INTERPOLATION ALGORITHM FOR FAST EVALUATION OF EM COUPLING BETWEEN WIRES

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Abstract: Efficient evaluation of the field radiated by a current flow along a wire is essential to solve electromagnetic coupling between wires. This paper describes a numerically efficient interpolation algorithm for the evaluation of coupling terms. Using standard routines, the field generated by a given wire is evaluated on a discrete grid of points within the observation region that contains the mantle of the coupled wires and interpolated. For computational efficiency, the function to be interpolated is smoothed by subtracting a properly defined term. The proposed algorithm is applied to enhance efficiency in the computation of coupling matrix elements for the analysis of mutual coupling between PEC wires discretized by the Method of Moment.

1. INTRODUCTION

The computation of electromagnetic coupling between arbitrary oriented wires is based on an evaluation of the electric field radiated by a current flowing along a single wire that induces a current on one or more other wires.

Focusing on the analysis of scattering from an arbitrary oriented wire, an Electric Field Integral Equation (EFIE) is solved by applying the Method of Moments (MoM) and has been successfully used [1]. However, even if efficient techniques are employed for the computation of impedance matrix elements of a single wire [2], [3], the evaluation of coupling matrix elements requires much more CPU time than the other parts of the program. The same problem has been encountered in the analysis of electromagnetic scattering from 3D objects where several techniques have been proposed to reduce the computational cost [4], [5].

This paper describes an efficient method to reduce CPU time needed. The electric field radiated by a current flowing along a wire can be sampled on a non–uniform grid defined in a way to follow the appropriate behavior [6] and then interpolated [7]. In order to further accelerate the generation of coupling matrices and to control the accuracy, a modified interpolation technique is applied to smoothen the interpolated function from which a properly chosen term is extracted.

2. FORMULATION

The mutual coupling between two arbitrary oriented wires can be described by considering the radiated field from one wire as an incident field on the other wire. Firstly we consider the current induced along a single thin wire by an impressed voltage and/or by an incident plane wave. Secondly, we derive an expression for the electric field radiated by such induced current distribution.

Single wire. We start considering the well–known Pocklington's equation with the Exact Kernel formulation [1]–[3] in the frequency domain for a single PEC wire in free space. By applying the Galerkin Method of Moment [8], with rooftop functions, the equation can be express as $\underline{\underline{\mathbf{Z}}}\underline{\mathbf{I}} = \underline{\mathbf{F}}_e$. The vector $\underline{\mathbf{F}}_e$ represents the weighted known excitation due to external sources, $\underline{\mathbf{I}}$ is the unknown current distribution vector and $\underline{\underline{\mathbf{Z}}}$ is a symmetric Toeplitz matrix known as "system impedance" matrix.

In order to find the radiated electric field by a straight thin wire, we write down the general expression of the electric field as a function of the magnetic vector potential as follows

$$\mathbf{E} = \frac{1}{j\omega\epsilon}\left(k^2\mathbf{A} + \nabla\left(\nabla\cdot\mathbf{A}\right)\right), \qquad \mathbf{A}(\mathbf{r}) = \int\limits_{z'=0}^{L}\int\limits_{\varphi'=0}^{2\pi}\frac{I(z')}{2\pi a}\frac{\exp\left(-jkR\right)}{4\pi R}\mathbf{i}_z a d\varphi' dz', \tag{1}$$

In particular, since the current is expanded in terms of N rooftop basis functions $B_i(z)$, the radiated electric field can be considered as a sum of N separate electric field contributions radiated by a current along the i–th wire segment and weighted by coefficient terms I_i, hence

$$\mathbf{E}(\mathbf{r}) = \sum_{i=1}^{N} I_i\mathbf{E}_i(\mathbf{r}), \qquad \mathbf{E}_i(\mathbf{r}) = \frac{1}{j\omega\epsilon}\frac{1}{8\pi^2}\int\limits_{z'=(i-1)\Delta z}^{(i+1)\Delta z} B_i(z')\int\limits_{\varphi'=0}^{2\pi}\frac{\exp(-jkR)}{R^3}\left\{-\left[(jkR)^2 + jkR + 1\right]\mathbf{i}_z\right.$$
$$\left. + \quad \left[3 + 3jkR + (jkR)^2\right]\frac{(\mathbf{r}-\mathbf{r}')}{R}\frac{(z-z')}{R}\right\} d\varphi' dz'. \tag{2}$$

Note the double integration representing electric current flow on the surface of the wire.

