ASAC 2003
Halifax, Nova Scotia

William C. Wedley
Eng Ung Choo
Faculty of Business Administration
Simon Fraser University

Diederik Wijnmalen
Physics and Electronics Laboratory
TNO, The Hague, The Netherlands

BENEFIT/COST PRIORITIES -- ACHIEVING COMMENSURABILITY [17]

Traditional Benefit/Cost analysis requires benefits and costs to be expressed in a common currency, usually dollars. More recently, benefits and costs have been expressed and compared as relative priorities. This process has been criticized because there is no guarantee that the two sources of priorities are in commensurate units. This paper illustrates simpler methods for achieving commensurability.

Introduction

The concept of benefit/cost analysis has been used for economic and business decision making for many decades. The basic concept is that the analyst will investigate a project and attempt to quantify all positive and negative aspects into a common currency such as dollars (Gramlich, 1981, Mishan, 1982). Once positives (benefits) and negatives (costs) are expressed in a common currency, their difference (net benefits) or ratio can be calculated to determine whether the benefits outweigh the costs. Those projects that have positive net benefits will also have benefit/cost ratios > 1. They are the ones that would be undertaken.

If only one project is to be selected from amongst several options, then the project with the highest net benefit yields the best incremental result. That is not necessarily the highest percentage return, because the generation of the net benefit may have required substantial costs. The project with the highest benefit/cost ratio gives the highest return for monies expended. If projects are interrelated, then an incremental or marginal analysis is required to determine if additional benefits warrant additional costs.

Provided all benefits and costs are expressed in the same unit of measure, benefit-cost analysis yields valid methods of analysis. The problem is that it is frequently difficult to transform all aspects of benefit and cost into the same measure, let alone measure them in the first place. For situations where measures are unknown and must be derived, the Analytic Hierarchy Process (AHP) has been suggested as a method of analysis. In AHP, relative ratio measures are derived from via pairwise questioning of the decision maker.

The AHP procedure generally involves two hierarchies for the same set of projects -- one measuring the project benefits and the other measuring the project costs. The resulting measurements are in terms of priorities (usually summing to one) that reflect the relative intensity of the benefits or costs of each project. The premise behind such benefit/cost analysis is that the AHP priorities are commensurate and in a common currency like dollars (Saaty and Kearns, 1994).

The first researchers to identify a problem with this approach were Bernhard and Canada (1990). Essentially, the problem occurs because the benefit priorities and the cost priorities come from separate hierarchies. While the benefit priorities are relative to each other for the benefit dimension and the cost priorities are relative to each other for the cost dimension, there is no necessary assurance that the two dimensions are measured in commensurable terms. It is possible for benefit/cost ratios to appear meaningful when in fact all projects have costs that exceed their benefits

---

[17]. The Natural Sciences and Engineering Research Council of Canada provided financial support necessary for this project

A solution to this problem has been proposed. Wedley et al (2001) suggested that there should be a comparison between the two dimensions to adjust their relative magnitudes. By doing that, the benefit and cost priorities can placed on commensurate terms so that they can be used just like a common dollar currency in regular benefit-cost analysis.

This paper extends the previous research by outlining new methods to link dimensions and make them commensurate as well as relative. It reviews the state of AHP benefit-cost analysis, it expands upon the defect in conventional approaches, and it illustrates new methods for achieving commensurability. It concludes with a discussion of the merits of each method.

AHP and benefit-cost analysis

AHP has been adapted in several different ways for benefit/cost analysis. In its most general form, two AHP hierarchies are used for the same set of projects, one for benefits and the other for costs (Azis, 1990; Clayton et al, 1993; Saaty and Kearns, 1994; Saaty and Turner, 1996). Through pairwise comparison of projects and synthesis of results, relative measurements in terms of composite priorities are derived for the benefits and costs. The benefit priorities are then compared to the cost priorities to see which option has the highest ratio.

Another variant similar to the above process is to introduce a third hierarchy for risk (Saaty, 1994a). With higher priorities measuring greater benefit, cost and risk, a Benefit/Risk or Benefit/[Cost*Risk] ratio is used as the selection criterion (Rabbani and Rabanni, 1996; D. Saaty et. al, 1996; Saaty and Shang, 1996). Risk in this context is like a modifier of the benefits. Since higher risk is measured by a higher priority, Benefit*1/Risk or Benefit*1/(Risk *Cost) has the effect of lowering benefits for a higher risk project. Like division by cost priorities, a problem remains because there is no assurance that the numerator and denominator are in commensurate units. For example, all risks could be trivial but their priorities, being measured relatively, look substantial.

