Calculating the 3-D Absorbed Power Distribution Inside a Human Body that is Illuminated by an Incident EM Field Using the WCG-FFT Method

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1. ABSTRACT

During the past several years considerable effort has been put into the development of computational techniques for handling the scattering and diffraction of electromagnetic waves by an object. We can distinguish between global techniques (e.g., the use of wave function expansion and integral equation) and local techniques (finite-difference and finite-elements methods).

In this paper we present a global technique to solve the full three-dimensional scattering problem by strongly inhomogeneous objects. This domain-integral is formulated in the frequency-domain. The strong form is weakened by using appropriate test functions and expansion functions. Subsequently, the domain-integral equation obtained is then solved using an iterative Conjugate Gradient scheme combined with an efficient computation of the convolutional integral involved by using the Fast Fourier Transform algorithm (WCG-FFT method). In order to show the accuracy of the method we first compare the numerical results associated with the scattering problem by an inhomogeneous dielectric sphere with the Mie-series solution. Then, numerical computations are carried out on a MRI-scan generated model of a human body inside a metallic enclosure with apertures. Here, we present the absorbed power density inside the human body.

2. WEAK FORM OF THE DOMAIN INTEGRAL EQUATION

The domain integral equation that is obtained in its strong form is weakened by testing it with appropriate testing functions and expanding the electric vector potential with appropriate expansion functions. The advantages of this procedure are, firstly, that the grad-div operator acting on the vector potential is integrated analytically over the domain of the object only, and secondly, that we have maintained the simple convolution structure of the vector potential. The integral equation is formulated in terms of the electric flux density. The continuity of the normal component of the electric flux density yields a correct implementation of the normal component of the electric field at the interfaces of (strong) discontinuity. No surface integrals that are directly related to surface-charges are introduced. The weak domain integral equation is solved using an iterative conjugate gradient scheme in which the convolution integral is computed efficiently by using the Fast Fourier Transform algorithm. This procedure is introduced by Zwamborn and van den Berg in [1] and we shall only resume the results.

We consider the problem of scattering by a lossy inhomogeneous dielectric object with complex permittivity

$$\varepsilon(\boldsymbol{x}) = \varepsilon_r(\boldsymbol{x})\varepsilon_0 + i\frac{\sigma(\boldsymbol{x})}{\omega}, \tag{1}$$

where ε_r denotes the relative permittivity of the object with respect to the lossless and homogeneous embedding with permittivity ε_0, and where σ denotes the electric conductivity of the object. The incident electric field is denoted as $\mathbf{E}^i = (E_1^i, E_2^i, E_3^i)$. In this paragraph, we formulate the scattering problem as a domain-integral equation for the unknown electric flux density $\mathbf{D} = (D_1, D_2, D_3)$ over the object domain $\mathbb{D}^S$ as

$$\mathbf{E}^i(\boldsymbol{x}) = \frac{\mathbf{D}(\boldsymbol{x})}{\varepsilon(\boldsymbol{x})} - (k_0^2 + \mathrm{grad\,div})\mathbf{A}(\boldsymbol{x}), \quad \boldsymbol{x} \in \mathbb{D}^S, \tag{2}$$

where $k_0 = \omega(\varepsilon_0\mu_0)^{\frac{1}{2}}$ and the vector potential $\mathbf{A} = (A_1, A_2, A_3)$ is given by

$$\mathbf{A}(\boldsymbol{x}) = \frac{1}{\varepsilon_0}\int_{\boldsymbol{x}'\in\mathbb{D}^S} G(\boldsymbol{x}-\boldsymbol{x}')\chi(\boldsymbol{x}')\mathbf{D}(\boldsymbol{x}')d\boldsymbol{x}', \tag{3}$$

in which the normalized contrast function χ is defined as

$$\chi(\boldsymbol{x}) = \frac{\varepsilon(\boldsymbol{x}) - \varepsilon_0}{\varepsilon(\boldsymbol{x})}. \tag{4}$$

