

# Antenna Diagnostics using Near Field Techniques

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## Abstract

The radiation characteristics of an antenna are fully determined by its aperture distribution. Measured Near Field data gives an impression but this is not good enough to detect small anomalies. For good antenna diagnostics, the field at the aperture plane is required.

The Near Field Measurement Technique calculates the Far Field of the AUT is by determining the so called Plane Wave Spectrum (PWS) from the measured Near Field data. Once the PWS is known, the electric field at any location can be calculated. In fact, the Far Field is only a limiting case of the PWS. Using the PWS, the field at the aperture of the AUT can be calculated. This aperture image is much "sharper" than the measured Near Field. Even small anomalies, hardly visible in the measured Near Field data, appear quite clearly. To determine the excitation coefficients of array elements, the Far Field data has to be corrected for the element radiation pattern. Unfortunately, some steps in the processing confine the accuracy of these methods.

## The Plane Wave Spectrum

Wang [1] gives a clear derivation of the Near Field to Far Field transformation. Part of this paper is reproduces here in brief.

The radiation of an antenna is described by the time-harmonic Maxwell equations for a source-free free-space region:

$$\nabla^2 \bar{\mathbf{E}} + k^2 \bar{\mathbf{E}} = 0 \quad (1a)$$

$$\nabla^2 \bar{\mathbf{H}} + k^2 \bar{\mathbf{H}} = 0 \quad (1b)$$

$$\nabla \cdot \bar{\mathbf{E}} = \nabla \cdot \bar{\mathbf{H}} = 0 \quad (2)$$

It can be shown that the following expressions constitute a solution to the above equations:

$$\overline{\mathbf{E}}(x, y, z) = \frac{1}{2\pi} \iint \overline{\mathbf{A}}(k_x, k_y) \cdot e^{-j\bar{\mathbf{k}} \cdot \bar{\mathbf{r}}} dk_x dk_y \quad (3)$$

$$\bar{\mathbf{k}} \cdot \bar{\mathbf{A}} = 0 \quad (4)$$

where  $k_x$  and  $k_y$  are real variables and

$$\bar{\mathbf{k}} = k_x \hat{\mathbf{x}} + k_y \hat{\mathbf{y}} + k_z \hat{\mathbf{z}} \quad (5)$$

$$k^2 = \bar{\mathbf{k}} \cdot \bar{\mathbf{k}} \quad (6)$$

$$\bar{\mathbf{r}} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}} \quad (7)$$

$$\overline{\mathbf{A}}(k_x, k_y) = A_x(k_x, k_y)\hat{\mathbf{x}} + A_y(k_x, k_y)\hat{\mathbf{y}} + A_z(k_x, k_y)\hat{\mathbf{z}} \quad (8)$$

$\bar{\mathbf{A}}$  is called the Plane Wave Spectrum because the expression

$$\overline{\mathbf{A}}(k_x, k_y) \cdot e^{-j\bar{\mathbf{k}} \cdot \bar{\mathbf{r}}} \quad (9)$$

represents a uniform plane wave propagating in the direction  $\bar{\mathbf{k}}$ .

Assume the AUT is placed in the region  $z \leq 0$ .

Expression (3) is then valid for  $z > 0$ .  $\bar{\mathbf{A}}$  is only a function of  $k_x$  and  $k_y$ . Assuming that the frequency and thus  $k$  ( $= 2\pi/\lambda$ ) is a given parameter,  $k_z$  can not be chosen. It has to fulfill (6) and the radiation condition for  $z > 0$ , which means:

$$k_z = \begin{cases} \sqrt{k^2 - k_x^2 - k_y^2} & \text{if } k_x^2 + k_y^2 \leq k^2 \\ -j\sqrt{k_x^2 + k_y^2 - k^2} & \text{otherwise} \end{cases} \quad (10)$$

A negative imaginary  $k_z$  corresponds to an evanescent PWS which is rapidly attenuated away from the  $z = 0$  plane. The area with a real  $k_z$  is called the visible space.

