Measured performance of a sequential two-step detection scheme

Abstract

Measurements have been carried out to compare a sequential two-pulse detection scheme with conventional single-pulse detection. Measured and computed results are in good agreement.

Theory

In a simple sequential detection scheme for search radar [1,V] a second pulse is transmitted in the same direction if the first detection threshold has been crossed after transmission of the first pulse. If the second detection threshold is crossed in the same range cell as the first threshold, an alarm is registered.

For conventional single pulse detection the probability of detection of a non-fluctuating target echo in normal white noise for a signal-to-noise power ratio (SNR) R is

$$P_d = Q\,(R\sqrt{2}, D) \tag{1}$$

where $$Q(A,D) = \int_D^\infty v \exp - \frac{A^2+v^2}{2} I_0(Av)\, dv \tag{2}$$

is Marcum's Q-function and D is the detection threshold.

If there are m range resolution cells (=bins), the false alarm probability per bin is

$$\alpha_{bin} = Q(0,D) = \exp - \frac{D^2}{2} \tag{3}$$

and the false alarm probability per sweep is

$$\alpha = 1 - (1-\alpha_{bin})^m \approx m\alpha_{bin} \tag{4}$$

For specified values of α and m the SNR corresponding to $Pd=\frac{1}{2}$ can be computed. It will be denoted by $R_C(\alpha)$.

For sequential detection the corresponding expressions are

$$P_d = Q\ (R\sqrt{2}, D_1).Q\ (R\sqrt{2}, D_2) \tag{5}$$

$$\alpha_{bin} = \alpha_{1bin}.\alpha_{2bin} = Q\ (0, D_1).Q(0, D_2) \tag{6}$$

and α follows from (4).

The probability for a second transmission in the case of noise only is

$$\alpha_1 = 1-(1-\alpha_{1bin})^m \approx m\alpha_{1bin} \tag{7}$$

For the same values of α and m as before the SNR's corresponding to $P_d=\frac{1}{2}$ can be computed for several values of α_1. They will be denoted by $R_S(\alpha,\alpha_1)$.

Now the gain of sequential detection over conventional detection can be expressed as:

$$G = 10\ \log \frac{R_C(\alpha)}{R_S(\alpha,\alpha_1)} - L \tag{8}$$

where L is a loss due to the increased average energy per test for sequential detection.

$$L = 10\ \log \left\{1 + \alpha_1 R_S(\alpha,\alpha_1)\right\} \tag{9}$$

The gain of sequential detection can be fully exploited in the case of a search radar, whose search volume does not contain any target.

In most practical situations the number of targets is so small relative to the number of cells that the gain needs no correction.

Curves of G versus α_1 have been obtained for m=1000 bins per sweep and for several values of α. (fig. 1a,b,c,d).
If the same analysis is done for a slowly fluctuating Rayleigh target the results are about the same.

Measurements

In order to check the theoretical results measurements have been made. A linear phased array (X-band, 40 elements illuminating a parabolic cylindrical reflector to obtain a pencil beam with 3 dB one-way beam-width of ~ 3^{o} at broadside) performed an azimuth scan over a small angle. The false alarm probability was estimated by counting alarms in the cross-hatched area, which was free of targets and clutter (fig.2). Each scan consisted of 100 target-free sweeps and,in order to estimate the probability of detection, 1 sweep in the direction of a target. Each run consisted of 1000 scans. The range interval under surveillance between r_{max} and r_{min} minus a range ring at the target range r_t, consisted of 1000 range bins.

X During one typical cycle of 5 runs the first and second thresholds were set at such levels D_1 and D_2 that the required probability of transmission of a second pulse for noise only α_1, and the required false alarm probability α were obtained. Then the radiated signal energy per pulse was chosen such that the probability of detection was 0.50 (run 1).

Then, for conventional single pulse detection, the threshold level D was chosen such that the same false alarm probability resulted and the detection probability was determined for three values of the signal energy (1 dB apart) (runs 2, 3 and 4).
Finally the first sequential run was repeated as a check on any change in average target strength, receiver drift, absence of interference etc (run 5).
By interpolation of the results of runs 2, 3 and 4, the signal energy required for a detection probability of 0.50 was obtained. The ratio of the required signal energies for conventional and sequential detection is equal to R_C/R_S.

Two targets have been used: the steel tower of a former radio and television station, located in the North Sea 9300 m off the coast (sea target) and a metal covered factory hall at a distance of 15500 m (land target). The results are shown in fig.1.
The main reason for the spread in the results is that the average echo strength was not constant during one cycle of measurements (5 runs with a total duration of 30 minutes), while this change could not be compensated for.

