LOCALISATION PERFORMANCE OF PASSIVE SONAR FOR SELF PROTECTION AGAINST FAST CRAFT

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Abstract: *Assets at sea are vulnerable to attacks from fast craft relying on the effect of surprise. These threats are not necessarily stealthy but leave a very limited time of reaction to the platform at risk. Early detection of the approaching vessel provides an opportunity for the attacked ship to react in time. Accurate localisation of the threat helps deciding for the best reaction and reducing false alarms. Localisation of such threats is often performed using sparse hydrophone arrays. Reliable passive localisation is challenging as the precision is determined by geometry and sensor positioning accuracy. In this paper, we present a localisation technique of time delay estimates integration using a Maximum Likelihood Estimator that estimates the position and speed of a moving target. Using the Fisher Information Matrix and the Cramèr Rao Lower Bound, we provide performance indicators for different array and measurement configurations. We further investigate the influence of hydrophone position uncertainty on the localisation performance. The results of this work can be used to assess the applicability of this method to locate fast moving craft and optimise sensor configurations to obtain reliable localisation.*

Keywords: *Passive sonar, sparse array, maximum likelihood, time delays*

1. INTRODUCTION

Assets at sea are vulnerable to attacks from fast craft relying on the effect of surprise. These threats are not necessarily stealthy but leave a very limited time of reaction to the platform at risk. Early detection of the approaching vessel provides an opportunity for the attacked ship to react in time. Accurate localisation of the threat helps deciding for the best reaction and reducing false alarms. Furthermore, an accurate and timely estimate of the speed of a suspicious craft can help determine whether this craft is a threat and decide for an appropriate response.

Passive sonar is often used for detection and localisation of noisy sources, especially at close range. Its main advantage over sensors such as radar or active sonar is its complete discretion. Depending on the configuration of the sonar, different signal processing algorithms are used.

Some passive sonars, such as low frequency towed arrays are usually “fully populated”, i.e. their hydrophones are spaced so as to sample the acoustic field according to Nyquist criterion. This type of configuration allows the application of beamforming techniques to the data collected by the sensor. Other sensors, such as passive ranging sonars are sparsely populated and rely on their baseline to estimate the position of a source. These techniques are often based on the estimation of time of arrival delays between the signal collected by a sensor and that of the other sensors [1-3]. These time delays are usually estimated by means of cross-correlation or an extension thereof [4]. The range and bearing of a target is then estimated using a single measurement, (usually a transient transmitted voluntarily or not by the source). Estimating the trajectory of a platform usually requires longer integration times which conflicts with the wish for a quick response. Here, we investigate the possibility of reducing the integration time by increasing the number of hydrophones in the array.

In this paper, we propose an extension of this method based on the assumption that the source is transmitting continuous broadband sound or, a frequent series of transients. We then derive localisation performance indicators and use them to analyse the influence of target position, array population and platform position uncertainty on the localisation performance and ability to quickly and reliably assess the speed of an incoming target.

2. LOCALISATION METHOD

2.1. Principle

Let us consider a sonar array composed of N_H hydrophones mounted rigidly on a platform. The platform position at any instant t_b is

$$\mathbf{r}_{P,b} = [x_{P,b} \quad y_{P,b}]^{\mathrm{T}}, \tag{1}$$

and the N_H hydrophone positions, relative to the position of the platform are

$$\mathbf{r}_{H,n} = [x_{H,n} \quad y_{H,n}]^{\mathrm{T}}, \tag{2}$$

where n is the hydrophone index. The sonar is observing a moving target radiating a random, stationary (in the statistical sense) broadband acoustic signal. This signal is measured at the hydrophones between frequencies $f_{\min}$ and $f_{\max}$. The source speed is assumed constant and its position at any time can therefore be written as

$$\mathbf{r}_{T,b} = [x_T + x_T t_b \quad y_T + y_T t_b]^{\mathrm{T}}. \tag{3}$$

If we assume that the propagation is cylindrical, the signal received by the n^{th} hydrophone can be expressed as

$$s_n(t) = s_1\left(t - \frac{1}{c}\left|\mathbf{r}_T - \mathbf{r}_{P,b} - \mathbf{r}_{H,1}\right| + \frac{1}{c}\left|\mathbf{r}_T - \mathbf{r}_{P,b} - \mathbf{r}_{H,n}\right|\right). \tag{4}$$

