A MODEL FOR DETERMINING CONDITION-BASED MAINTENANCE POLICIES FOR DETERIORATING MULTI-COMPONENT SYSTEMS

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Abstract

We discuss a method to determine strategies for preventive maintenance of systems consisting of gradually deteriorating components. A model has been developed to compute not only the range of conditions inducing a repair action, but also inspection moments based on the last known condition value so as to minimize expected costs per time unit. In our cost optimization, we include costs of inspection, repair, failure and operation, while accounting for savings that can be obtained by combining inspection or repair of two or more components. Deterioration is modelled as a stochastic process; an independence assumption allows us to model the decision problem as a (semi-)Markov decision process. The solution procedure for the system as a whole is a heuristic one as we use only aggregate information about the other components while determining a maintenance strategy for a particular component. This strategy is of the opportunistic control-limit type: upper limits induce mandatory actions, lower limits allow anticipatory action if a combination with a mandatory action on at least one other component is possible. In this paper we describe the principles of this heuristic. We show how other performance indicators can be calculated besides costs and give a numerical example of a system of components made of concrete.

1. Introduction to the problem

We focus on the maintenance of structures or equipment which perform specific functions on a permanent basis and which consist of one or more components. One might think of a bridge with components made of steel, concrete, etc. and mechanical components. We investigate each component individually, and thus do not take any technical dependencies between components into account. We do wish, however, to model economic dependencies explicitly. The cost of maintenance actions

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consists of set-up costs and component-specific costs. If actions involving different components are combined, the set-up costs can be charged only once. This would be possible in the case of inspection and repair. An inspection has the character of a technical inspection and reveals the exact condition of the inspected component. Depending on the degree of deterioration, it may be followed by a repair. Repair returns the component to the new condition and may be considered, therefore, as equivalent to replacement. Component failure involves damage costs dependent on the duration of failure, and additional costs charged only once per failure. Operating costs for properly functioning components (perhaps including regular visual inspection) may depend on the actual (but possibly unknown) condition. The process of deterioration is modelled as a stochastic process. Research results from the field of civil engineering show that some well-known deterioration processes can be described by explicit mathematical functions with parameter values that should be provided by the expert but can be updated using information obtained by successive inspections.

The problem is to determine inspection and repair strategies for each component of the system so as to minimize total expected maintenance costs of the system per time unit, taking into account discounts on inspection and repair costs should maintenance actions for a number of components be combined. An inspection strategy involves deciding when to inspect, depending on the component's last known condition and on opportunities for combining this inspection with that of other components. A repair strategy involves deciding whether or not to repair, depending on the condition found by inspection and on opportunities for combining this repair with that of other components.

The original model and solution algorithm for a one-component system were developed by Tijms & Van der Duyn Schouten (1984), and subsequently improved by Wijnmalen & Hontelez (1992). In Burger & Hontelez & Wijnmalen (1994) more general deterioration processes than the initial one-step Markov chain assumption are introduced into the model. Wijnmalen & Hontelez (1994) presents the theoretical model extension to multi-component systems with discounts on maintenance costs.

2. The model and its solution

For modelling purposes, we divide the components of the system into one or more groups of identical components, called 'component types'. Each type consists of one or more identical components. A specific component, however, can only belong to a single type. We focus on an arbitrarily chosen component of each type; input data, maintenance strategy and all results pertaining to this component are equal to those of any other component of the same type. Maintenance actions on any two or more components may be combined. One might introduce the notion of 'clusters' of types in order to allow for the case that components in different clusters, and, therefore, of specific types, cannot be combined. This does not change the essence of the model, however, and for this reason we shall not consider clusters in this paper.

We divide the (infinite) planning horizon into planning intervals of equal length; one time unit coincides with one planning interval. Opportunities for inspection and repair occur at discrete points in time. In order to avoid scheduling problems, we assume for the multi-component model, unlike the one-component model, that maintenance actions do not take time. As a consequence, the following events can take place without time delay: the decision to inspect, the inspection, the decision to repair based on the inspection result, and the repair action itself.

