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Reliability based structural design

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ABSTRACT: According to ISO 2394, structures shall be designed, constructed and maintained in such a way that they are suited for their use during the design working life in an economic way. To fulfil this requirement one needs insight into the risk and reliability under expected and non-expected actions. A key role in this respect is played by the structural reliability analysis (SRA). In this paper the present state of the art will be summarised, including the simplifications to semi-probabilistic calculations as being used in daily practice. Although in principle the adopted Bayesian reliability approach should be able to take care of all uncertainties involved, present practice still uses other safety concepts like Robustness Design and Quality Assurance as tools for achieving the safety objectives of design and assessment.

Keywords: Risk based decisions, structural reliability, time variant and time invariant models, structural system reliability, uncertainty modelling, code calibration

1 INTRODUCTION

Structural reliability analysis (SRA) comprises a set of methods and models that can be used for the probability and risk based decision making with respect to design and assessment of structural systems. The widest application may be found in building, civil and offshore engineering, sometimes explicit but in most cases in the form of so called semi probabilistic procedures.

The core business of SRA is the estimation of the lifetime (or annual) failure probability for a given structure. As input the calculation procedure requires structural behaviour models as well as a probabilistic description of all relevant actions, material properties and geometrical parameters. The establishment of these models themselves is in fact already an essential part of the SRA. Failure may be defined with respect to all kinds of structural performances, but usually it is related to collapse or to the violence of serviceability limits.

One should realise that the calculation of the failure probability is not a goal is itself. The final goal is to make decisions with respect to the design of new structures and inspection or repair programs for existing ones. This means that we also need to have insight into the consequences of failure, the corresponding risks and cost optimisation. This final goal also determines the choice of Bayesian probability theory as the basis of SRA. We will return to this statement later on in this paper.

2 SRA IN A NUTSHELL

The response of a structural system depends on the loading characteristics and the geometrical and material properties of the structure itself. Two main categories of structural responses may be distinguished: the desired state and the adverse or undesired state. The boundary between the two is referred to as the limit state. A structure being in the undesired state is considered as having failed. Given the scatter and uncertainties in the various loading and structural parameters as well as in the models, we may derive the probability of failure according to:

$$P_f = P(g(X) \le 0) = \int_{g(x)\le 0} f_X(x)\,dx \qquad (1)$$

where $\boldsymbol{X}$ is the vector of basic random variables, $g(\boldsymbol{x})$ is the limit state function for the failure mode considered and $f_X(\boldsymbol{x})$ is the joint probability density function of the random variables $\boldsymbol{X}$. The limit state function $g(\boldsymbol{x})$ is defined in such a way that negative values correspond to failure and positive values to non-failure.

Instead of the failure probability P_f the reliability of the structure may also be expressed by means of the so called reliability index β, which may be obtained from:

$$\beta = -\Phi^{-1}(P_f) \qquad (2)$$

where Φ (..) is the standard normal cumulative distribution function. For β in the range from 1 to 4 the approximation $P_f = 10^{-\beta}$ is adequate and serves the purpose of getting a first impression of the relation between these two equivalent measures of reliability.

In many SRA-techniques all random variables $\boldsymbol{X}$ are transformed into the so called U-space, that is a set of independent normal variables with zero mean and unit standard deviation. If the variables X_k are independent, the transformation may be performed by:

$$\Phi(U_k) = F_{Xk}(X_k) \tag{3}$$

for every variable X_k. For a Gaussian distributed variable, the relation between X_k and U_k is simply given by $X_k = \mu_k + U_k\, \sigma_k$. In case of dependent variables the transformation is more complex and usually referred to as the Rosenblatt transformation (Rosenblatt, 1952, Hohenbichler and Rackwitz 1981).

