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Abstract

A class of digital feed-forward filters is developed, satisfying the requirement of a maximally flat stopband characteristic at zero frequency if the pulse repetition frequency is modulated. A simple algorithm is given to obtain the filter coefficients. Some properties are summarized.

INTRODUCTION

High-pass digital feed-forward filters with alternating binomial weighting coefficients are frequently discussed in literature. A property of these filters is that they cancel input signals which are polynomials of degree N-2 (N = number of weighting coefficients). They provide good relative attenuation in the stopband and are therefore often proposed as clutter suppression filters in MTI radar (e.g.[3]). Besides many authors refer to binomial filters for comparing the performance of other types of filters. However if the p.r.f. is staggered to eliminate unwanted stopbands at multiples of the p.r.f. ("blind velocities" in MTI radar) the performance of the binomial filter degrades heavely [5] because the typical properties are no longer valid. In this letter a generalization of the binomial filter for staggered p.r.f. is developed from the requirement of a maximally flat stopband at zero frequency.

Theory

Let the weighting coefficients of a digital feed-forward filter with N elements be denoted by $\{w_n\}$ = w_0 ..... w_{N-1}. Then the transfer function y(f) of this filter for a non-uniformly sampled signal is (see figure)

$$y(f) = \sum_{n=0}^{N-1} w_n \, e^{j2\pi f t_n} \qquad (1)$$

where t_0 ..... t_{N-1} are the sample moments with $t_0=0$.

The Taylor series expansion of $y(f)$ around $f=0$ is

$$y(f) = y(0) + \frac{y'(0)}{1\,!} f + \frac{y''(0)}{2\,!} f^2 + \ldots\ldots \qquad (2)$$

with $$y^{(k)}(0) = (j2\pi)^k \sum_{n=0}^{N-1} t_n{}^k\, w_n; \; k = 0,\, 1,\, 2,\ldots \qquad (3)$$

Now we develop a class of filters with maximally flat stop-band by requiring that the first M coefficients $y^{(k)}(0)$, $k=0 \ldots\ldots M-1$ in (2) be zero.
Then the remaining series has a leading term which is proportional to f^M. If M is maximum then $y(f)$ is maximally flat around $f=0$. Each coefficient being set to zero yields a homogeneous linear equation in $\{w_n\}$. As there are N variables w_n we can form a system of maximally N-1 linearly independent equations with a non-trivial solution, so M = N-1. This yields (using (3)):

$$\sum_{n=0}^{N-1} t_n{}^k\, w_n = 0, \qquad k = 0,\, 1,\, \ldots\ldots N-2 \qquad (4)$$

Since $t_0=0$ only the first equation of the system in (4) has a term containing w_0. Then we have

$$\left.\begin{array}{l} w_1 + w_2 + \ldots\ldots\ldots\ldots + w_{N-1} = -w_0 \\ t_1\, w_1 + t_2\, w_2 + \ldots\ldots\ldots\ldots + t_{N-1}\, w_{N-1} = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ t_1^{N-2}\, w_1 + t_2^{N-2}\, w_2 + \ldots\ldots\ldots\ldots + t_{N-1}^{N-2}\, w_{N-1} = 0 \end{array}\right\} \qquad (5)$$

or $$T\,\underline{w} = -w_o\,\underline{u}_1 \qquad (6)$$

with

$$\underline{w}' = (w_1, w_2, \ldots\ldots w_{N-1})$$
$$\underline{u}_1' = (1, \ 0, \ \ldots\ldots 0 \ \ )$$

and

$$T = \begin{pmatrix} 1 & 1 & \ldots\ldots & 1 \\ t_1 & t_2 & \ldots\ldots & t_{N-1} \\ \ldots & \ldots & \ldots & \ldots \\ t_1^{N-2} & t_2^{N-2} & \ldots\ldots & t_{N-1}^{N-2} \end{pmatrix} \tag{7}$$

The system (6) can be solved by inverting the matrix T if T^{-1} exists:

$$\underline{w} = -w_0 \, T^{-1} \, \underline{u}_1 \tag{8}$$

The matrix T is a Vandermonde matrix [6] and T^{-1} exists since $t_i \neq t_j$ for $i \neq j$.

Let the inverse of T be denoted by R with elements r_{ij}.

Then in the appendix it is shown that

$$r_{ij} = (-1)^{N-1-j} \prod_{\substack{n=1 \\ n \neq i}}^{N-1} (t_i - t_n)^{-1} \sum_{C_{N-2}^{N-1-j}} t_{l_1} \cdot t_{l_2} \cdots t_{l_{N-1-j}} \tag{9}$$

$$i, \ j = 1, \ \ldots N-1$$

Here C_{N-2}^{N-1-j} denotes that the sum has to be taken over all $\binom{N-2}{N-1-j}$ products having a combination of N-1-j indices $(l_1, l_2, \ldots\ldots l_{N-1-j})$ out of a set of N-2 values 1, 2, ... i-1, i+1, ... N-1. Each product comprises N-1-j factors (a product comprising 0 factors being 1).

Since $\underline{u}_1$ in (8) has only one non-zero component only the first column of R needs to be evaluated:

$$w_i = -w_o \cdot r_{i,1} \qquad i=1 \ \ldots\ldots \ N-1 \tag{10}$$

This is the column with j=1 (see (9)).

Since $C_{N-2}^{N-2} = \binom{N-2}{N-2} = 1$, (10) simplifies to

$$w_i = -w_0 \prod_{\substack{n=1 \\ n \neq i}}^{N-1} \frac{t_n}{t_n - t_i} \qquad i=1, \ldots\ldots N-1 \tag{11}$$

(c.f. [1], p.44 and [2], p. 349).