Furthermore, we divide the range of potential conditions of a component

into intervals. The number of condition levels may be chosen arbitrarily and need not be of equal length. By making the assumption that the increment of deterioration over a given period of time is independent of deterioration in previous periods, we transform continuous mathematical deterioration functions into discrete processes with $r_{ij}(t)$ as the probability of deterioration from condition level i to j during t time units, see also Burger & Hontelez & Wijnmalen (1994).

The state of a component in the decision model is defined by:

- the last known condition level i, with $i=1$ (new);2,...,N (failure),
- and the number m of time units passed since we obtained this knowledge, with $m=0,...,M_i$,
- the opportunity k of a discount $V[k]$ on maintenance costs presenting itself, with $k=1,...,K_R$ or K_I (K_R pertains to repair and K_I to inspection).

The quantity M may depend on i and has been introduced to keep the number of states finite. We assume for the sake of simplicity that repair and inspection costs consist of a set-up part (which can be saved in combined actions) and an action part. When focussing on a particular component, we then have three cases: no combination (no savings), a combination with components of a different type (saving of system set-up costs), and a combination with at least one component of the same type (saving of both system and type set-up costs). This gives us $K_R=K_I=3$. Possible decisions on maintenance actions in relation to a given component are the following:

- leave the component as it is, allowed in states $\{(i,m,k) \mid i=1,...,N\text{-}1;\ m=0,...,M_i\text{-}1\}$
- inspect the component, allowed in states $\{(i,m,k) \mid i=1,...,N\text{-}1;\ m=1,...,M_i\}$
- repair the component, allowed in states $\{(i,0,k) \mid i=2,...,N\}$

We can now formulate the decision problem concerning a single component as a (semi-)Markov decision process. The process is called Markov because the transitions between the states of the decision process depend only on the current state where the decision is taken and on the decision itself. It is a semi-Markov process because the time intervals between decisions are not of equal length (namely, zero or one). We are, in fact, dealing with a finite, irreducible, homogenous Markov chain. Probabilities, costs, and durations of state transitions can be calculated in a straightforward way, see Wijnmalen & Hontelez (1994). The optimal strategy is calculated by applying the policy iteration method using a special search algorithm. Essentially, the method optimizes the expected costs divided by the expected length of a complete "life cycle" (from new condition to new condition through one repair). We have chosen this modelling approach because it offers great flexibility in handling different cost categories, in computing simple but powerful and differentiated maintenance rules for practical application, and in modelling varying levels of detail and types of deterioration processes (notwithstanding the independence assumption mentioned above).

From the above definitions it will be clear that a model encompassing all relevant components of the system at the same time would be far too large to solve within acceptable (or even feasible) memory and/or execution time limits. The number of states would grow exponentially with the number of components encompassed. For this reason, we have developed a heuristic approach based on aggregation and decomposition. We consistently consider each component type (i.e. an arbitrary component of each type) separately, whilst taking into account aggregate information about inspection and repair of the other components. Except when computing total system costs, we regard the system not as a totality but in a decomposed way. The third state parameter k denotes an opportunity for a combination with one or more other components

yielding a discount $V[k]$ on costs. For each k we compute a probability of that opportunity (and thus of the discount $V[k]$) presenting itself, based on the aggregate probability of inspection and repair of other components at a decision moment. The aggregate probabilities are computed per component.

Before describing the solution procedure, we introduce the class of maintenance strategies on which we are focussing. The maintenance strategies that we are considering belong to the class of opportunistic control-limit rules, written as $R=(p_0;p_1,\ldots,p_{p0-1})$, where:
p_0 is the repair rule $(p_0[1],p_0[2],\ldots,p_0[K_R])$; $p_0[1]$ denotes the upper limit value which is the condition level (and worse) where repair is mandatory, $p_0[k]$ denotes the k-th lower limit value where repair is allowed when a combination opportunity with discount $V[k]$ on costs presents itself; thus the range of condition values $1,\ldots,p_0[K_R]$-1 does not induce repair, the range $p_0[K_R],\ldots,p_0[1]$-1 may induce repair if a combination is possible, and the range $p_0[1],\ldots,N$ induces a mandatory repair. Note that $p_0[1]>=p_0[2]>=\ldots>=p_0[K_R]$. p_i is the inspection rule $(p_i[1],p_i[2],\ldots,p_i[K_i])$; as with repair, the first element of this vector is the upper limit inducing mandatory inspection if it is $p_i[1]$ time units since we knew that the condition level was i; the other elements are the lower limit values, expressed in time units, with $p_i[1]>=p_i[2]>=\ldots>=p_i[K_i]$.