The integral (1), of course, can be solved by straightforward numerical integration. This, however, requires a large number of so called "limit state function evaluations". If the limit state function $g(\,)$ is a simple 'one liner', we may still be able to handle problems with say a maximum of 7–10 random variables. For more complex limit state functions, requiring the call of large computer codes, the method may hardly be feasible. An alternative is Monte Carlo simulation. This method at least is not sensitive to the number of random variables, but also requires a large number of limit state function evaluations in case of small probabilities. Some special Monte Carlo techniques exist to reduce this problem (Rubinstein, 1981). Examples are Directional Sampling, Importance Sampling, Latin Hypercube sampling, possibly in combination with response surface techniques. However, for large systems this still may be cumbersome.

Another way to reduce the calculation time is provided by first and second order approximations (FORM, SORM) that have become very popular in SRA (Hasofer and Lind, 1974; Hohenbichler and Rackwitz, 1983). Standard FORM is based on a linearization of the limit state function in the U-domain and SORM provides a local second order correction (Breitung, 1994). The method works very well for linear or almost linear failure surfaces in the U-domain. FORM usually is formulated as an optimisation problem. The reliability index β, found by the method can be interpreted as the minimum distance in the U-space from the origin to the limit state equation $g(\boldsymbol{u}) = 0$:

$$\beta = \min \| \boldsymbol{u} \|, \quad \text{sub } g(\mathbf{u}) = 0 \tag{4}$$

The point in the U-space where the minimum occurs is called beta point or design point. It has to be found using an iterative calculation scheme. In the case of n variables every calculation step requires $(n + 1)$ limit state function evaluations (one at the design point and one for finding all derivatives). The number of iterations depend on the type of problem and the calculation method. In the course of time a large variety of possible calculation procedures have been developed and can be found in the literature (see Rackwitz 2001). When using FORM, three types of problems may however be encountered: (1) no convergence is achieved, (2) convergence is achieved but at a local (instead of a global) minimum and (3) convergence is achieved but there is loss of accuracy due to high non-linearity. The first problem at least has the advantage that it is recognised, the other two may go unnoticed.

In some cases it may be advantageous to use a mix of methods, including analytical results. As an example, consider a case where one or two variables are known to be responsible for a heavy non-linear aspect. In that case we may combine the method of numerical integration and FORM by the using Total Probability Theorem:

$$P_f = \int P(g(\boldsymbol{U}) < 0 \mid u_n))\; \varphi(u_n)\, du_n \tag{5}$$

In this approach we may derive the conditional probabilities $P(g(\boldsymbol{U}) < 0 \mid u_n)$ using FORM for a series of u_n-values.

3 TIME DEPENDENCY

The parameter time plays an important role in reliability engineering. Practically all statements on the reliability of a structure are meaningless without making reference to a certain period of time. It is helpful to state this explicitly in the problem statement and to reformulate (1) as:

$$P_f = P(\min\{g(\boldsymbol{X}, t)\} < 0) \tag{6}$$

The elements of $\boldsymbol{X}$ may refer to random variables as well as to time dependent random processes. The minimisation is performed over the period of consideration, say $0 < t < T$. The time dependency may be the result of variation in the loads, but also of various degradation processes in the structure.

Loads on a structure may have short and long scale fluctuations. Slowly varying, long term fluctuations may represent changes over hours of even years. In SRA the slow component is often modelled as a time series of constant blocks (FBC processes). Fast and short scale mechanical load fluctuations are often modelled as Poisson pulse processes or continuous Gaussian processes. The short term fluctuations may cause dynamic effects in the structure. In that case spectral analysis techniques are often used to deal with them. In some load models both types of fluctuations may be present. Figure 1 shows a schematized wind load as the sum of a slowly changing hourly mean vale and a fast fluctuating Gaussian gust process.

Also the resistance is usually not constant in time. In most cases structural properties may deteriorate under the influence of (random) mechanical, chemical, physical or biological influences. Typical examples are fatigue (mechanical), corrosion (chemical) and freezing-thawing (physical) mechanisms. In all these cases it is preferred to have physics based models for the deterioration processes to feed the limit state functions in the reliability analysis. Many deterioration processes, however, are so complex that no physics based models exists. In that case a classification system of structural states as indicated in Fig. 1 could be useful. Markov models may be applied to describe the transition probabilities from one state to another in an empirical way. Another option is to formulate an empirical function for the conditional failure rate h (see next section).