Assume the Planar Near Field measurement plane is located at  $z = z_t > 0$ .  $\overline{E}$  follows from (3a):

$$\overline{E(x, y, z_t)} = \frac{1}{2\pi} \iint \overline{A(k_x, k_y)} \cdot e^{-jk_z z_t} \cdot e^{-j(k_x x + k_y y)} dk_x dk_y \quad (11)$$

This can be written as an inverse Fourier Transform:

$$\overline{E(x, y, z_t)} = F^{-1} \left\{ \overline{A(k_x, k_y)} \cdot e^{-jk_z z_t} \right\} \quad (12)$$

Now it is quite clear how to determine the PWS from the measured Near Field data:

$$\overline{A(k_x, k_y)} = e^{jk_z z_t} \cdot F \left\{ \overline{E(x, y, z_t)} \right\} \quad (13)$$

The Far Field of the AUT is the limit of  $r \rightarrow \infty$  of expression (3). This limit can be expressed as:

$$\lim_{r \rightarrow \infty} \overline{E(x, y, z)} = j \frac{e^{-jkr}}{r} k_z \overline{A(k_x, k_y)} \quad (14)$$

with

$$\begin{aligned} k_x &= k \cdot \sin(\theta) \cos(\varphi) = k \cdot \sin(AZ) \cos(EL) \\ k_y &= k \cdot \sin(\theta) \sin(\varphi) = k \cdot \sin(EL) \\ k_z &= k \cdot \cos(\theta) = k \cdot \cos(AZ) \cos(EL) \end{aligned} \quad (15)$$

In practical situations the PWS as determined by (13) will be the product of the PWS of the AUT and the probe antenna. Probe correction is a common technique in Near Field measurements. However, it has consequences for the calculation of the aperture field as will be shown later.

The aperture field at  $z = 0$  follows from (3):

$$\begin{aligned} \overline{E(x, y, 0)} &= \frac{1}{2\pi} \iint \overline{A(k_x, k_y)} \cdot e^{-j(k_x x + k_y y)} dk_x dk_y \\ &= F^{-1} \left\{ \overline{A(k_x, k_y)} \right\} \end{aligned} \quad (16)$$

Expression (16) clears also why this aperture field calculation is often called "backtransformation". Note that the PWS in (16) should be the PWS of the AUT only!

An example of the measured Near Field and the corresponding backtransformed aperture field is shown in figure 1.

## Array Element Excitation

The Far Field of an array antenna is the product of the element radiation pattern and the array factor. When the PWS of an element is given by  $\overline{P_{elem}}$ , then the total PWS of the array is:

$$\overline{A(k_x, k_y)} = \overline{P_{elem}(k_x, k_y)} \cdot C(k_x, k_y) \quad (17)$$

The array factor  $C$  of a rectangular array is given by expression (18):

$$C(k_x, k_y) = \sum_m \sum_n c_{mn} \cdot e^{j(mk_x d_x + nk_y d_y)} \quad (18)$$

$d_x$  and  $d_y$  are the element spacings in the  $x$ - and  $y$ -direction.

This expression can be rewritten as a Digital Fourier Transform (DFT):

$$\begin{aligned} C(k_x, k_y) &= \frac{2\pi}{d_x d_y} \frac{1}{2\pi} \sum_m \sum_n c_{mn} \cdot e^{j(mk_x d_x + nk_y d_y)} d_x d_y \\ &= \frac{2\pi}{d_x d_y} \cdot \text{DFT} \{ c_{mn} \} \end{aligned} \quad (19)$$

The element excitation factors can be calculated from the array factor by inverse DFT:

$$c_{mn}(md_x, nd_y) = \frac{d_x d_y}{2\pi} \cdot \text{DFT}^{-1} \{ C(k_x, k_y) \} \quad (20)$$

It looks as if the array factor  $C(k_x, k_y)$  can be derived easily from (17) as

$$C(k_x, k_y) = \frac{\overline{A(k_x, k_y)}}{\overline{P_{elem}(k_x, k_y)}} \quad (21)$$

but the quotient of two quantities is not necessarily a scalar. A pragmatic way to tackle this problem is to restrict (21) to the co-polar component.

More details about this technique, including a sensitivity analysis for most parameters, e.g. the S/N-ratio of the Near Field data, can be found in [2]. An example of a comparison of the backtransformation and a direct measurement is shown in fig. 2.