Figure 1 shows the flow chart of the solution procedure which we are about to describe. We refer to Wijnmalen & Hontelez (1994) for a formal mathematical presentation of the procedure and its computational models.

As we do not have any information on repair or inspection of components when starting the procedure, the *first step* is to look at each component separately while ignoring the others. As each component type consists of

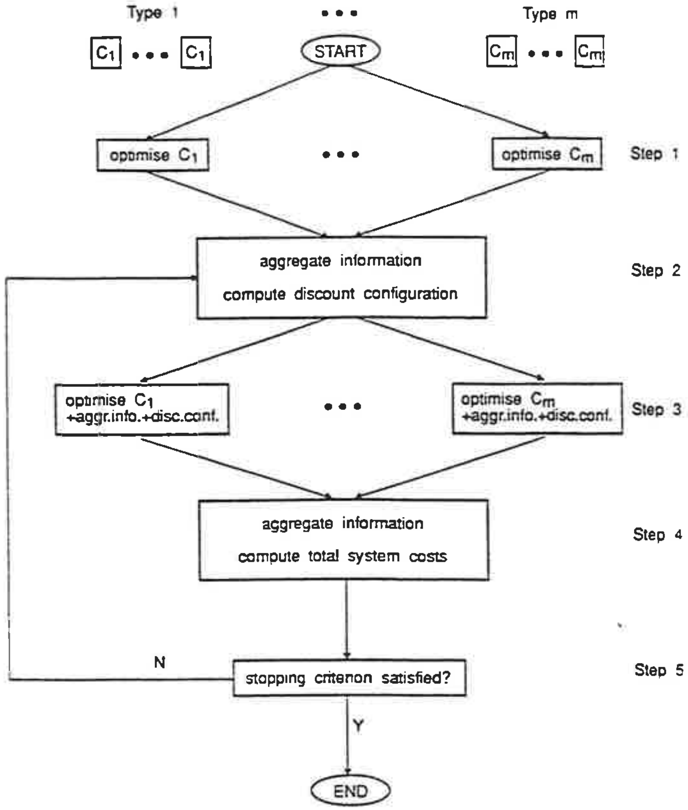

Figure 1: Flow chart of the iterative solution procedure

a number of identical components, we take an arbitrary component of each type in turn and solve the repair and inspection model without discounts for that type. For this, we apply the model described in Wijnmalen & Hontelez (1992), which uses the policy iteration method. The result is an optimal strategy with one repair limit p_0 and p_0-1 inspection limits. Under this strategy, the steady-state probabilities of taking the decision to repair a component and of the decision to inspect a component are computed.

In the *second step*, fixing the attention on an arbitrary component of each component type, the probability of repair of at least one other component

of the same type and then of any other type is computed. This is done using the steady-state repair and inspection probabilities of each component, computed in the first step (or fourth step when iterating). The amounts of discount on repair are equated to the appropriate set-up costs of the component considered: these set-up costs will not be charged when the component considered is repaired, because they are already covered due to the simultaneous repair of other components. The same is done with regard to inspection. The result is four vectors defining the discount configuration per component type: two discount probability vectors (for repair and for inspection) and two discount value vectors (for repair and for inspection).

In the *third step*, each component type is considered again, separately and successively. We start with lower control-limit values equal to the control-limit values ('upper values') in the first step. But this time the discount possibilities are taken into account, and optimal lower-limit vales are computed, again using the policy iteration method.

In the *fourth step*, total system costs are computed. This is essentially a summation of the costs (per unit of time) of the upper and lower-limit strategy for each individual component. One correction is, however, necessary. It follows from the description of step two that set-up costs are discounted once too often, and this needs to be corrected.

Steps two, three and four are repeated up to the point that the stopping criterion of step five is satisfied:

In *step five*, several stopping criteria can be considered: whether the current upper and lower-limit strategy is equal to the previous one (for each component type); whether the relative change in the discount probability vectors is less than some threshold value; whether the relative change in total system costs is less than some threshold value. Which particular criterion is to be used may be left to the user. Changes in the discount probability vectors produce changes in upper and lower-limit values of the strategy for each component, and changes in total system costs. This should, therefore, be the primary criterion. A maintenance planner is, however, primarily interested in strategies and costs and will, therefore, focus on the first and third criterion.