A hybrid procedure is to use just a single benefit hierarchy to get priorities for benefits. Then, these benefit priorities are compared to the actual dollar costs of the projects to get a measure of benefits per dollar of expenditure. Although the denominator and numerator in this calculation are different measures, benefit/dollar is an easy way to compare the contribution of projects. The objective here is to maximize benefit from each marginal dollar expended. A problem arises, however, if some of the costs are not expressed in dollars.

Another approach is to deal directly with net benefits to determine the project that has the highest cash flow (Peniwati, 1994). In this procedure, benefits are interpreted as being the difference between income and costs. A single benefit or cash flow hierarchy is used with costs subsumed within the benefit hierarchy. The project that is considered best is the one with the highest positive difference between income and cost.

The last approach is an incremental one that determines where to spend marginal resources (Saaty, 1994b; Peniwati, 1994). Using separate hierarchies for benefits and costs, existing projects are ranked in the order of their cost priorities. The benefit/cost ratio (b_i/c_i) for the project with the lowest cost priority (c_i) is then utilized as the starting point for comparing the incremental benefit/cost improvement. Thus, $(b_{i+1}–b_i)/(c_{i+1}–c_i)$ is the ratio of incremental difference between benefit priorities and those of cost priorities, where b_i and c_i are replaced by b_{i+1} and c_{i+1} as higher cost alternatives are considered. Projects that have negative ratios yield fewer incremental benefits than those given by projects with lower costs. They would be deleted as potentially funded projects because their marginal return is negative.

From an allocation perspective, the above objectives could be formulated into different types of knapsack or linear programming problems (Bennett and Saaty, 1993). For example, benefit/cost ratios, benefits, or net benefits of a package of projects could be maximized subject to different types of resource constraints. Alternatively, the objective could be to minimize costs subject to resource constraints.

The Benefit-Cost Problem

In order to illustrate problems with the use of benefit/cost priorities, we have selected an example from Saaty (1994a) that involved the choice of word processing equipment. The problem was structured into the benefit and cost hierarchies shown in Figure 1. To simply the example, we removed an intermediate level of benefit factors that were extraneous to the exposition.

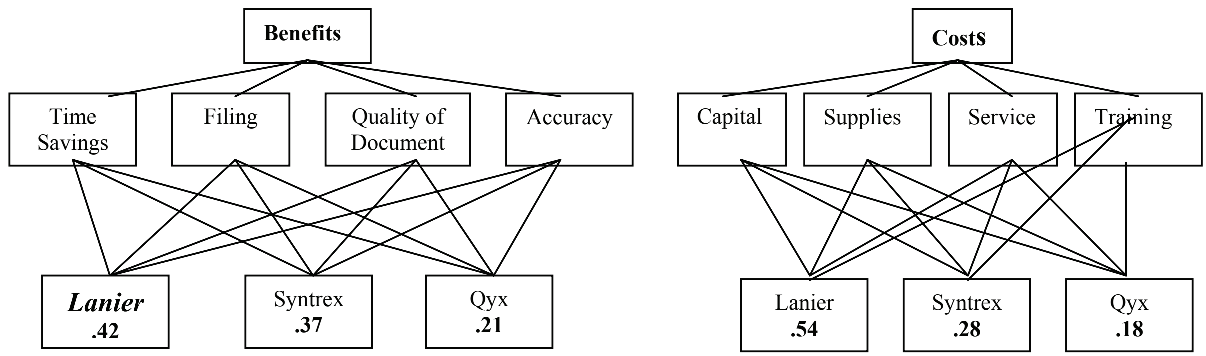


Figure 1 -- Separate Benefit and Cost Hierarchies

The composite benefit and cost priorities shown at the bottom of the hierarchy of Figure 1 are for the three different types of word processing equipment. The respective AHP benefit/cost ratios are:

	Benefit Priority	Cost Priority	Benefit/Cost ratio
Lanier	.42	.54	.42/.54 = .78
Syntrex	.37	.28	.37/.28 = 1.32
Qyx	.21	.18	.21/.18 = 1.17

Based upon the benefit/cost ratios, we would conclude that Syntrex is superior, followed closely by Qyx. But before making that conclusion, we should look at the underlying process for generating the priorities.