Properties

1. (Frequency domain)

   According to the definition (4) a filter defined by (11) has a maximum number of derivates of the transfer function in f=0 being zero. (The first N-1 terms of the Taylor series expansion of the frequency characteristic are zero).

2. (Time domain)

   Suppose that the filter input $\{x_n\}$ consists of N samples of a function $x=t^k$ (k integer):

$$x_n = t_n^{\,k}, \qquad n=0, \ldots\ldots N-1 \tag{12}$$

   Let the output at t_{N-1} be denoted by z. Then:

$$z = \sum_{n=0}^{N-1} t_n^{\,k}\, w_n \tag{13}$$

   Comparing (13) with (4) learns that a filter defined by (11) rejects signals that are proportional to t^k if $k \leqq N-2$.

   So the filter has the property of rejecting the first N-2 terms of the Taylor series expansion of the input signal (c.f. [1], p.43).

3. (Uniform p.r.f.)

It is easily verified that if $t_j = j$, $j=0 \ldots\ldots N-1$ then (11) can be written as

$$w_k = (-1)^k \, w_0 \binom{N-1}{k} \tag{14}$$

This expression can be recognized as the binomial filter which can be obtained by cascading N-1 single delay line cancelers.

Application of this type of filters in MTI radar for suppression of echoes from stationary targets (clutter) is discussed in [5].

APPENDIX

The inverse of the Vandermonde Matrix.

(c.f. [6], p.125).

Let T be a Vandermonde matrix of dimension N-1.

$$T = \left\{ t_j^{\,i} \right\} \quad i=0 \;.....\; N-2; \quad j=1 \;.....\; N-1 \qquad (15)$$

and let the inverse of T be denoted by $R = \left\{ r_{ij} \right\}$.

Then

$$RT = I \qquad (16)$$

Observe the fundamental polynomials

$$\pi_i(t_k) = \prod_{\substack{j=1 \\ j\neq i}}^{N-1} (t_k - t_j) \qquad i,\ k=1 \;.....\; N-1$$

$$\triangleq \sum_{j=0}^{N-2} c_{ij}\, t_k^{\,j} \begin{cases} \neq 0 & \text{if} \quad i=k \\ = 0 & \text{if} \quad i\neq k \end{cases} \qquad (17)$$

or

$$\sum_{j=1}^{N-1} c_{i,j-1}\, t_k^{\,j-1} = \delta_{ik} \cdot \pi_i(t_i); \quad i,k=1 \ldots N-1 \quad (18)$$

where δ_{ik} is the Kronecker delta and (from (17))

$$c_{i,j} = (-1)^{N-2-j} \sum_{C_{N-2}^{N-2-j}} t_{l_1} \cdot t_{l_2} \cdots t_{l_{N-2-j}} \qquad (19)$$

In (19) C_{N-2}^{N-2-j} denotes that the sum has to be taken over all $\binom{N-2}{N-2-j}$ products having a combination of N-2-j indices $(l_1,\ l_2,\ .....\ l_{N-2-j})$ out of a set of N-2 values 1, 2, ... i-1, i+1, ... N-1. Each product comprises N-2-j factors (a product comprising 0 factors being 1).

On the other hand, since the i,j-th element of RT must equal δ_{ij} in order to satisfy (17) we can write

$$\sum_{j=1}^{N-1} r_{ij}\, t_k^{\,j-1} = \delta_{ik} \qquad i,\ k = 1 \,.....\, N-1 \qquad (20)$$

Comparing (20) with (18) and using (19) we find that

$$r_{ij} = \frac{c_{i,j-1}}{\pi_i(t_i)} = (-1)^{N-1-j}\left\{\pi_i(t_i)\right\}^{-1} \sum_{C_{N-2}^{N-1-j}} t_{l_1}\cdot t_{l_2}\cdots t_{l_{N-1-j}}$$

$$i,\ j = 1,\ .....\ N-1 \qquad (21)$$

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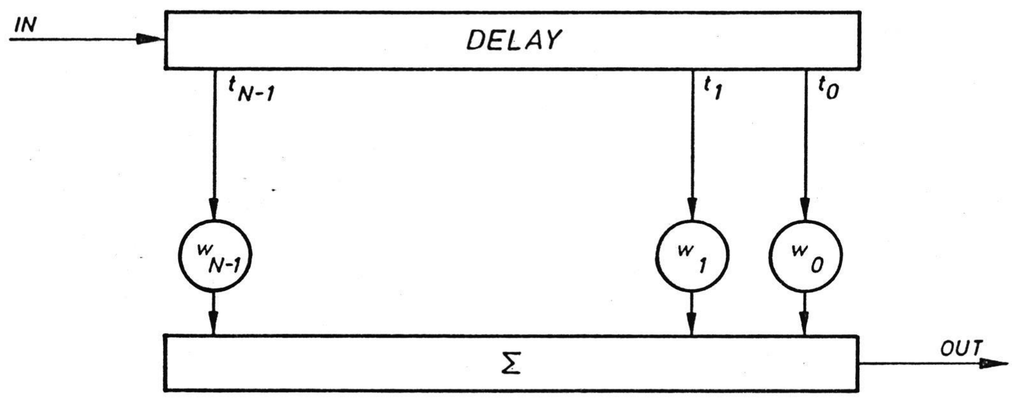
IN
DELAY
t_{N-1}
t_1
t_0
w_{N-1}
w_1
w_0
Σ
OUT