## Practical problems

There are some problems, inherently connected with the theory described above. In the FFT-processing the sampling spacing and the spectral extends are related as follows:

$$\begin{aligned} k_x &\in [-k_{xm}, k_{xm}] & \text{with } k_{xm} &= \pi/d_x \\ k_y &\in [-k_{ym}, k_{ym}] & \text{with } k_{ym} &= \pi/d_y \end{aligned} \quad (22)$$

The spectral spacings and the spectral extends are also related:

$$\begin{aligned} x &\in [-x_m, x_m] & \text{with } x_m &= \pi/dk_x \\ y &\in [-y_m, y_m] & \text{with } y_m &= \pi/dk_y \end{aligned} \quad (23)$$

The number of sampling points in both the spectral and spatial domain are equal in FFT-processing. Let there be  $N_x$  and  $N_y$  sampling points in the x- and y-direction. Then the spacings and extends are related as

$$\begin{aligned} 2x_m &= N_x d_x \\ 2y_m &= N_y d_y \\ 2k_{xm} &= N_x dk_x \\ 2k_{ym} &= N_y dk_y \end{aligned} \quad (24)$$

In Near Field measurements the sampling spacing is chosen near to  $\lambda/2$ . From (22) it follows that the k-space is limited by  $[-k, k]$  in both directions. With this sampling spacing, the k-space encloses the whole visible space as defined by (10). However, the corners of this k-space are outside the visible space ( $k_x^2 + k_y^2 > k^2$ ). This is the region of the evanescent waves which don't exist anymore in the Far Field. Besides that, the Far Field of the probe antenna is measured in an anechoic chamber so there is no knowledge about spectral components outside the visible space. In practice, one can not do anything else than eliminate the spectral components outside the visible space. From a mathematical point of view, this is a multiplication with a window- or block-function. The theory of Fourier Transforms learns that a multiplication in one domain results in a convolution in the other domain. The 1-dimensional case is well-known. The transform of a block-function is then a  $\sin(x)/x$ -function. Those

who are familiar with measurements on a HP8510 NWA probably have seen this phenomenon in the time-domain. A band-limited frequency measurement can be described as a product of a block-function and a frequency-unlimited response. In the 2-dimensional case, the transform of a circular block-function is a first order Bessel-function.

In this way the backtransformed field at the aperture plane is influenced by values of the points around it. Of course, the same happens in the calculation of the element excitations conform (20). For elements, spaced at  $\lambda/2$ , the interaction of neighboring elements is -15 dB [2].

Since the evanescent waves don't exist anymore in the Far Field, these cannot be present in the backtransformed field. The backtransformed field is only an image of the radiating field components, not of the total field at the aperture.

Another practical problem arises in the calculation of the element excitations. When the spacing of the elements is not equal to the sampling spacing of the Near Field data, the k-space has to be resampled to fulfill the relations of (22-24). With the interpolation, involved with the resampling, some accuracy is lost.

There are methods to overcome these problems. First, one can use the fact that the convolution function is known. One can try to calculate a deconvolution. This requires quite some computational power since one has to deal with large matrices. However, similar techniques have been used in optics with defocused pictures. Sometimes it is also described as super-resolution technique. Another approach is to apply a beam synthesis approach. Originally, this technique was developed to calculate a best-fitting set of excitation coefficients to approximate a prescribed field, given the array configuration and the element radiation patterns. This technique uses an iterative scheme in which only the forward transform is used. The difference between the calculated field and the prescribed field is used to adapt the excitation coefficients. This technique too requires quite some computational power. By using the measured Far Field as the prescribed field, this technique can be used to determine the excitation coefficients. At TNO-FEL such a program is developed [3] but up

to now not applied to Near Field measurements. The program has to be adapted to the k-plane format of the Near Field calculations. The number of points might be a problem.

### Conclusions

As the total Far Field of an AUT is known, the radiating field at any place can be calculated by means of backtransformation. For antenna diagnostics the aperture field is very interesting. Planar Near Field measurements are most suitable for this technique because the data formats match the processing requirements. When the element radiation pattern is known, the element excitation coefficients can be obtained in a similar way. This is useful for alignment of active elements in an array. The accuracy is confined by some convolution effects, introduced by the processing.

### References

- [1] J.J.H. Wang, "An Examination of the Theory and Practices of Planar Near Field Measurement", IEEE Trans. AP., Vol. AP-36, no. 6, June 1988, pp. 746-753.
- [2] M.R. van der Goot, A.P.M. Zwamborn, "Reconstruction of Array Antenna Excitation Coefficients using Planar Near-Field Measurements", TNO-FEL report no. 93A229, April 1994.
- [3] A.P.M. Zwamborn, "Iterative Synthesis of the Radiation Pattern for Array Antennas consisting of Non-identical Elements", TNO-FEL report no. 94A300, March 1994.