One popular option to elaborate (6) is by discretizing the time axis and looking to the survival probability:

$$\begin{aligned} 1-P_f &= P(g(t_1) > 0 \cap g(t_2) > 0 \cap \ldots.) \\ &= P(g(t_1) > 0)\, \Pi\{P\{g(t_i) > 0 \mid \cap g(t_j) > 0\} \end{aligned} \quad (7)$$

where i = 2 … n, j runs from 1 to (i – 1) and n is the number of time intervals. Within one time interval, one may consider the g-functions as being constant in time. By taking load parameters on their maximum and resistance parameters on their minimum an approximation on the safe side is obtained. The result of (7) can be further developed into:

$$P_f(0,T) = 1 - \exp\left\{-\int_0^T h(\tau)d\tau\right\} \quad (8)$$

where $h(\tau)\, d\tau$ is the probability to fail in the interval (τ, $\tau + d\tau$), conditional upon no failure before that interval. In terms of the discrete time intervals and limit state functions this comes down to:

$$h(t_i) = \lim P(g(t_i) < 0 \mid g(t_j) > 0 \text{ for } 0 < t_j < t_i)/\Delta \quad (9)$$

with $\Delta = t_i - t_{i-1}$. The function h is referred to as the conditional failure rate; in many engineering mechanics applications it is often taken as a constant value: $h = \lambda$.

Cramer and Leadbetter (1967) showed that if the random vector process $\mathbf{X}(t)$ is a sufficiently mixing process, the above expressions may be approximated by replacing the conditional failure $h(\,)$ rate by the outcrossing rate $\nu(\,)$:

$$\begin{aligned} P_f(0,T) &= 1 - \exp\left\{-\int_0^T \nu(\tau)d\tau\right\} \\ \nu(t_i) &= \lim P(g(t_i) < 0 \mid g(t_{i-1}) > 0)/\Delta \end{aligned} \quad (10)$$

Note that (10) usually is obtained by assuming that individual outcrossings are independent events. If failure at the start of the period ($t = 0$) is also taken into account we arrive at:

$$\begin{aligned} P_f(0,T) &= P_f(0) \\ &\quad + [1 - P_f(0)]\left(1 - \exp\left[-\int_0^T \nu(\tau)d\tau\right]\right) \end{aligned} \quad (11)$$

The outcrossing approach is quite accurate in case of ergodic processes. So when substantial non ergodic elements are present (e.g. the resistance) one can better take care of them separately. The same holds for slowly varying processes. So, for instance, in the case of windloading we may have:

$$P_f = E_R\{P_f(0) + (1 - \exp n\, E_Q(1 - \exp(-\int \nu_{RQ})\, d\tau))\} \quad (12)$$

where E_R and E_Q indicate expectations over the constant variables $\boldsymbol{R}$ and the intensities $\boldsymbol{Q}$ of the slowly varying processes respectively and ν_{RQ} is the

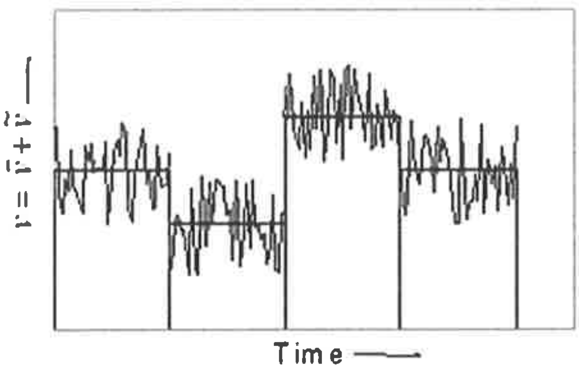

Figure 1. Example of the sum of an FBC Block process and a Gaussian process.