The priorities of the two independent hierarchies in Figure 1 come from paired comparisons regarding the relative worth of different components of benefits and costs. For example, the benefit priorities were generated by comparing the three types of equipment between themselves for each of the types of benefits. Through an eigenvector routine applied to the comparison matrix, the comparisons are converted to ratio priorities (x_i) that sum to one for each option under each criterion. These are called local priorities.

Next, the same procedure is applied to comparisons of the relative worth of the various benefit factors. Again, priorities (w_j) summing to one are generated to measure the relative importance of each factor. Then, part worth values or global priorities (w_jx_i) for each of the options are calculated under each of the factors. Finally, these weighted values are summed across all criteria to yield the composite values .42, .37 and .21. Exactly the same process is used to get the composite cost priorities of .54, .28, and .18.

These composite priorities for benefits and costs are in ratio form, relative to each other. For example, Lanier has twice (.42/.21) the overall benefit as Qyx and 1.13 times (.42/.37) the benefit of Syntrex. Notice that the sum of the benefit priorities equals one, which implies that the unit of the ratio scale is the sum of the benefit of the three items. Through proportional transformation, the unit of the ratio scale could be the benefit on any one of the items. For example, .42/.42 = 1, .37/.42= .88 and .21/.42=.5 re-expresses the scale in the unit of Lanier's overall benefit. The same type of transformation can be applied to the cost scale.

Relative ratio scales that sum to one without the explicit specification of the unit can be deceiving. Although the benefits and costs are each measured in relation to the total benefit and total cost of the options under consideration, those quantities are only measures between options and not between the types of measures. Thus, there is no assurance that the benefit priorities are commensurate with the cost priorities. Without any type of linkage between the two hierarchies of Figure 1, there is no way of knowing the relationship of the magnitude between total benefits and total costs. Only in the rare case where the sum of the all benefits in one hierarchy exactly equals the sum of all costs in the other hierarchy would the benefit and cost priorities be in the same unit (i.e. sums of relative values are the same).

An easy way to see this problem is to ascribe actual dollar figures to benefits and costs as though we are performing regular benefit/cost analysis. In Table 1, the total benefits are \$10,000: \$4,200 from Lanier, \$3,700 from Syntrex and \$2,100 from Qyx, giving priorities of .42, 37, and .21 respectively. For costs, we have used three different scenarios, each yielding the same cost priorities of shown in Figure 1 (.54, .28, .18). In the first scenario, costs are less than benefits and all Bp/Cp ratios are positive: Lanier, 1.56; Syntrex, 2.64; Qyx, 2.33. Syntrex is the best choice.

In the second scenario, costs are higher such that the sum of total benefits (\$10,000) equals the sum of total costs (\$10,000). Notice that when this occurs, the Bp/Cp ratios (.78, 1.32, 1.17) are the same as the ratios that would have been calculated from the priorities in Figure 1. This is verification of the point made above that the benefit and cost priorities from unlinked hierarchies are in common units only when the sum of the all benefits in one hierarchy exactly equals the sum of all costs in the other.

Table 1 --Benefit/Cost Ratios with Different Dollar Amounts

Measure	Lanier	Syntrex	Qyx
Aggregate Benefit \$	\$4,200	\$3,700	\$2,100
AHP Benefit priority	0.42	0.37	0.21
1. Aggregate Cost, B\$ > C\$	\$2,700	\$1,400	\$900
2. Aggregate Cost, \$ Mixed	\$5,400	\$2,800	\$1,800
3. Aggregate Cost, B\$ < C\$	\$8,100	\$4,200	\$2,700
AHP Cost priority for all 3 scenarios	0.54	0.28	0.18
Benefit \$/Cost \$ ratio (B\$/C\$)			
1. Aggregate Cost \$ B\$ > C\$	1.56	2.64	2.33
2. Aggregate Cost \$ Mixed	0.78	1.32	1.17
3. Aggregate Cost \$ B\$ < C\$	0.52	0.88	0.78
Benefit priority/Cost priority (Bp/Cp)	0.78	1.32	1.17

In the third scenario, costs are higher again such that the cost for each option exceeds the benefits. Using dollars to calculate the ratios, all B/C ratios are less than one. However, both Syntrex and Qyx with AHP Bp/Cp ratios are >1

We should note that Syntrex is the best option under all three cost scenarios. Thus, if we are choosing just the best alternative from the lot, relative ratios of benefit/cost priorities always gives the best choice. But the best of the lot may not be the correct decision. As we can see from Scenario 3, Syntrex is a bad choice when its cost exceeds its benefits. In effect, a benefit-cost ratio of priorities is deceiving if it is composed of two sets of measures that have not been adjusted relative to each other. In order to emulate and get all of the advantages of conventional benefit/cost analysis, the AHP priorities for the various dimensions must be transformed to a common unit of measure.