Conclusion

The measurements confirm the theoretical results. A higher gain of sequential detection can be obtained by applying such methods as range dependent thresholds to concentrate the improvement at the end of the range, reduced range resolution at the first transmission and increased signal energy at the second transmission.

For instance a gain of 3.6 dB may be obtained for a false alarm probability per sweep $\alpha = 10^{-4}$ and m=1000 range bins, if the resolution during the first transmission is reduced to 50 bins and the second pulse has 3 or 4 times the energy of the first pulse.

Reference

1. J.J. Bussgang, "Sequential methods in radar detection", Proc IEEE, vol. 58, pp 731-743, May 1970.

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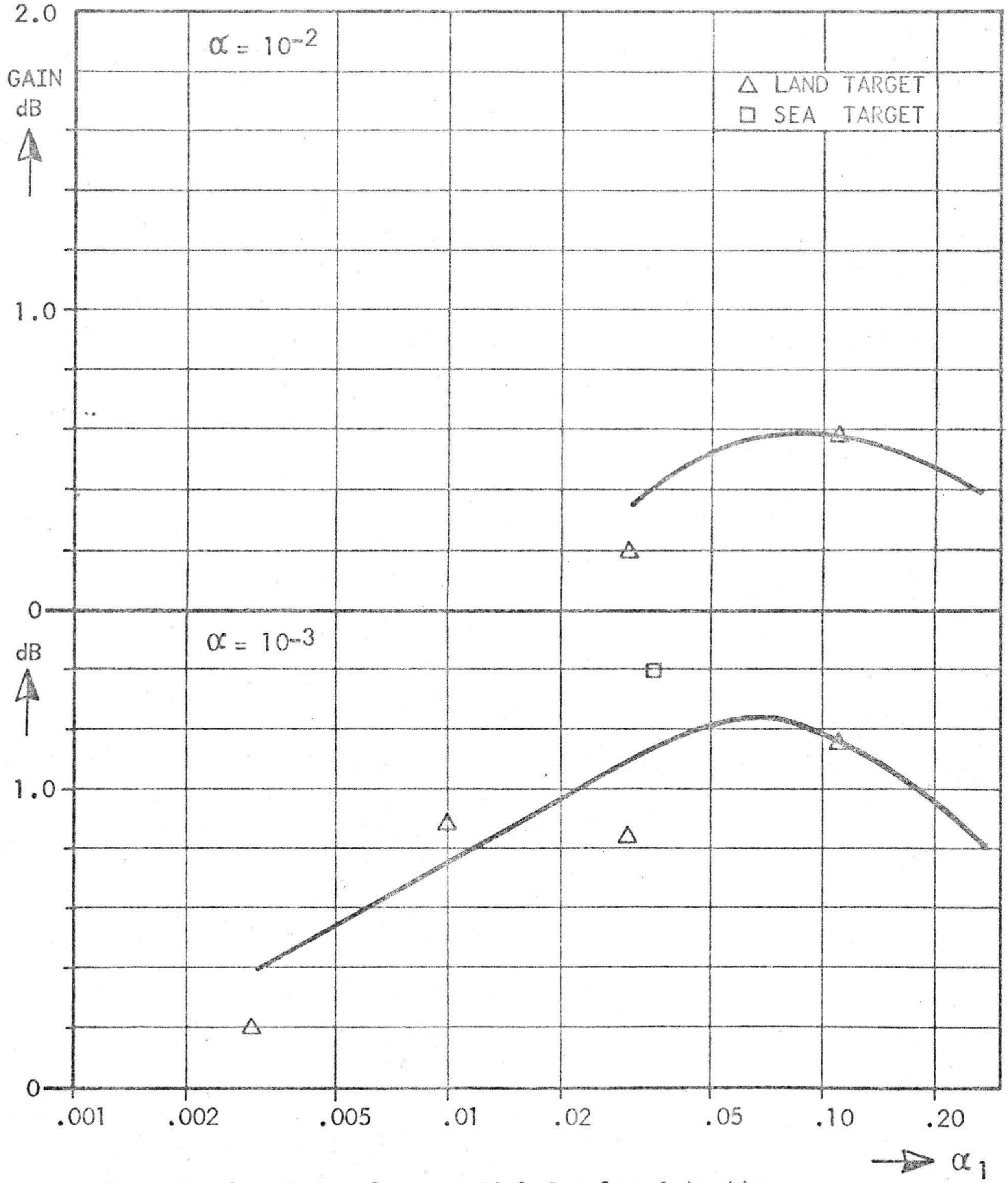

Fig. 1 a,b. Gain of sequential 2-pulse detection over conventional 1-pulse detection as a function of the probability of first false alarm for false alarm probabilities per sweep $\alpha = 10^{-2}$ and 10^{-3}