The time delay between the signal at the source and the signal received by the hydrophones cannot be estimated without knowledge of the radiated signal or the target range. We can however estimate the time of arrivals difference (TOAD) between each hydrophone and the first hydrophone over an integration time T_B. Note that the integration time for cross-correlation is short enough so that the target and hydrophone positions can be assumed constant over the integration time T_B. A compensation for target and receiver movement for time delay estimation is given in [5]. Let us write this time delay as:

$$\tau_{n,b} = \frac{1}{c}\left|\mathbf{r}_T - \mathbf{r}_{P,b} - \mathbf{r}_{H,1}\right| - \frac{1}{c}\left|\mathbf{r}_T - \mathbf{r}_{P,b} - \mathbf{r}_{H,n}\right|. \tag{5}$$

We estimate the time delays of the signal measured by the first hydrophone and that of the other hydrophones through cross-correlation over an integration time T_B:

$$\tau_n = \max_{t_k}\{s_n(t_k) * s_1(t_k)\}, \tag{6}$$

where $*$ denotes the convolution operator. This estimation is performed for a batch of N_B measurements. We then note $\tau_{n,b}$ for the TOAD of the n^{th} hydrophone of the b^{th} measurement.

Quazi [4] gives expressions for the Cramèr Rao Lower Bound (CRLB) of time delay estimation for such a signal at low Signal to Noise Ratios (SNR),

$$\sigma_\tau = \frac{1}{SNR}\sqrt{\frac{3}{8\pi^2 T_B\left(f_{\max}^3 - f_{\min}^3\right)}}, \tag{7}$$

where SNR is the signal to noise ratio at hydrophone level. In the rest of the paper, we assume a TOAD precision of 10^{-4} s, which corresponds to a SNR of about -23 dB for a signal between 500 Hz and 1000 Hz and an integration time of one second.

We will now present a Maximum Likelihood Estimator for the position and speed of the target, using the time delays as measurement, similarly to the approach presented by Farina in [6] for bearing only target motion analysis. This method will be referred to as Time Delay Target Motion Analysis (TD-TMA) in the rest of the paper. Let us write the state vector as

$$\mathbf{x} = [x_T \quad y_T \quad x_T \quad y_T]^{\mathrm{T}} \tag{8}$$

The observation vector consists of the measured TOADs, for each hydrophone, for each measurement:

$$\mathbf{y} = \left[y_1 \cdots \; y_m \cdots y_{N_H \times N_B}\right] \tag{9}$$

where

$$m = n + (b - 1) \tag{10}$$

and

$$y_m = \tau_{n,b}. \tag{11}$$

The measurement and state vectors are related through the observation function

$\mathbf{h}: \mathbb{R}^4 \to \mathbb{R}^{N_H \times N_B}$

$$\mathbf{x} \mapsto h_m(\mathbf{x}) = \frac{1}{c}\left|\mathbf{r}_T - \mathbf{r}_{P,b} - \mathbf{r}_{H,1}\right| - \frac{1}{c}\left|\mathbf{r}_T - \mathbf{r}_{P,b} - \mathbf{r}_{H,n}\right|, \forall m, m \in [1, N_H N_B]. \quad (12)$$

Let

$$\mathscr{L}(\mathbf{x}|\mathbf{y}) = \Pr(\mathbf{x}|\mathbf{y}) = \prod_{m=1}^{N_B N_H} \frac{1}{\sigma_\tau \sqrt{2\pi}} \exp\left(-\frac{(y_m - h_m(\mathbf{x}))^2}{2\sigma_\tau^2}\right), \quad (13)$$

the likelihood function of the estimate given the measurement, where Pr is a probability. We define the log-likelihood, the maximisation of which results in the MLE of the state vector:

$$\ell(\mathbf{x}|\mathbf{y}) = \ln\mathscr{L}(\mathbf{x}|\mathbf{y}) = -N_H N_B \ln\left(\sigma_\tau \sqrt{2\pi}\right) - \frac{1}{2\sigma_\tau^2}\sum_{m=1}^{N_H N_B} \left(y_m - h_m(\mathbf{x})\right)^2. \quad (14)$$

In our practical implementation we used a Differential Evolution algorithm [7], to minimise $\ell(\mathbf{x}|\mathbf{y})$, kindly provided by Van Moll [8].