Achieving Commensurability

There are several methods for adjusting AHP priorities to make the numerator and denominator commensurable. To illustrate these methods, we have assigned dollar amounts and associated priority values for case 1 of Table 1 where benefits exceed costs. The dollar analogs to priorities in Table 2 enable the various methods to be validated. We should recognize, however, that we rarely have the dollar amounts in actual applications where AHP is used to measure intangibles.

Table 2 -- Benefits and Costs in Dollars and Priorities

Measure	Benefits and Costs in dollars			Benefits and Costs in priorities		
	Lanier	Syntrex	Qyx	Lanier	Syntrex	Qyx
Aggregate Benefits	$4,200	$3,700	$2,100	0.42	0.37	0.21
Time Savings	$2,100	$1,000	$1,050	0.51	0.24	0.25
Filing	$900	$1,000	$450	0.38	0.43	0.19
Quality of Document	$1,000	$1,000	$500	0.40	0.40	0.20
Accuracy	$200	$700	$100	0.20	0.70	0.10
Aggregate Costs	$2,700	$1,400	$900	0.54	0.28	0.18
Capital	$700	$400	$200	0.54	0.31	0.15
Supplies	$700	$300	$200	0.58	0.25	0.17
Service	$1,000	$300	$300	0.63	0.19	0.19
Training	$300	$400	$200	0.33	0.44	0.22

Previous Suggestions for Magnitude Adjustment

Bernhard and Canada (1990) were the first researchers to recognize the need for AHP benefit/cost adjustment. They proposed a cut-off rate that a Bp/Cp ratio has to exceed before it is acceptable. They defined the cutoff as k'/k where k' is the sum of all costs measured in dollars and k is the sum of all benefits measured in dollars. A project would be attractive if Bp/Cp $\geq k'/k$ or equivalently Bpk/Cp$k' \geq 1$. Applying this formula to Lanier, we would get .42*(4200+3700+2100)/.54*(2700+1400+900) = 1.56 which is the correct underlying B/C ratio based on dollars.

In effect, Bernhard and Canada's test helps assure that the Bp/Cp ratio is >1. It's drawback, however, is that it requires the decision-maker to know or estimate k' and k in dollar terms. If that were possible, it would make sense to forget AHP and use conventional B/C analysis from the start. Normally, AHP is used when some cost and benefit factors are intangibles or when our utility for tangibles is non-linear.

Saaty (1994a, p. 151), the founder of AHP, has also proposed solutions. He suggested that the decision-maker should first answer the question "In this problem, do the benefits justify the costs?" If the benefits are much more important than the costs then Saaty suggests two alternative approaches: (1) ignore costs and base the decision solely on benefits or (2) consider both the benefits and costs in separate hierarchies (as in Figure 1) and compare their ratios. Later, Saaty recognized the necessity of better calibration. He suggested that the benefits and costs must be sufficiently commensurable to produce meaningful ratios (Saaty, 1996, p. 40).