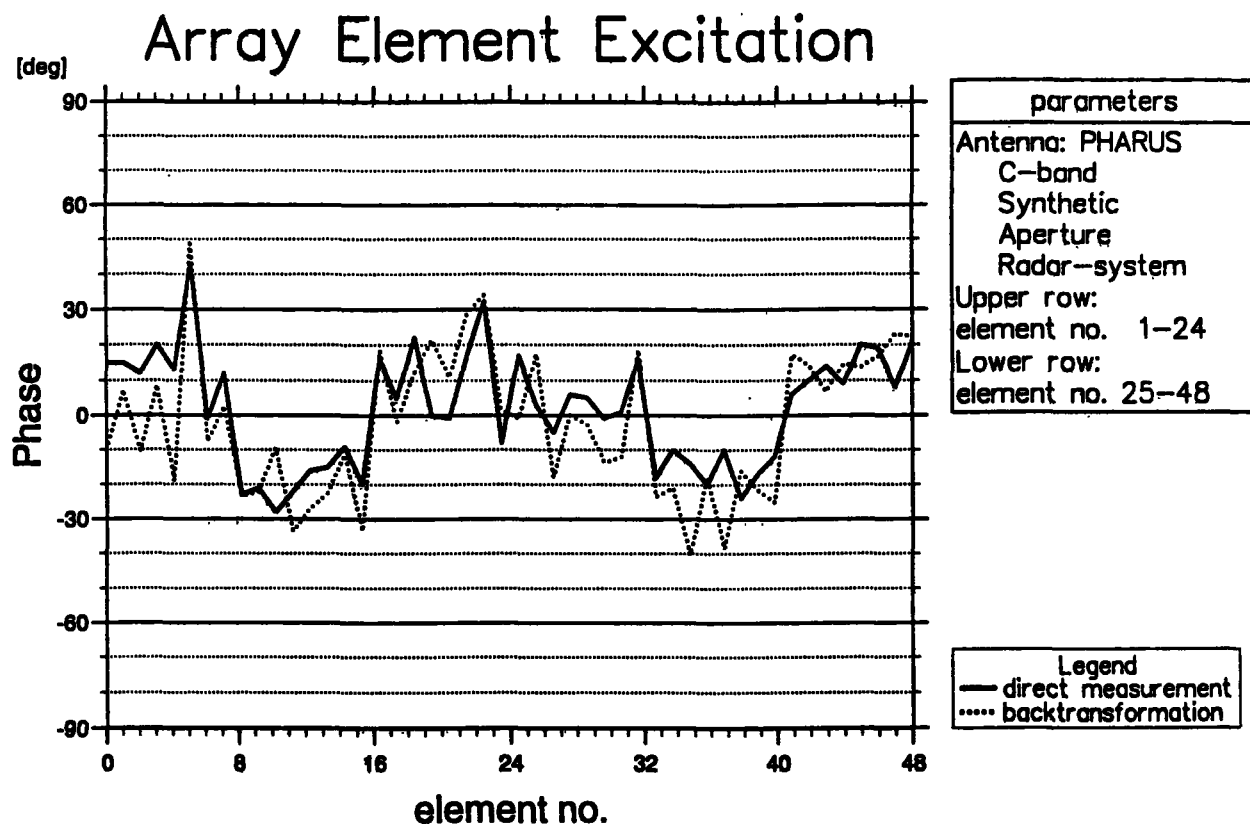
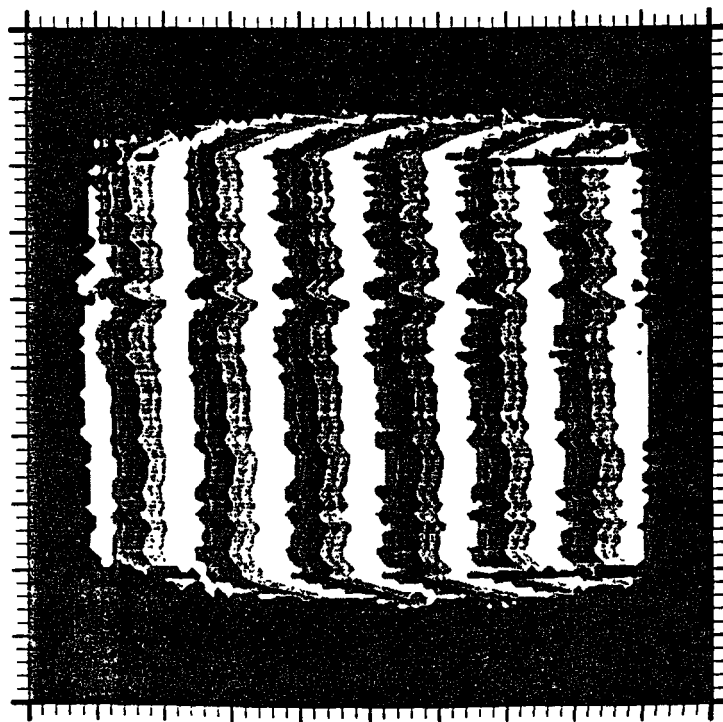
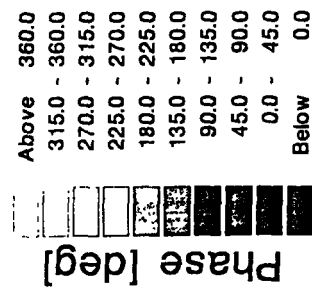