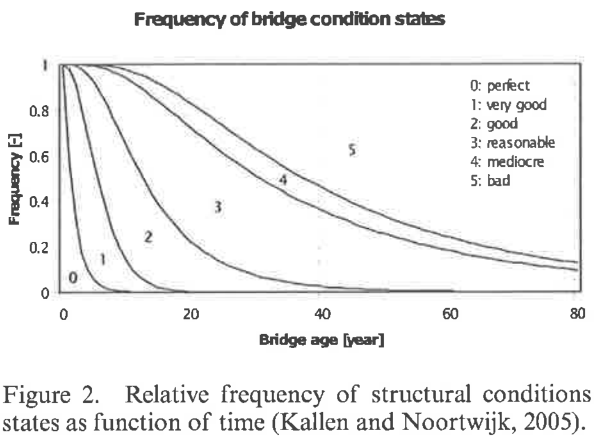

Figure 2. Relative frequency of structural conditions states as function of time (Kallen and Noortwijk, 2005).

outcrossing rate of the fast fluctuating gust process conditional upon **R** and **Q**. The integration runs over the duration T_s of an individual FBC block (single storm, sea state..) and $n = T/T_s$ is the number of FBC-blocks.

4 SYSTEM FAILURE

In reliability theory a system failure is a failure defined as a combination of events and conditions, using logic operators like AND and OR. Structural system behaviour is often represented using elementary parallel and series models, fault trees, event trees, failure trees, Bayesian Networks, and so on.

The distinction between a single mode failure and a system failure however is often very subtle. Consider for instance a series system of two elements 1 and 2, for which we have:

$$\text{Failure} \equiv \{g_1 < 0 \cup g_2 < 0\} \tag{13}$$

One may now simply introduce the function $g = \min(g_1 < 0, g_2 < 0)$ and claim that this is a single mode limit state function. It would not help to "forbid" the use of "min" of "max" operators, because we can easily produce analytical functions, continuous and infinitely many times continuously differentiable, but showing effectively the same behaviour as (13).

Monte Carlo and Numerical Integration can easily take system complications on board, but FORM-procedures are not giving appropriate results in the case of strong system effects, hidden or not. In those cases one needs to address the system effects explicitly. The general procedure is that as a start for all individual modes a FORM analysis is performed, resulting in a vector of reliability indices β_i and a matrix of influence coefficients α_{ij}. Based on these results a treatment of the system effects is possible. Methods for this type of System Analysis are widely available in the literature (Stevenson and Moses 1973, Ditlevsen 1979, Hohenbichler and Rackwitz 1983). One of the advantages of such an explicit system procedure compared to direct calculation is that valuable intermediate results become available.

In the case of continuum structural systems (beams, plates, slopes) the notion of one or more dimensional random fields enters the game. For relative simple problems the outcrossing approach mentioned before, but then in space domain, may be of value. For more complex systems the stochastic FEM (Finite Element Method) may be applied. Many publications exist, below we give a short overview on the basis of (Karadeniz et al, 2004).

In the linear elastic case the focus is on the derivation of mean values and the covariance matrix of the response (displacements, strains and stresses). In fact this is comparable to the mean value approximation of the FORM family. Suitable methods based on perturbation techniques are outlined in (Sudret and Der Kiureghian, 2000, Chakraborty and Bhattacharyya, 2002). In the Neumann series expansion (Haldar and Mahadevan, 2000) the random stiffness matrix is split up into its mean and deviatory parts as:

$$K = K_0 + K_\theta \tag{14}$$

where K_0 is the stiffness matrix associated with the mean value of the structural properties and K_θ is the stiffness matrix which contains deviatory part of the random properties. The resulting displacement vector **w** may be found from a recursive solution of:

$$K_0\, \mathbf{w}^{(i)} = \mathbf{F} - K_\theta\, \mathbf{w}^{(i-1)} \tag{15}$$

In deriving the solution one should keep track of the correlation between the (random) load vector **F** and displacement vector **w**.

For the nonlinear structural behaviour a variety of options exist. (Morutso, (1983), Gierlinski et al (1991), Guiterez (1999)). Very popular is the use of the Response surface method (Bucher and Bourgund, 1990): the idea is that the limit state function is generated for a number of selected points in the *u*- or *x*-space. One option is to set up the response surface first and do the reliability calculations afterwards. Another option is to develop the surface as a part of the reliability procedure and to adapt the set of selected points in order to get better and more accurate results. An advanced and interesting calculation scheme by mixing Directional Sampling and an Adaptive Response Surface Technique (DARS) has been proposed by Waarts (2000). He proved that the total number of actually needed limit state function evaluations may be as low as $7n$, where n is the number of random variables.