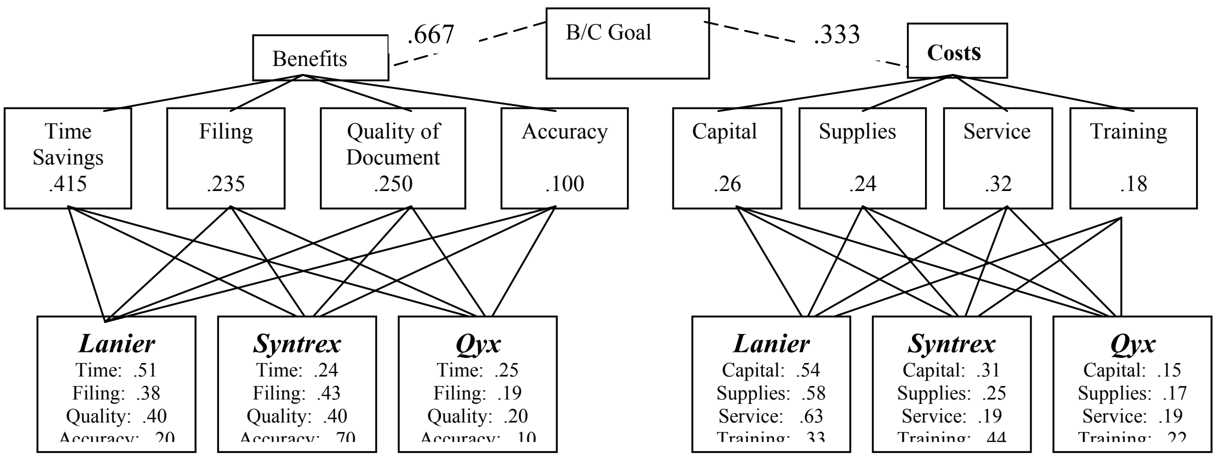
The difficulty with Saaty's question is that the user is seldom aware that the magnitude check must be done before benefit/cost ratios are used. Also, benefits and costs are seldom commensurable on their own. It is for this reason that Wedley et al (2001) suggested a formal magnitude adjustment procedure that converts the benefit and cost hierarchies to a common unit. They suggested two different questioning methods between the two hierarchies for assessing overall benefits vs. overall costs:

1. Which perspective is more important, the aggregate benefits or the aggregate costs and by how many times?

2. Which perspective is more important, the average alternative benefit or the average alternative cost and by how many times?

Based upon the dollar analogs in Table 2, the answers are as follows: Aggregate benefits ($10,000) /Aggregate costs ($5,000) = 2 times or Average benefits($3,333)/Average costs($1,667) = 2 times.

Irrespective of which method is used, the resulting priorities (.667, .333) represent the overall benefits to the overall costs. These priorities are then applied to the separate benefit and cost hierarchies to adjust their priorities to commensurate terms (Figure 2). The resulting adjusted priorities capture the correct underlying B/C ratios listed in Table 1: .280/.180 = 1.56, .247/.093 = 2.66, and .140/.060 = 2.33



Benefits: .42 .37 .21 Costs: .54 .28 .18

Adjusted .280 .247 .140 Adjusted .180 .093 .060

Figure 2: Adjusted Benefit and Cost Hierarchies

To understand why this overall magnitude adjustment generates B/C ratios that are analogous to dollar figures, we must look at the separate hierarchies that generate the benefit and cost priorities. For example, consider just the benefit hierarchy in Figure 2. Since the aggregate benefit priorities (.42, .37, .21) are in ratio form (Lanier possesses .42/.21 = 2 times the benefit of Qyx), they are relative to each other. However, their unit of measure is obscure. The fact that they sum to one indicates that their unit of measure consists of themselves and represent the total benefits that they possess (Wedley and Choo, 2001). The same applies to the cost priorities (.54, .28, .18). They are in the unit of the total relative cost that they possess.

The problem is that the unit of total benefit is seldom equal to the unit of total cost. Thus, it is very rare that the benefit and cost priorities are like scenario 2 of Table 1 where both have the same unit of measure. Accordingly, an adjustment is needed to form a new common unit or to place one set of priorities in the unit of the other. In Figure 2, the dotted lines at the top signify that the hierarchies must be somehow linked to establish that common unit. The relative priorities for total benefits (.667) to total costs (.333) are treated as global priorities that adjust all lower level priorities. The resulting adjusted priorities that sum to one across both hierarchies signify that they are in the unit of all benefits and cost. With such a common unit, a value greater than unity for Bp/Cp indicates that the breakeven point has been exceeded. As Bennett and Saaty (1993) recognized, unadjusted Bp/Cp ratios do not have that desirable property.