3. Performance indicators

The solution produced by the procedure of section 2 also includes calculation of some additional performance indicators. This is a welcome by-product of our steady-state semi-Markov decision model. We use the value determination step of the policy iteration procedure while observing that the new condition is the renewal state of the process. Given the final inspection and repair strategy of a particular component, we can assign a carefully chosen cost value to each state and thus to the decision taken in that state and interpret this value as a time duration or as an indication of an event which does or does not occur. The value of the average 'costs' per time unit, which is part of the solution of the set of equations in the value determination step, should then be interpreted as the expected time fraction of a repair cycle or as the expected number of events per unit of repair cycle time. We shall give the appropriate cost values per performance indicator. Except for the cost quantities mentioned below, all cost quantities are zero.

Expected length of a repair cycle:

> make all repair costs equal to one; the average 'costs' per time unit from the solution of the set of equations represents the number of repairs per time unit; as there is by definition only one repair per cycle, the reciprocal value is the expected cycle length.

Expected life time:

> make the failure cost per time unit equal to one; the average 'costs' per time unit from the solution of the set of equations represents the duration of failure as a fraction of the length of the repair cycle; one minus this fraction multiplied by the expected repair cycle length yields the expected life time; should repair from the failure condition take time, then the repair costs should in addition be made equal to the duration of repair.

Expected number of inspections during a complete repair cycle:

> make all inspection costs equal to one; the average 'costs' per time unit from the solution of the set of equations represents the number of inspections per time unit of a repair cycle; multiplication by the expected length of the repair cycle yields the expected number of inspections.

Probability of failure during a complete repair cycle (reliability):

> make the repair/replacement costs from the failure condition equal to one; the average 'costs' per time unit from the solution of the set of equations represents the number of failures per time unit, which can be interpreted as the steady-state failure probability.

Availability:

> as time durations of all maintenance actions are assumed to be negligible, availability is equal to expected life time divided by the length of the repair cycle; otherwise, if the component is not 'available' during inspection and repair, make inspection and repair costs equal to their respective time durations, and then divide the average 'costs' per time unit from the solution of the set of equations by the length of the repair cycle.

4. A numerical example

In our example, we focus on a simple bridge for pedestrians. It mainly consists of concrete components. We assume that they can be devided into two groups of more or less identical components. The first component type consists of three and the second component type of four components. Deterioration is due to carbonation. The propagation of this (stochastic) process can be described by the following formula:

$$\text{carbonation depth} = A * \text{sqrt}(t) + b * U * \text{sqrt}(t)$$

where A and b are the parameters of the process, U is the standard

Normal distribution, and t is the time parameter (in years). We give a summary of the input data for both types in table 1; operating costs are zero.

Table 1: Input data for each of two component types

Component-type	1		2	
Number of comp.	3		4	
Parameter values	$A=3.22, b=0.6$		$A=3.22, b=1.0$	
Costs per event:				
Inspection				
total	Dfl. 1200		Dfl. 800	
system set-up	Dfl. 150		Dfl. 150	
type set-up	Dfl. 100		Dfl. 100	
Repair				
total per cond. level	0 - 4 mm	Dfl. 7200	0 - 5 mm	Dfl. 2500
	4 - 8 mm	Dfl. 13000	5 - 10 mm	Dfl. 8000
	8 - 12 mm	Dfl. 16000	10 - 15 mm	Dfl. 10000
	12 - 16 mm	Dfl. 19000	15 - 20 mm	Dfl. 12000
	16 - 20 mm	Dfl. 45000	20 - 25 mm	Dfl. 27500
	> 20 mm	Dfl. 57000	> 25 mm	Dfl. 40000
system set-up	Dfl. 1000		Dfl. 1000	
type set-up	Dfl. 3000		Dfl. 1000	
Failure	Dfl. 350000		Dfl. 100000	

Firstly, we have computed an optimal inspection and repair strategy for each component type separately, without taking into account combinations of repairs or inspections. The results are shown in table 2.