2.2. Simulation

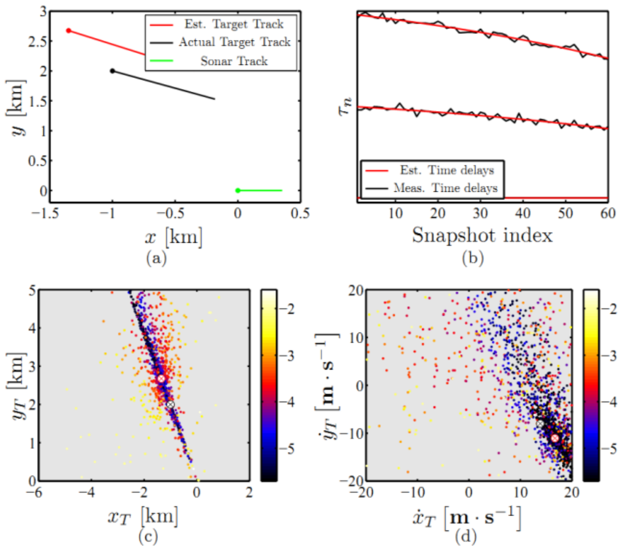


Fig. 1 Results of the MLE for a one minute scenario with a time delay precision of 10^{-4} s: (a) Geographical display of the sailed track, the estimate and the ground truth. The dots mark the first point of each track. (b) Measured and estimated time delays for all three hydrophones. (c) and (d) Scatter plots of all energy function evaluations used in the optimisation for the target position and speed, respectively. The colour axis corresponds to a measure of the likelihood function. The ⊗ symbols mark the estimated (red) and actual (black) target position and speed.

We will now briefly present simulation results. TOADs series were generated for an array of three hydrophones spaced by 15 m. Three hydrophones at least are needed for range estimation. These TOADs were measured with a centred Gaussian error of variance $\sigma_\tau = 10^{-4}$ s. The correlation integration time T_B is 1 s and the number of snapshots N_B is 60, resulting in a total integration time of one minute. The target assumes a course of 120 ° at a speed of 16 m · s^{-1}, while the measuring platform is sailing along a course of 90 ° at a speed of 6 m · s^{-1}. The target is situated at a range from the platform of 2.24 km at the beginning of the simulation and 1.61 km at the end of the simulation. A single realisation of this simulation is shown in Fig. 1. A hundredfold repetition of the simulation reveals that the estimate is biased:

$$\mathbf{b}_{\mathbf{x}} = \frac{1}{100}\sum_{r=1}^{100} (\mathbf{x}_{\boldsymbol{MLE}} - \mathbf{x}) \tag{15}$$

$$= [-93\ \mathrm{m} \quad 182\ \mathrm{m} \quad 0.6 \quad 0.5\ \mathrm{m}\cdot\mathrm{s}^{-1}]^{\mathrm{T}}, \tag{16}$$

where $\mathbf{x}$ is the actual state, $\mathbf{x}_{\boldsymbol{MLE}}$ is the state estimate and $\mathbf{b}_{\mathbf{x}}$ is the estimate bias.

3. PERFORMANCE ANALYSIS

We will now present two performance indicators and examine them while varying design and configuration parameters.