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Further, the three-dimensional Green's function G is given by

$$G(\boldsymbol{x}) = \frac{\exp(ik_0|\boldsymbol{x}|)}{4\pi|\boldsymbol{x}|}, \quad \boldsymbol{x} \in \mathbb{R}^3. \tag{5}$$

Testing and expansion procedure

We first introduce a discretization in the spatial domain $\boldsymbol{x} = (x_1, x_2, x_3)$. We use a uniform mesh with grid widths of Δx_1, Δx_2 and Δx_3 in the x_1, x_2 and x_3 directions, respectively.

In order to cope with the grad-div operator in Eq. (2), we test the strong form of Eq. (2) by multiplying both sides of the equality sign by a vectorial testing function $\boldsymbol{\Psi}^{(p)}_{M,N,P}(\boldsymbol{x})$, $p = 1, 2, 3$, and integrate the result over the domain $\boldsymbol{x} \in \mathbb{D}^{\mathrm{S}}$. The testing function $\boldsymbol{\Psi}^{(p)}_{M,N,P}(\boldsymbol{x}) = \psi^{(p)}_{M,N,P}(\boldsymbol{x})\boldsymbol{i}_p$ is a volumetric rooftop function.

Using these functions we obtain the following weak formulation of the domain-integral equation

$$\mathrm{e}^{i,(p)}_{M,N,P} = \sum_{I,J,K \in \mathbb{B}^3} \sum_{q=1}^{3} \left\{ \mathrm{D}^{(q)}_{I,J,K} u^{(p,q)}_{M,N,P;I,J,K} - \mathrm{A}^{(q)}_{I,J,K} \left[k_0^2 v^{(p,q)}_{M,N,P;I,J,K} + w^{(p,q)}_{M,N,P;I,J,K} \right] \right\}, \tag{6}$$

in which $\mathbb{B}^3$ denotes the set of discrete points located inside the object domain and

$$\mathrm{e}^{i,(p)}_{M,N,P} = \sum_{I,J,K \in \mathbb{B}^3} \sum_{q=1}^{3} \mathrm{E}^{i,(q)}_{I,J,K} v^{(p,q)}_{M,N,P;I,J,K}, \tag{7}$$

$$u^{(p,q)}_{M,N,P;I,J,K} = \int_{\boldsymbol{x} \in \mathbb{D}^{\mathrm{S}}} \varepsilon^{-1}(\boldsymbol{x}) \boldsymbol{\Psi}^{(p)}_{M,N,P}(\boldsymbol{x}) \cdot \boldsymbol{\Psi}^{(q)}_{I,J,K}(\boldsymbol{x}) d\boldsymbol{x}, \tag{8}$$

$$v^{(p,q)}_{M,N,P;I,J,K} = \int_{\boldsymbol{x} \in \mathbb{D}^{\mathrm{S}}} \boldsymbol{\Psi}^{(p)}_{M,N,P}(\boldsymbol{x}) \cdot \boldsymbol{\Psi}^{(q)}_{I,J,K}(\boldsymbol{x}) d\boldsymbol{x}, \tag{9}$$

$$w^{(p,q)}_{M,N,P;I,J,K} = \int_{\boldsymbol{x} \in \mathbb{D}^{\mathrm{S}}} \mathrm{div}\boldsymbol{\Psi}^{(p)}_{M,N,P}(\boldsymbol{x}) \, \mathrm{div}\boldsymbol{\Psi}^{(q)}_{I,J,K}(\boldsymbol{x}) d\boldsymbol{x}. \tag{10}$$

In order to cope with the singularity of the Green's function we employ the spherical mean of Eq. (5) and compute the vector potential by using a FFT algorithm. More details can be found in [1].