Clearly, at the operational edge of SRA is the combined nonlinear dynamic analysis. The spectral approach is usually restricted to linear systems but linearization techniques for nonlinear frequency domain analysis are available (Robert and Spanos, 1990, Schuëller et al, 1991). Also time domain analysis is an option, again in combination with FORM or Monte Carlo. In the latter case an interesting reduction in calculation time may be obtained if use is made of so called Constrained Simulation (Harland et al, 1999), which may be conceived as a kind of importance sampling.

5 INSPECTION AND MAINTENANCE

Once a structure has been built, it can be inspected. In order to combine the data from the measurements with the original data, Bayesian updating procedures can be used. In principle two procedures for updating are available (JCSS, 2000). In the first procedure one updates the probability density distributions of all random variables and then recalculates the probability of failure. One may also directly update the probability of failure. The first procedure is more informative, but sometimes more complex.

An essential point in the updating procedure is the reliability of the inspection method itself. First of all there is the Probability of Detection (PoD), for instance in the case of inspecting fatigue cracks in steel structures. For many techniques PoD curves are available but not for all. At present this is a typical omission in the reliability assessment of existing structures. A second important piece of information is the accuracy of the inspection method: if we measure a fatigue crack of 3 mm, could in reality this also be 2 mm or 4 mm? The same holds for most non-destructive test methods estimating the strength of materials.

Apart from explicit inspections, all kinds of observed behaviour of the structure should be included in the updating analysis like permanent deformations, settlements, cracks, loose elements, corrosion and so on. It should be realized that these phenomena may require a double treatment in the SRA: (1) it directly affects the structural model as such (e.g. a cracked beam instead of a non-cracked beam as a starting model) and (2) a change in the statistical models of possibly all random variables: the presence of a crack might for instance may be an indication that the load is higher than originally expected.

Given the extra data we may update the probability of failure or the reliability index beta. Figure 3 gives an example of the original and

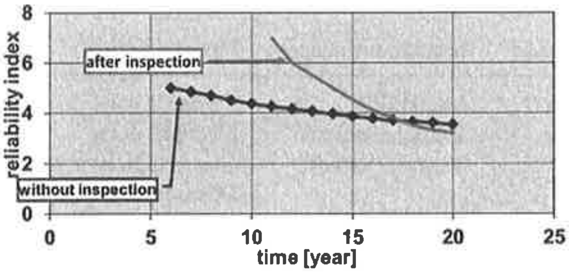

Figure 3. Effect of (fatigue) inspection at year 10 on the annual reliability index beta (example with disappointing inspection result leading to a lower reliability index after some time).

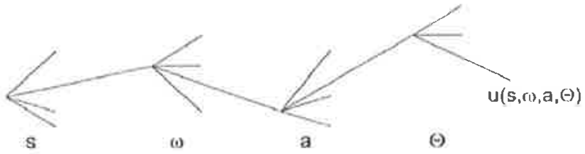

Figure 4. Decision–event tree for inspection planning.

updated reliability index after inspecting a crack in a steel structure, both as functions of time. It is interesting to see that the reliability index first increases, because of reduced uncertainty, but later on (in this example) becomes lower than the original one, because of a somewhat disappointing measurement result.

The ultimate test is the proof load. Although it looks an easy concept, it is recommended to consider carefully the uncertainties still present.