3.1. Performance indicators

3.1.1. Precision

The MLE is unbiased and asymptotically reaches the CRLB for precision. The CRLB is derived through the inversion of the Fisher Information Matrix (FIM). We reuse the derivations of the CRLB for Bearing Only Target Motion Analysis from [6] to deduce the CRLB for the estimator presented here. The Fisher Information Matrix (FIM) for TD-TMA is equal to

$$\mathbf{J}(\mathbf{x}) = \mathbf{E}\left(\frac{\partial \ell(\mathbf{x}|\mathbf{y})}{\partial \mathbf{x}}\frac{\partial \ell(\mathbf{x}|\mathbf{y})}{\partial \mathbf{x}}^{\mathrm{T}}\right) = \frac{1}{\sigma_\tau^2}\frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}}\frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}}^{\mathrm{T}}. \tag{17}$$

Let us consider $\frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}}$ the Jacobian of $\mathbf{h}(\mathbf{x})$, and define

$$r_{x,n,b} = \left(x_T + T_B\frac{b}{N_B}x_T - x_{H,n,b}\right) \tag{18}$$

and

$$r_{y,n,b} = \left(y_T + T_B\frac{b}{N_B}y_T - y_{H,n,b}\right) \tag{19}$$

and express the terms of the Jacobian, using the expression of $\mathbf{h}(\mathbf{x})$ in equation (12) :

$$\frac{\partial h_m(\mathbf{x})}{\partial x_T} = \frac{r_{x,n,b}}{c\sqrt{r_{x,n,b}^2 + r_{y,n,b}^2}} - \frac{r_{x,1,b}}{c\sqrt{r_{x,1,b}^2 + r_{y,1,b}^2}}$$
$$\frac{\partial h_m(\mathbf{x})}{\partial y_T} = \frac{r_{y,n,b}}{c\sqrt{r_{y,n,b}^2 + r_{y,n,b}^2}} - \frac{r_{y,1,b}}{c\sqrt{r_{y,1,b}^2 + r_{y,1,b}^2}}$$
$$\frac{\partial h_m(\mathbf{x})}{\partial x_T} = T_B\frac{b}{N_B}\frac{\partial h_m(\mathbf{x})}{\partial x_T}\ , \qquad \frac{\partial h_m(\mathbf{x})}{\partial y_T} = T_B\frac{b}{N_B}\frac{\partial h_m(\mathbf{x})}{\partial y_T} \tag{20}$$

By injecting these expressions in equation (17), we easily obtain the FIM for TD-TMA and the corresponding CRLB.

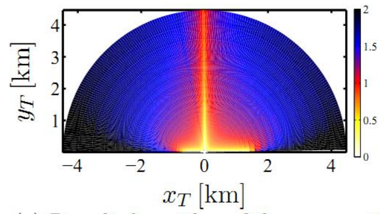
3.1.2. Bias

As we mentioned earlier, the MLE is asymptotically unbiased, as the number of measurements increases. The scope of this article concerns fast threatening targets (which imply short measurement times) and sparse arrays with a limited number of sensors. The total number of measurements ($N_B N_H$) is therefore limited and can lead to biases in the estimates. Expressions for the biases are given in [9] but are not derived in this paper. We chose to estimate the bias through repeated simulations of a given scenario.

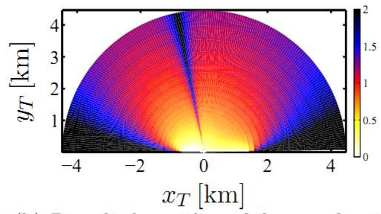
3.2. Performance analysis

3.2.1. Effect of target position

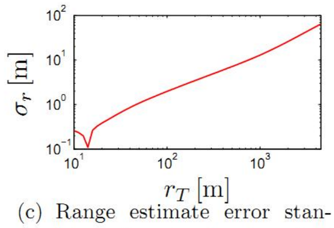
For these computations, we assumed an attacking target sailing towards the initial position of the platform at a speed of 16 $\mathrm{m \cdot s^{-1}}$. For initial target ranges (r_T) of 0 km to 4 km and bearings from 0 ° to 180 °, we computed the FIM and deduced the CRLB. This is represented in Fig. 2 . One can note that the expected range precision (σ_r) increases linearly with target range. As can be expected from a linear array, the performance at forward and endfire bearings is worse than at broadside. With the given time delay accuracy, an acceptable range precision of the target of about 60 m is attained at ranges up to 4 km. The speed estimate errors are however of the order of 10 $\mathrm{m \cdot s^{-1}}$ at similar ranges which makes it difficult to estimate the target trajectory. A threat assessment based on the speed (high versus low) is also infeasible with such speed uncertainties. As we will see in 3.2.2, the speed accuracy can be improved by increasing the number of hydrophones in the array.