3. NUMERICAL RESULTS

In this chapter we briefly discuss the applicability of WCG-FFT for the numerical approximation of three-dimensional scattering problems, such as the scattering by a strongly inhomogeneous dielectric sphere and the computation of the absorbed power distribution inside a human body that is illuminated by an incident plane wave. The human body is located inside a metallic structure with apertures. The human model is obtained from a MRI-scan generated dataset [3]. It should be noted that is not the objective of the paper to discuss countermeasures against an electromagnetic health treat. Instead, we want to show the applicability of the method for complex electromagnetic scattering problems or studies concerning the analysis of possible hazards caused by non-ionizing radiation.

3.1 Scattering by the Dielectric Sphere

In this paragraph, we present the case of a plane wave incident on a radially inhomogeneous lossy dielectric sphere. For this test case, analytical results are available. The electric field is polarized parallel to the x_1-axis and propagates along the negative x_3-axis. The origin of the sphere is located in $\{0, 0, 0\}$. We present the magnitude of the electric field strength inside a lossy inhomogeneous sphere with $\varepsilon_{r;1} = 72, \sigma_1 = 0.9$ S/m and $k_0 r_1 = 0.163$ and $\varepsilon_{r;2} = 7.5, \sigma_2 = 0.05$ S/m and $k_0 r_2 = 0.314$, where r_1 and r_2 denote the radius.

In the WCG-FFT method we have used mesh sizes of $15 \times 15 \times 15$ and $29 \times 29 \times 29$ to discretize the sphere. For the $29 \times 29 \times 29$ case, the WCG-FFT method needed 404 iterations to obtain a residual error less than 0.1 percent. On the Convex C230 this calculation took 15500 s CPU time and required 18 Mbyte of storage.

In Figures 1a and 1b we present the magnitude of the electric field inside the sphere. Compared to the Mie-series solution, it has been observed that the iterative WCG-FFT method produces very accurate results, even for a coarse mesh. In case the mesh becomes more dense, it has been shown that the numerical scheme approximates the analytical solution more accurate.

3.2 Scattering by a Complex Strongly Inhomogeneous Structure

Let us yet consider a human body located inside a metallic enclosure. This enclosure has three apertures. One small aperture in front of the eyes and the other two apertures at both hand-sides. With respect to the human body the Cartesian reference frame is chosen such that the x_1-axis is parallel with front to back, the x_2-axis is parallel with left to right and the x_3-axis is parallel with feet to head. The frequency of operation is taken to be 450 MHz. The electromagnetic field is polarized parallel to the x_2-axis and propagates along the positive x_1-axis. The human body is discretized into $21 \times 31 \times 53$ subdivisons with mesh widths $\Delta x_1 =$ 1.676 cm, $\Delta x_2 =$ 1.601 cm and $\Delta x_3 =$ 3.123 cm. For muscle we have taken $\varepsilon_{r,M}$ =71.5, σ_M =0.83 S/m, for