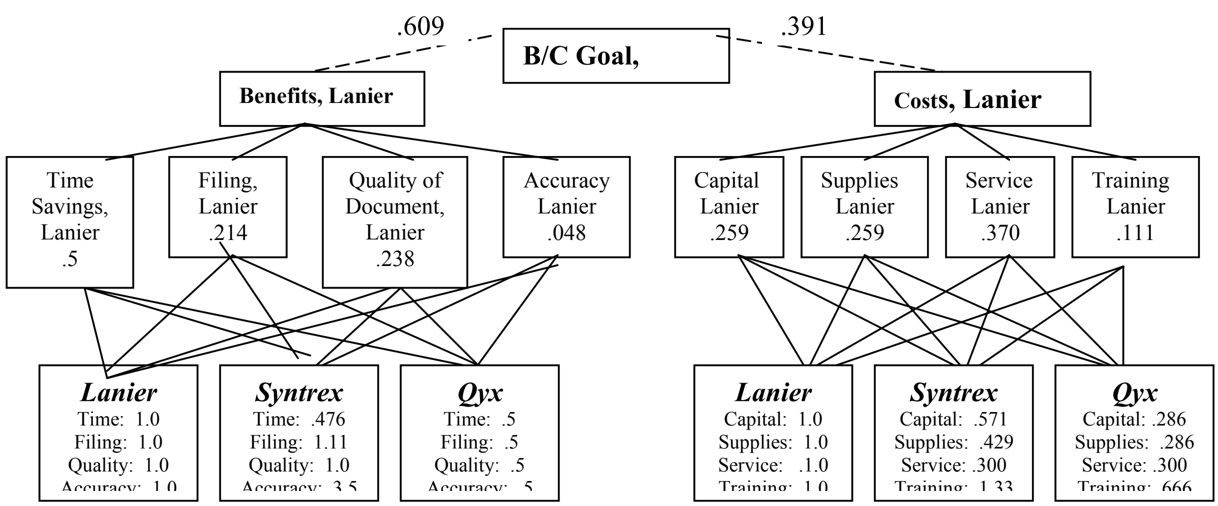
Linking Pin Methods

These adjustment mechanisms proposed by Wedley et al (2001) enable B/C priorities to achieve all the desirable attributes of regular B/C analysis, including incremental analysis and unity equal to breakeven. The disadvantage, however, is that it is cognitively difficult to compare aggregate (or average) benefits to aggregate (average) cost. Simpler techniques for achieving commensurability are desirable.

The fact that AHP hierarchies are based upon ratio measurement makes them amenable to simpler procedures. Notice in Figure 2 that the benefit node is the unit for the benefit scale and the cost node is the unit for the cost scale. Since the unit of a ratio scale is arbitrary (Barzilai,1998), a proportional transformation of the scale can put the unit in any node. Furthermore, the link between hierarchies does not have to be at topmost node for aggregate benefits and costs (Schoner et al, 1993). Realizing these two facts, any node and any level can be the link to achieve commensurability. Below are two examples that illustrate the possibilities.

Links with one alternative, Lanier.

Figure 3 presents benefit and cost hierarchies where Lanier is used as the benchmark or frame of reference. Rather than comparing aggregate benefits and costs of all alternatives at the top, an easier comparison is taken between the aggregate Benefits of Lanier vs. the aggregate Costs of Lanier. If done as accurately as the dollar analog, the ratio of benefits to costs would be $4200/$2400 or .609 to .391. Since Lanier is the link between the two hierarchies, the comparison of the various aspects of benefits and costs at the next lower level are done for just Lanier. Again, comparing just the aspects for Lanier rather than all alternatives is an easier cognitive task. Finally, at the bottom level, the various alternatives are compared amongst themselves for each aspect, but the priorities are normalized so that Lanier takes a unit value. That way, the priorities established for Lanier at the higher levels will flow down to Lanier and the other alternatives via the unit value. The final composite priorities for benefits and costs sum to one, because they are no linger in units relevant to themselves. Rather, they are in the unit of Lanier for both benefits and costs. As can be seen from the correct Bp/Cp at the very bottom of Figure 3, Lanier as the common unit yields the correct ratios.