Performing inspections and processing the results is a costly matter. So one should find out which inspections are worth the effort and which are not. The theoretical tool to support those decisions is the so called Preposterior Bayesian Analysis (JCSS, 2000). Preposterior means that we try to find out beforehand whether the result will be profitable or not. As the outcome of the inspection beforehand is random, we can only optimise on the basis of expectations. The idea is often presented in the form of an event-decision tree as in Figure 4. We first have to choose an inspection plan or strategy "s". Once we have chosen our strategy, nature will come with an inspection result "ω". Given this inspection result we have to choose an action "a" (do nothing, repair option A, B, C.., reduce the use, demolish the structure, etc.) and finally this may result in costs (or utilities) "u" depending on the state of nature "θ". For instance, we may choose to check some strength parameter, decide (in case of a positive result) to do nothing, but have a failed structure in the end after all. Taking the expectations over all random outcomes (ω,θ) and the optimum over all decisions (s,a) we arrive at the optimal strategy. In practice this might prove to be a tedious procedure, but even when using a more pragmatic procedure, it always pays to keep the principles in mind. A simple example is: try in advance to find out which outcome is necessary to change a decision and estimate how likely such an outcome is. The profit then should outweigh the product of inspection costs and the probability of a good outcome.

6 TYPES OF UNCERTAINTY

In the previous sections we discussed the elaboration of equation (1). The structural models and probability distributions were assumed to be given and correct. But, how sure can we be about these models themselves. We might easily forget some important action or mechanism and we may have very limited technical and statistical information.

For the sake of discussion we will split the hazards (actions or mechanisms) into the following three categories:

- Foreseeable and dealt with.
- Known in principle, but unrecognized or ignored.
- Unknown or unforeseeable.

Table 1 gives an overview of the category of foreseeable hazards in structural engineering. The list is not claimed to be complete, if such completeness would ever be possible. In principle similar lists for degradation and collapse mechanisms exist on the resistance side. Information on the most relevant (deterministic) load and resistance models can be found in codes and text books. Information on the statistical modelling of the variables in those regular types of loads and resistances may be found in the literature or data bases and in particular in the JCSS Probabilistic Model Code (JCSS, 2001).

However, all models are approximations of reality for which reason we have to introduce the notion of so called model uncertainties and statistical uncertainties Given the usually present substantial lack of substantial information to come to objective quantifications, a frequentistic probability in SRA interpretation is not meaningful. Therefore, in SRA the Bayesian intuitive degree of belief approach is generally adopted as the basis to step forward (Ditlevsen, 1988). The advantage of this approach (above other options) is that treating aleatory and epistemic uncertainties in the same way opens the possibilities to combine them in a coherent probabilistic decision making processes. Further-more, when new or additional information becomes available, it can easily be incorporated using the concepts of Bayesian updating.

If we move to Column 4 of Table 1, we see a list of human influences, actions that are not accidental but deliberate. These actions are extremely difficult to model as in general the aim is to get a load higher than the resistance. We will not discuss them here. Finally the last column shows the various types of human errors. Methods of human reliability analysis (HRA) are under progress but a meaningful interaction between SRA and HRA is still far away. In structural design procedures Quality Assurance and SRA, until now, live next to each other a separate life.

In addition to the uncertainties and errors in the treatment of the known and recognised hazards we must face the fact that certain phenomena may be completely overlooked or, until now, objectively unknown (the so called black swans). Let us first observe that the completely black swans are very seldom. More important, but essentially not very different, are the forgotten, neglected or underestimated hazards or mechanisms. To deal with those issues the notion of robustness has been developed.

Technically speaking, robustness is related to scenarios where due to unintentional or unforeseen exposures the resistance of the structural system has been reduced. An illustration is presented in Figure 5 (from Eurocode EN 1991-1-7). Due to an exposure of any nature (a), local damage (b) may occur. Given the direct or local damage the structure may survive or (partly) collapse. Robustness requirements are especially related to the step from (b) to (c), i.e. to avoid that a local damage, regardless its origin, develops to total collapse (Faber, 2011).

Estimating some reasonable number for the pattern and probability of the initial damage may give the opportunity to bring robustness design again within the scope of structural optimisation.

Table 1. Overview of foreseeable actions.