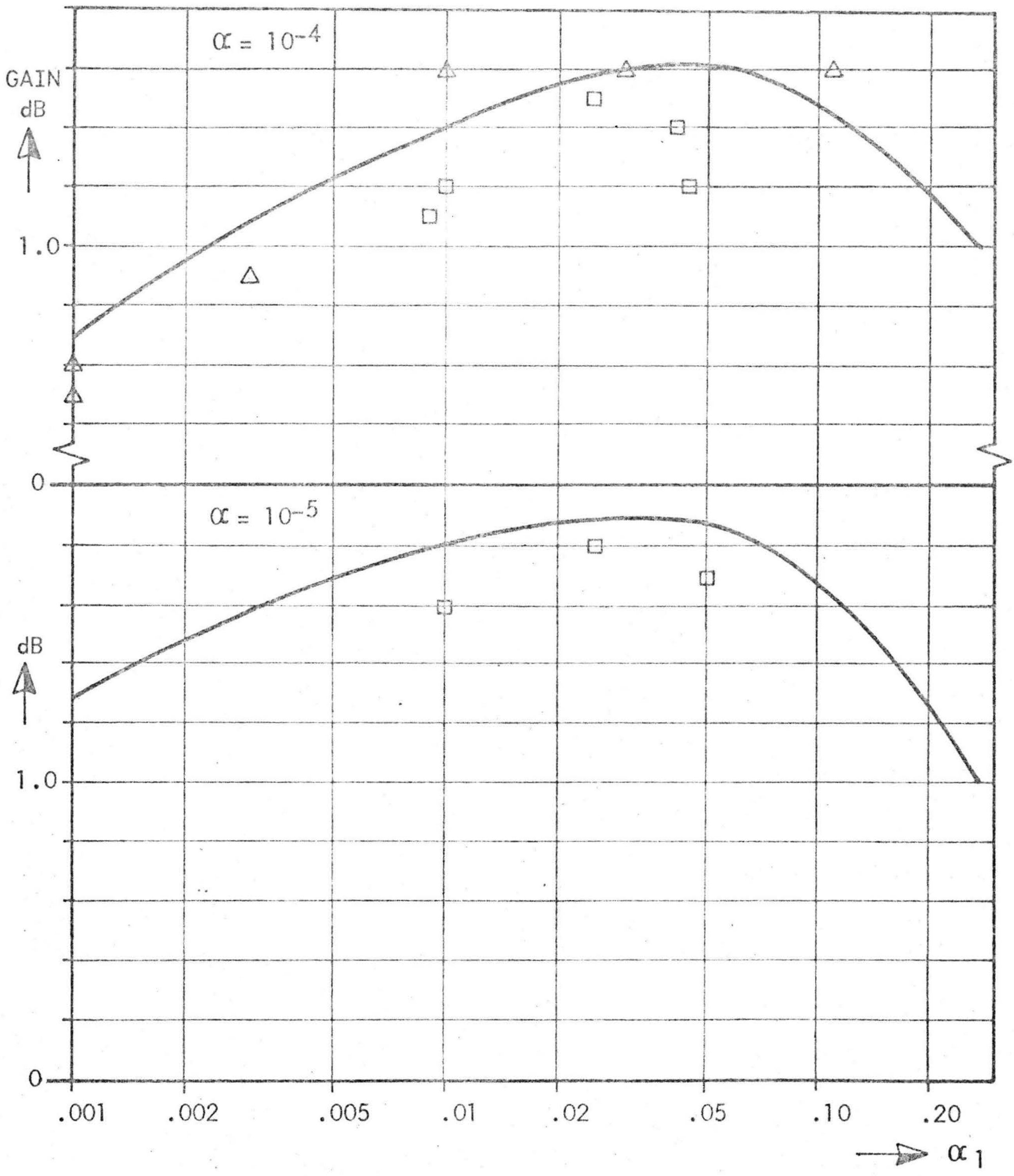

Fig. 1 c,d. Idem for false alarm probabilities per sweep $\alpha = 10^{-4}$ and 10^{-5}

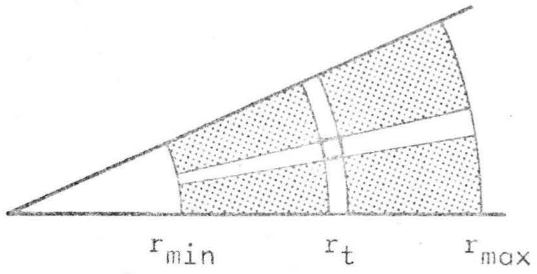

Fig. 2 Scan plane