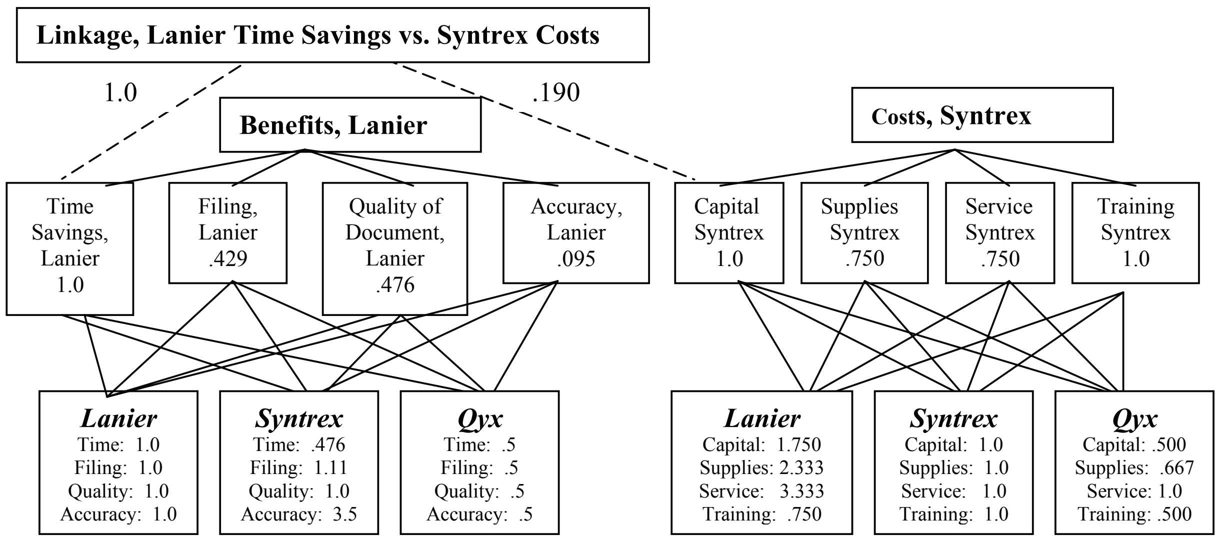
Benefits: .609 .536 .304 Costs: .391 .202 .130

Bp/Cp: Lanier: .609/.391 = 1.56 Syntrex: .536/.202 = 2.64 Qyx: .304/.130 = 2.33

Figure 3: Benefit and Cost Hierarchy with Lanier as the Link and Unit

Links with two alternatives, Lanier and Syntrex.

Perhaps an even easier procedure is to focus on a specific benefit factor and a specific single cost factor that is well known or easy to measure (perchance in monetary units). In Figure 4, we have selected Time Savings for Lanier and Capital Cost of Syntrex as linking nodes. These are made the unit of measure for their respective hierarchies. Other benefits are measured relative to the Time Savings for Lanier and other costs are measured relative to the Capital Cost of Syntrex. Similarly, for each benefit factor, the alternatives are measured relative to Lanier and for each cost factor the alternatives are measured relative to Syntrex. Thus, before linkage, the benefits are expressed in the unit of Time Savings of Lanier and costs in the Capital Cost of Syntrex.



Benefits: 2.0 1.762 1.0 Costs: 1.286 .667 .429

Bp/Cp: Lanier: 2.0/1.286 = 1.56 Syntrex: 1.762/.667 = 2.64 Qyx: .1.0/.429 = 2.33

Figure 4: Benefit and Cost Hierarchy linked by Lanier Time Savings and Syntrex Capital Costs

The next step is to link the two hierarchies. We do this by asking which is more: the Time Savings of Lanier or the Capital Cost of Syntrex. From the dollar figures of Table 2, the correct answer is Time Savings by 2100/400 or 1.0 to .19. When these priorities are applied to their respective hierarchies, the unit of both is converted into units of Time Savings of Lanier. As can be seen from the very bottom of Figure 2, this common unit across both hierarchies produces the correct Bp/Cp ratios.

Discussion and Conclusion

In the last example, it may have been easier to compare between hierarchies had we chosen the same alternative for Time Savings and Capital Cost. We chose two different alternatives to demonstrate that there are many possible links and reference alternatives.

Although linking via aggregate benefits and aggregate costs is possible, we believe it is a cognitively easier task if the links are across a common alternative or specific factors. Before linking, one merely has to identify a node that becomes the unit of measure for each hierarchy. Other nodes are then expressed in terms of that unit. When linking, the unit nodes are compared and one is converted to the unit of the other or both are converted to commensurate terms. Either way, commensurability occurs.

The need to express intangible benefits and costs in commensurable terms is crucial. Without commensurability, the Bp/Cp ratios can be deceiving and lead to incorrect decisions.

With commensurability, the Bp/Cp assume all of the desirable properties of conventional benefit-cost analysis.