Fig. 2 Example of the determination of the Array Element Excitation by means of backtransformation. The data is compared to a direct measurement of the excitations on the modules. The antenna is a Phased Array SAR antenna, consisting of 2 rows of 24 elements.

# Measured Near Field Data



parameters
Antenna Under Test
Phased array of slotted waveguides
4 of 64 ferrite phase shifter DRIVERS removed
NOTICE: leaky waveguide flange at right side



# Aperture Field Distribution

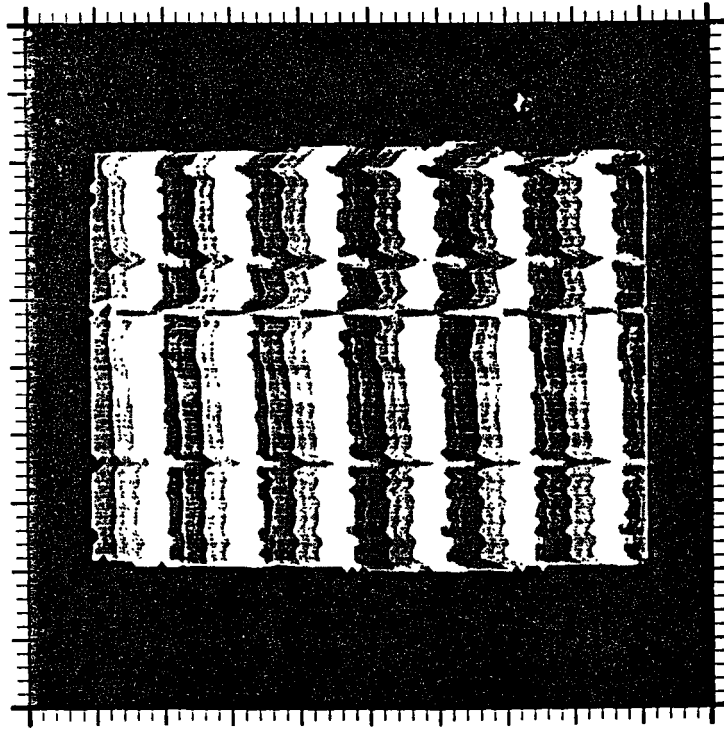
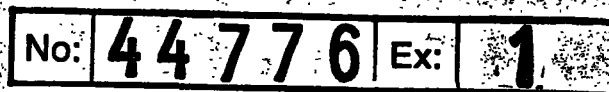
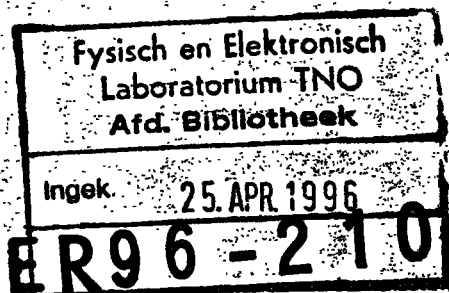


Fig. 1 Example of measured Near Field data and the corresponding backtransformed aperture field. The antenna is a Phased Array, consisting of 64 vertical slotted waveguides, each fed by a ferrite phase shifter at the bottom. The phase shifters are fed by a horizontal waveguide underneath them. The input is at the right side of the antenna at approximately 20% of the bottom. The drivers of 4 phase shifters were disconnected. Those 4 waveguides still radiate, but with the wrong phase.

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