We conclude from table 3 that the discount on repair costs is not so high as to make anticipatory repair profitable. The expected discounts do not offset the otherwise higher costs of an anticipatory repair strategy. By combining inspection of two or more components, however, savings can apparently be obtained. The scale is now turned in favour of an opportunistic strategy. If, for example, a component of type 1 appears to be in condition 2 (4-8 mm) then one should wait 8 years, unless a combination with one or more components of type 2 would present itself

after 7 years (first lower limit).

Table 2: Optimal strategy, costs and performance indicators for each component type, without anticipatory actions

Component type	1		2	
Repair limit	Cond. level 4 (> 12 mm)		Cond. level 4 (> 15 mm)	
Inspection intervals	0 - 4 mm	11 years	0 - 5 mm	15 years
	4 - 8 mm	8 years	5 - 10 mm	12 years
	8 - 12 mm	2 years	10 - 15 mm	7 years
Component costs	Dfl. 1173 per year		Dfl. 567 per year	
Performance indicators:				
Repair cycle	23.5 years		27.0 years	
Life time	23.5 years		27.0 years	
Numb. of inspections	7.0		2.5	
Probab. of failure	$2.32\ 10^{-6}$		$1.01\ 10^{-3}$	
Availability	100%		100%	
Total system costs without savings	Dfl. 5787 per year			
Total system costs with savings	Dfl. 5653 per year			

Table 3: Steady-state opportunistic strategy, costs and performance indicators for each component type

Component type	1		2	
Repair limits				
All limits equal to	Cond. level 4 (> 12 mm)		Cond. level 4 (> 15 mm)	
Inspection intervals				
Upper limits	0 - 4 mm	11 years	0 - 5 mm	15 years
	4 - 8 mm	8 years	5 - 10 mm	12 years
	8 - 12 mm	2 years	10 - 15 mm	7 years
First lower limits	0 - 4 mm	10 years	0 - 5 mm	14 years
	4 - 8 mm	7 years	5 - 10 mm	11 years
	8 - 12 mm	1 years	10 - 15 mm	6 years
Second lower limits	0 - 4 mm	9 years	0 - 5 mm	13 years
	4 - 8 mm	6 years	5 - 10 mm	10 years
	8 - 12 mm	1 years	10 - 15 mm	5 years
Component costs	Dfl. 1072 per year		Dfl. 539 per year	
Performance indicators:				
Repair cycle	38.0 years		26.7 years	
Life time	38.0 years		26.6 years	
Numb. of inspections	22.6		2.7	
Probab. of failure	$2.93\ 10^{-7}$		$6.75\ 10^{-4}$	
Availability	100%		99.6%	
Total system costs with savings	Dfl. 5584 per year			

If a combination with one or more components of the same type 1 would be possible after as early as 6 years (second lower limit) then inspection is already profitable after 6 years. It took three iteration rounds of the solution procedure before the repair and inspection strategies did not change anymore and the first stopping criterion (see section 2) was satisfied. We continued iterating until total system costs per time unit did not change significantly (i.e. change < 0.02%) anymore: the third stopping criterion was satisfied. This took another four iteration rounds. The steady-state probabilities also converge to limiting values, but more slowly (results not shown here).

5. Final remarks

The final solution obtained by this procedure is an approximation of the optimal solution. While the strategy the model produces for a specific component (type) is optimal, given the current values of the discount probabilities (which are updated in each iteration round of the solution procedure) and given the fixed upper limits, the strategy will not necessarily be optimal at system level. This is mainly due to the use of the approximate steady-state probability values. On the other hand, we now have a repair-and- inspection model that suggests strategies for individual components in the context of the whole system. We are now investigating the quality of those strategies. We have found a way out of the dimensionality problem. The size of the model is determined by the number of condition levels, the maximum number of time units until the next inspection, and the number of discount opportunities. The latter can be reduced by adopting a cost structure like the one in this paper (set-up costs to be discounted and action costs). It does not depend, therefore, on the number of components. All in all, we conclude that the model presents a nice application of the policy iteration method, embedded in a heuristic iteration procedure at a higher level. The model results appear to be usable, although one might wish to have a closer link with reality.

Further research will, therefore, deal with (partial) repair as well as replacement, modelling more complex deterioration processes, and with time-dependent planning and scheduling of maintenance.

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