As has been shown herein, AHP can be used to calculate cost and benefit priorities. With caution to make sure that both sets of priorities are in commensurable units, such priorities can be used for benefit/cost comparisons. With magnitude adjustments and linkages to achieve commensurability, deceptive benefit/cost ratios can be avoided. When the measures are commensurate and sufficiently consistent, decision makers can be confident that Bp/Cp<1 identifies bad alternatives that should be ignored.

References

Azis, I.J. (1990), "AHP in the Benefit-Cost Framework: A Post-Evaluation of the Trans-Sumatra Highway Project," *European Journal of Operational Research*, 48, 38-48

Barzilai, J (1998) "Measurement Foundations for Preference Function Modeling," Proceedings of the IEEE International Conference on Systems, Man and Cybernetics, 4038-4044.

Bennett J. P and Saaty, T. L. (1993), "Knapsack Allocation of Multiple Resources in Benefit-Cost Analysis by way of the Analytic Hierarchy Process," *Mathematical and Computer Modelling*, 17, 4/5, 55-72,

Bernhard, Richard H. and Canada, John R. (1990), "Some Problems in Using Benefit/Cost Ratios with the Analytic Hierarchy Process," *The Engineering Economist*, 36, 1, 56-65.

Clayton, W.A., Wright, M. and Sarver, W.S. (1993), "Benefit Cost Analysis of Riverboat Gambling," *Mathematical and Computer Modeling*, 17, 187-194

Gramlich, Edward M. (1981), *Benefit-Cost Analysis of Government Programs*, Englewood Cliffs, New Jersey: Prentice-Hall Inc.

Mishan, E. J. (1982), *Cost-Benefit Analysis*, London: George Allen & Unwin.

Peniwati, Kirti, (1994) "Misuses of Clustering and Absolute Scales of Measurement in the Analytic Hierarchy Process." *Proceedings of the Third International Symposium on the Analytic Hierarchy Process*, George Washington University, Washington, D. C., 239-250.

Rabbani, Simin Jalali R. and Rabanni, Soheil Rahnemay (1996) "Resource Allocation to Road Safety and the Analytic Network Process." *Proceedings of the Fourth International Symposium on the Analytic Hierarchy Process*, Simon Fraser University, Burnaby, B. C. Canada, 94-105.

Saaty, Daniel; Charlson Joseph; Clauss, Beth; Lin, Young-Chi; and Markle, Ron (1996) "The Future of the University of Pittsburgh Medical Center: Strategic Planning with the Analytic Hierarchy Process." *Proceedings of the Fourth International Symposium on the Analytic Hierarchy Process*, Simon Fraser University, Burnaby, B. C. Canada, 107-121.

Saaty, Thomas L. (1994a), *Fundamentals of Decision Making and Priority Theory,* RWS Publications, Pittsburgh.

Saaty, Thomas L. (1994b), "How to Make a Decision: The Analytic Hierarchy Process," *Proceedings of the Third International Symposium on the Analytic Hierarchy Process*, George Washington University, Washington, D. C. vi–xxvi.

Saaty, Thomas L. (1996), *The Analytic Network Process, Fundamentals of Decision Making and Priority Theory,* RWS Publications, Pittsburgh.

Saaty, Thomas L. and Kearns, Kevin P. (1994), *Analytic Planning,* RWS Publications, Pittsburgh.

Saaty, Thomas L. and Shang, Jen S. (1996) "The Analytic Hierarchy Process and the Voting System." *Proceedings of the Fourth International Symposium on the Analytic Hierarchy Process*, Simon Fraser University, Burnaby, B. C. Canada, 505-517.

Saaty, Thomas L. and Turner, David S. (1996) "Prediction of the 1996 Superbowl: an Application of the AHP with Feedback (The Supermatrix Approach)." *Proceedings of the Fourth International Symposium on the Analytic Hierarchy Process*, Simon Fraser University, Burnaby, B. C. Canada, 409-420.

Schoner, B., Wedley, W. C. and Choo, E. U. (1993), "A Unified Approach to AHP with Linking Pins", *European Journal of Operational Research*, 64, 345-354.

Wedley, William C and Choo, Eng Ung. "A Unit Interpretation of Multi-Criteria Ratios," *Proceedings of the Sixth International Symposium on the Analytic Hierarchy Process*, Berne, Switzerland, 2001, 561-569

William C. Wedley, Eng Ung Choo, and Bertram Schoner, "Magnitude Adjustment for AHP Benefit/Cost Ratios", *European Journal of Operational Research*, 133/2, 2001, 342-351.