(a) Decadic logarithm of the range estimate error standard deviation for TD-TMA



(b) Decadic logarithm of the speed estimate error standard deviation for TD-TMA

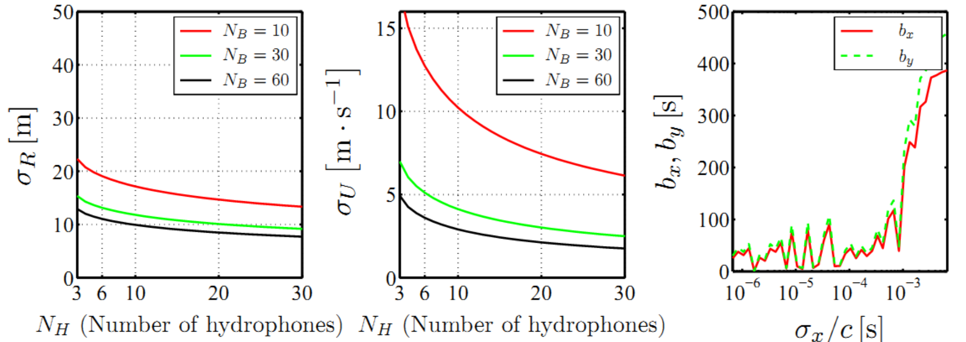


(c) Range estimate error standard deviation at a bearing of 45 deg

Fig. 2 Precision parameters for TD-TMA. The lower plot (c) is a section of the other (a) for a target of initial bearing 45 °

3.2.2. Effect of array population

A target incoming from 45 ° at 16 m · s^{-1} at an initial range of 1 km was considered. For a fixed array baseline (30 m), the number of hydrophones was increased and the effect on range and speed estimates precision observed. The integration time was varied as well (10 s, 30 s and 60 s). The effect is especially visible on the speed estimate for which a difference between 5 m·s^{-1} and 2 m·s^{-1} in precision (for 3 and 20 hydrophones) makes the speed estimate usable. By comparing the black curve with the green curve in the right plot, one can notice that doubling the number of hydrophones would allow obtaining the same speed estimate accuracy within half the integration time.



(a) Range error standard deviation as a function of array population

(b) Speed error standard deviation as a function of array population

(c) Bias in source position estimate as a function of platform position perturbation standard deviation.

Fig. 3: Effect of array population and platform position uncertainty on the localisation performance.

3.2.3. Effect of platform position uncertainty

An expected limitation of all synthetic aperture related methods, such as TD-TMA, is the effect of the relatively poor knowledge of hydrophone positions on performance. To evaluate this effect on the method presented here, we performed repeated simulations and considered the bias resulting from an increase in the hydrophone position error standard deviation. A random Gaussian perturbation was applied to the position of the platform (the respective position of the hydrophones were not affected, i.e. the array was still considered rigid). We can see in Fig 3 (c) that as long as the hydrophone position uncertainty (σ_x) stays under the TOAD input precision (10^{-4} s), little effect is seen on the bias of the target position estimate, but very quickly rises afterwards.

4. SUMMARY AND CONCLUSION

A Maximum Likelihood method for acoustic source localisation using Time Delay of Arrivals was presented. We showed that this method could not only provide an estimate of the range of a source, but also of its speed and course. The Cramér Rao Lower Bound was derived for this measurement model. This CRLB is a useful tool for the dimensioning of a sonar and the choice of an integration time for a given type of target, and finally gives a measure of the estimates accuracy.

This CRLB as well as indicators of bias and observability were used to assess the influence of different parameters on localisation performance. It was shown that increasing the number of hydrophones in a sparse array was improving the estimate of a target's speed. This means that, by increasing the number of hydrophone in an array, one can reduce the integration time and therefore the reaction time. Furthermore, simulations showed that the bias in the estimates was increasing due to hydrophone position perturbations only if the standard deviation of the latter was of the order of the accuracy of the measured time delay of arrivals.

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