Normal loads (including tail values)	Accidental/natural	Accidental/manmade	Human influences	Human Errors
Self-weight	Earth-quake	Internal explosion	Vandalism	Design error
Imposed loads	Landslide	External explosion	Demonstrations	Material error
Car park loads	Hurricane	Internal fire	Terrorist attack	Construction error
Traffic	Tornado	External fire		Misuse
Snow	Avalanche	Impact by vehicle etc.		Lack of maintenance
Wind	Rock fall	Mining subsidence		Miscommunication
Hydraulic	High ground-water	Environmental attack		
	Flood			
	Volcano eruption			

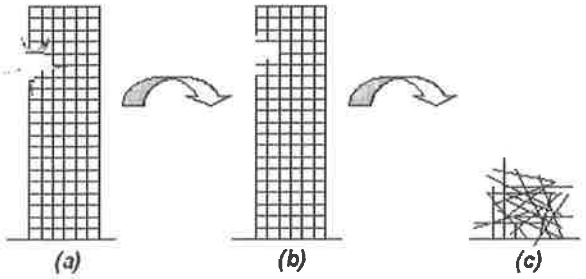

Figure 5. Illustration of the basic concepts in robustness.

7 TARGETS AND OPTIMISATION

According to ISO 2394, structures shall be designed, operated, maintained and decommissioned such as to support societal functionality. Mathematically expressed one may state that one should aim at the optimisation of the total benefits, incorporating all relevant socio-economic cost items. In the elementary case one may search for the minimum of the sum of direct building costs and the risk:

$$C_{tot} = C_b + P_f\, C_f \tag{16}$$

Such an optimisation may be performed over the intended lifetime of the structure, but one should keep in mind that the design lifetime of the structure may be an object of optimisation in itself.

Note that although the crisp limit state concept is very dominant in structural analysis (both deterministic and probabilistic), one may also think of applications where the distinction between desired and undesired states is more gradual/diffuse or where the consequences of failure depend heavily on X. In those cases we may want to combine the probability and risk calculations, leading to:

$$C_{tot} = C_b + \int C(x) f_x(x) dx \tag{17}$$

In many cases the function $C(\mathbf{x})$ is so complex that FORM methods for reliability analysis cannot be applied and only the more time consuming Numerical integration or Monte Carlo will work. One interesting exception is if C is a monotonically increasing function from zero to one. In that case the integral can be interpreted as a convolution and a corresponding artificial limit state function can be formulated, enabling FORM to solve the case (Gollwitzer, 2004).

Optimising the costs for a given fixed design lifetime leads approximately to the result that the optimal life time failure probability is independent from the length of the design working life. (Holický, 2012). This means that the economically

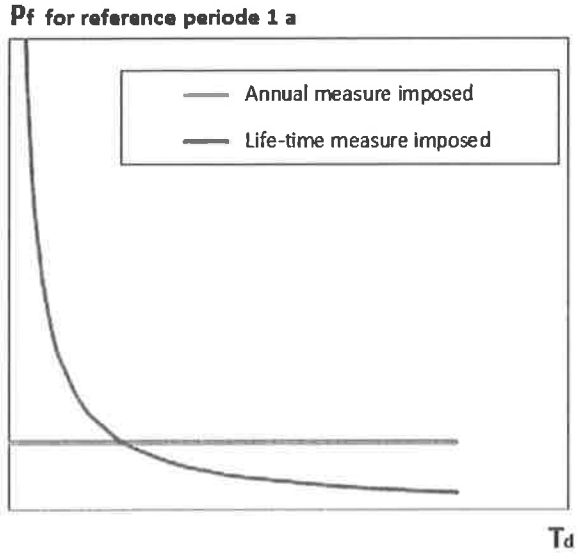

Figure 6. Schematic relationship between the annual and lifetime failure probability Pf and the design working life (from Bigaj, 2013).

optimal annual failure probability is high for short periods and low for longer design lives. Spending money on safety makes sense if one can have a longer period to profit from it.

When human safety is at stake, one should add an amount of money to the damage costs (either a formal compensation value or some real economic value). Techniques exist to make proper estimates (Nathwani, J. et al, 2009). However, there may also be ethical or legal reasons to consider limits on human safety. Usually they will be formulated as a maximum acceptable value per year. So, the target curves may be presented as in Figure 6: for short design periods the annual value form human safety is governing, for longer periods the economic optimum.

