In this case, the two algorithms find different solutions, and the iterative least square method is, in fact, trapped in a local minimum. The better performance of the proposed algorithm with respect to the iterative least square synthesis is showed by a reduction of $\approx 6dB$ of the mean square error.

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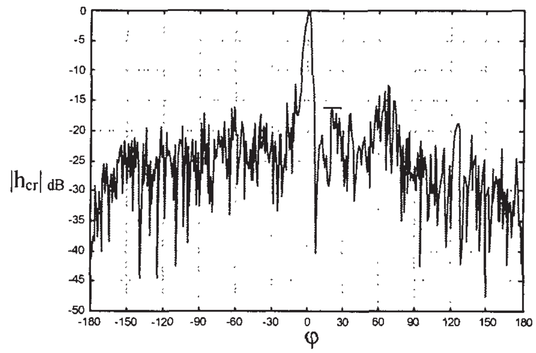

Fig. 1 – Amplitude pattern using mean-field neural net synthesis.

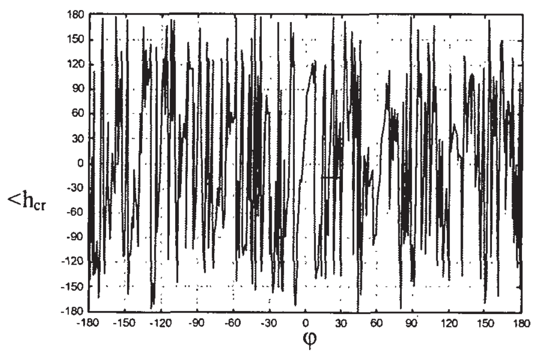

Fig. 2 – Phase pattern using mean-field neural net synthesis.

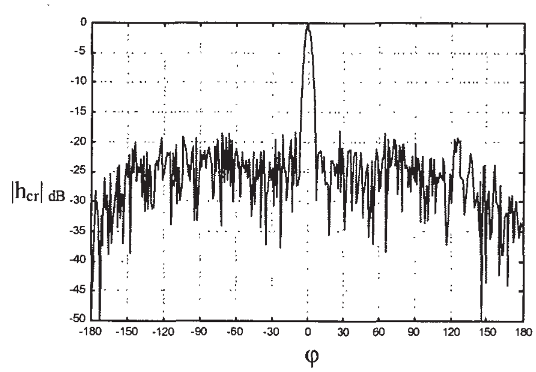

Fig. 3 – Amplitude pattern using iterative least square synthesis.

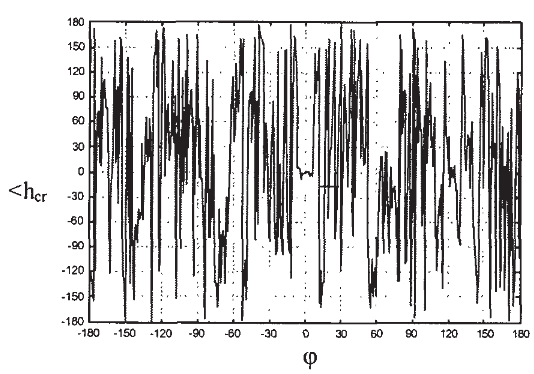

Fig. 4 - Phase pattern using iterative least square synthesis.

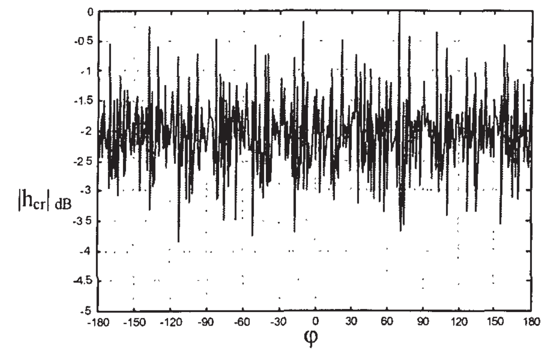

Fig. 5 – Amplitude pattern using mean-field neural net synthesis.

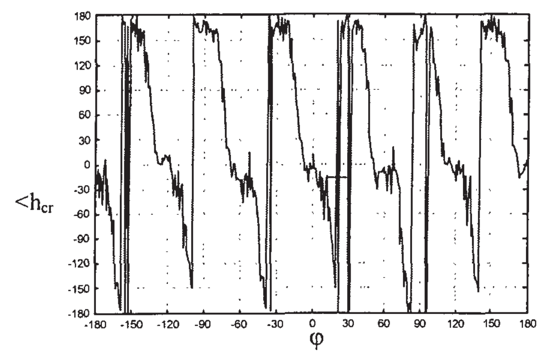

Fig. 6 – Phase pattern using mean-field neural net synthesis.

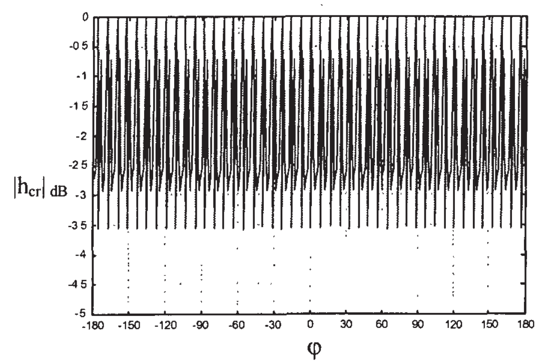

Fig. 7 - Amplitude pattern using iterative least square synthesis.

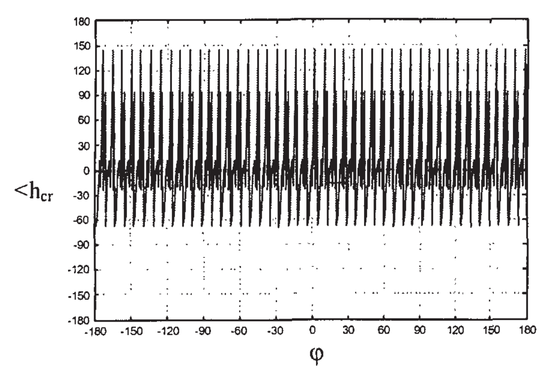

Fig. 8 – Phase pattern using iterative least square synthesis.