Mutual Coupling. In the following we will discuss the mutual coupling between two wires in the configuration depicted in Fig.1–(a). We suppose that two perfectly conducting thin wires, wires $m = 0$ and $m = 1$ with their own local coordinate system are divided in N^m segments. This may easily be generalized for the case of more than two wires. To describe the mutual coupling, we consider that a current distribution along segment i of wire $1-m$ radiates an elementary electric field $\mathbf{E}_i(\mathbf{r})$ as in Eq. (2). This field impinges on the other wire m and induces a current along each of its segments. The total incident field on the ℓ–th segment of wire m can be written as the sum of two parts

$$\mathbf{E}_\ell^m(\mathbf{r}) = \mathbf{E}_{ext}^m(\mathbf{r}) + \mathbf{E}_i^{m,1-m}(\mathbf{r}). \tag{3}$$

in which $\mathbf{E}_{ext}^m(\mathbf{r})$ denotes the field due to external sources and $\mathbf{E}_i^{m,1-m}(\mathbf{r})$ the induced field on wire m due to the current flowing along the i–th segment of wire $1-m$, with $m = 0, 1$. By applying the Galerkin MoM procedure to this problem, the system matrix equation $\underline{\underline{\mathbf{Z}}}\underline{\mathbf{I}} = \underline{\mathbf{F}}_e$, is elegantly extended as follows

$$\begin{bmatrix} \underline{\underline{\mathbf{Z}}}^0 & -\underline{\underline{\mathbf{C}}}^{0,1} \\ -\underline{\underline{\mathbf{C}}}^{1,0} & \underline{\underline{\mathbf{Z}}}^1 \end{bmatrix}\begin{bmatrix} \underline{\mathbf{I}}^0 \\ \underline{\mathbf{I}}^1 \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{F}}_e^0 \\ \underline{\mathbf{F}}_e^1 \end{bmatrix}, \tag{4}$$

in which each known vector on the right–hand side represents the weighted field of external origin. The diagonal "self matrices" $\underline{\underline{\mathbf{Z}}}^0$, $\underline{\underline{\mathbf{Z}}}^1$, represent the interaction between segments of the same wire, while, the off–diagonal "coupling matrices" $\underline{\underline{\mathbf{C}}}^{0,1}$, $\underline{\underline{\mathbf{C}}}^{1,0}$, describe the interaction between segments of different wires.

3. EFFICIENT MATRIX ELEMENTS EVALUATION

Elements of self matrices and known vectors are efficiently evaluated as explained in [3]. Our attention is here focused on the computation of coupling matrix elements. Each matrix element involves two times a double integral evaluation. Consider the (ℓ, i)–th element of matrix $\underline{\underline{\mathbf{C}}}^{0,1}$. To simplify notation, we will place the radiating segment i in the center of the cylindrical coordinate system (ρ', φ', z') as shown in Fig.1–(b). A first double integration has to be carried out to evaluate the field $\mathbf{E}_i^1$ radiated by a current distribution along the i–th segment of wire 1, see Eq. (2). An additional double integral has to be computed to determine the mutual interaction between this field and the induced current distribution along the ℓ–th segment of wire 0. This kind of straightforward calculation is a time–consuming process since many integrals are involved. Thanks to the rotational symmetry of $\mathbf{E}_i^1$, the observation region, which contains the mantle of wire 0, can be defined in a plane (ρ', z') as in Fig.1–(b). In order to accelerate the coupling matrix elements computation, by making use of standard routines, we modify a numerical integration routine [6] to generate a set of points used in a subsequent interpolation step [7]. In this way, the routine will choose the distribution of points according to the behavior of the field. Further efficiency is gained by considering a function difference between this field and the electric field $\mathbf{E}^p$ radiated by a point dipole placed in the origin of the coordinate system. Function $\mathbf{E}^p$ behaves asymptotically as $\mathbf{E}_i^1$ and is singular when the distance R vanishes. Thanks to these properties, the resulting function $\mathbf{D}_i^1 = \mathbf{E}_i^1 - \mathbf{E}^p$ has a behavior smoother than $\mathbf{E}_i^1$ and is determined in a numerically easier way with a controlled accuracy.

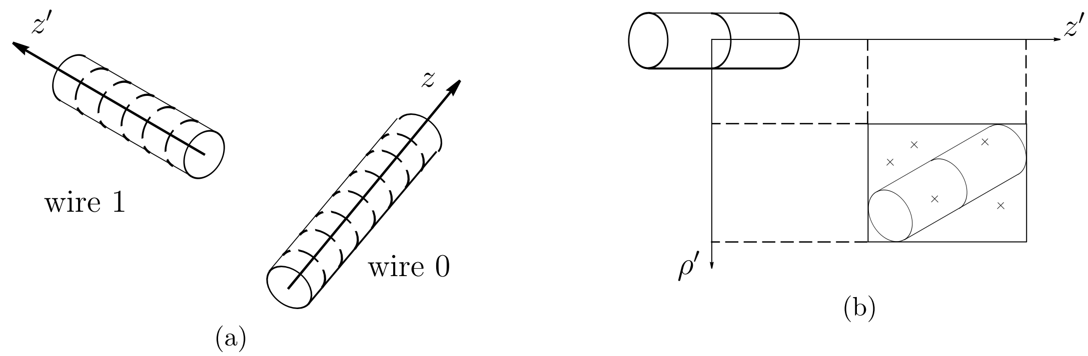

Figure 1: (a)–*Wires geometry;* (b)–*Interpolation area for em interaction evaluation.*

4. CONCLUDING REMARKS

The paper describes a novel interpolation algorithm that relies on non–uniform polar grid sampling of a properly defined function extracted from the electric field radiated by a wire current density. Within a prescribed accuracy, coupling matrix elements can be computed more efficiently than by straightforward double integrations. During the presentation, the two methods will be compared by means of some numerical test-cases and conclusions will be drawn.

5. REFERENCES

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