Eurocode EN 1990 gives a standard value of $\beta_t = 3{,}8$ ($P_{ft} = 8 \cdot 10^{-5}$) for a design working life $T_d = 50$ a. The target value of 3,8 is raised to 4.3 for high failure consequences and lowered to 3,3 for less consequences. The value does not depend on the structural costs involved to reach a higher safety level as theory demands. Also no information is given as to which criterion (economy of human safety) is the governing one and no guidance on what to do for shorter or longer design life time. This indicates that in the world of code makers this is still an unsolved problem.

8 CODIFICATION USING SEMI-PROBABILISTIC METHODS

Most codes of practice offer the reliability requirements in the form a Partial Factor (or similar) format, where well defined characteristic values for loads and resistances are combined with corresponding partial factors, combination factors, importance factors, and so on. To some extent these factors have been calibrated to consequence dependent reliability targets. This is also referred to as the semi-probabilistic method.

The partial factor method is based on the following assumptions:

- strength and loads are independent random variables with known distributions;
- characteristic values of strength and loads (X_k) are defined as specified fractiles of the respective distributions.
- uncertainties are taken into account by transforming characteristic values (X_k) into design values (X_d), by applying partial factors γ (multiplying with load factors, dividing by material factors) or additional elements for geometrical properties;
- the assessment of safety is considered as sufficient if the design action effects do not exceed the design strengths.

A relatively simple way (JCSS, 1996) of deriving partial factors is by first calculating the design or beta point values for each variable according to (assuming normal distributions):

$$X_d = \mu(1 - \alpha \beta_t V) \qquad (18)$$

where μ is the mean value, V is the coefficient of variation, β_t the target reliability index for structure and α the averaged FORM sensitivity factor ($-1 < \alpha < 1$); averaging is over a well-chosen set of representative structural elements. Given for the same variables the characteristic value X_k,

$$X_k = \mu(1 - k V) \qquad (19)$$

(usually with $k = 0$ for actions and $k = 1.64$ for material properties) the partial factor for resistance respectively actions follow from:

$$\gamma = X_k / X_d \quad \text{or} \quad \gamma = X_d / X_k \qquad (20)$$

If loads of different types are involved several load combinations may have to be checked.

In the case of non-normal distributions the formulas become more complex, but the principle remains the same. More advanced methods for receiving partial factors exist. One may for instance minimize the sum of $(\beta - \beta_{target})^2$ for a large set of structures in the area of application (Faber and Sorensen, 2003) or require that the averaged failure probability is less than the target.

9 CLOSURE

In the course of the past half century, Structural Reliability Analysis has developed into an impressive set of computational techniques. High speed computers enable the use of these techniques for quite realistic structures, in particular in a research environment. Daily practice still uses primarily semi probabilistic methods, but good calibration procedures, linking safety factors to reliability requirements, are available.

Although much progress has been made, there still remains important research and development work to be done. To start with, an increased level of user friendliness of presently available computer codes is necessary in order to reach a larger group of engineers. Reliability methods should be coupled to standard engineering calculation software in the same way as is available for partial factor methods. Consider by way of example an engineer who has assessed his structure using the standard code requirements and found that some of the unity checks were not satisfactory. In the ideal case it should then be not more than a relatively simple job to start up a reliability calculation for the same structure and the same limit states. The reliability analysis should use the same structural (FEM) model and automatically transform the semi-probabilistic input into input formulated in terms of means, standard deviations and correlation patterns. The use of data bases like the JCSS Probabilistic model code could be of help. The user, of course, should select methods and strategies of calculation, but even there an expert system could give valuable suggestions.

Next to such integration of semi—and full probabilistic methods, the probabilistic methods themselves still require improvements, both from the theoretical as from the operational point of view. The necessary research involves a wide spectrum of topics, like model uncertainties, degradation processes, fire safety, inspection and monitoring, repair, failure consequences and risk estimates. But also more fundamental issues like human safety considerations, cost optimisation, robustness requirements and interaction between SRA and Quality Assurance will ask in the years to come our full attention.

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