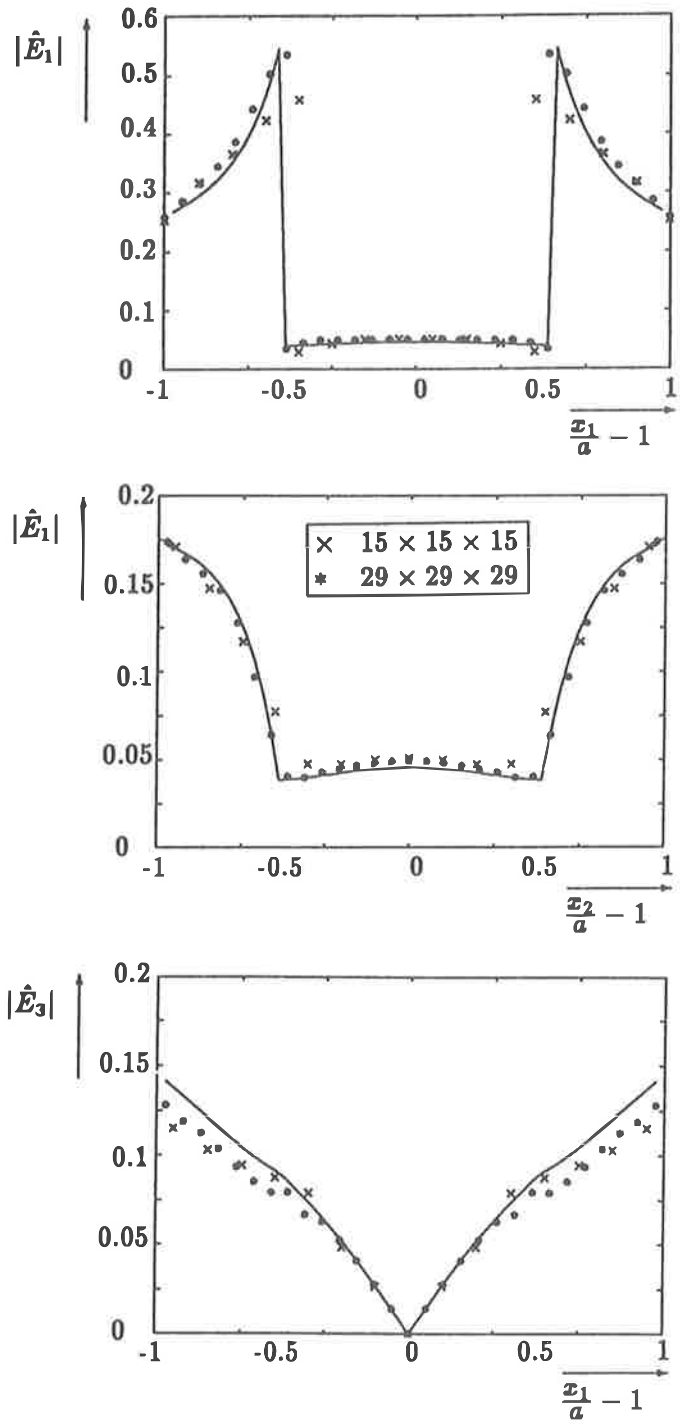

Figure 1: The magnitude of the electric field inside a strongly inhomogeneous sphere with $\varepsilon_{r,1}$ =72, σ_1 =0.9 S/m and $k_0 r_1$ =0.163 and $\varepsilon_{r,1}$ =7.5, σ_1 =0.05 S/m and $k_0 r_1$ =0.314. The results pertaining to a mesh of 15 × 15 × 15 are presented by the symbols x and the results pertaining to a mesh of 29×29×29 are presented by the symbols *. The analytical solution (Mie series) is presented by the solid line.

fat we have taken $\varepsilon_{r,F}$ =15, σ_F =0.22 S/m and for metal we have taken $\varepsilon_{r,I}$ =1, σ_I =1000 S/m. Note that the subscripts M, F and I denote muscle, fat and metal, respectively. The WCG-FFT method needed 862 iterations to obtain a residual error norm of less than 0.1 percent. On the Convex C230 this calculation took about 59000 s CPU time and required 37 Mbyte of storage.

In Figure 2 we present the absorbed power density normalized to the maximum occuring power density in the human body. It should be noted that the absorbed power density is calculated as follows

$$\dot{w} = \frac{1}{2}\sigma|\mathbf{E}|^2. \tag{11}$$

Figure 2a shows the x_1x_2-plane at x_3 = 78.2 cm and in Figure 2b the x_2x_3 plane is shown at $x_1 = -1.676$ cm. It has been observed that the maximum values of the absorbed power density are located in the direct surroundings of the eye and brain tissue. Further, the absorbed power density is located at the outer boundary of the human body. This is expected because the penetration depth in the human body is limited for electromagnetic fields at 450 MHz.

4. CONCLUSIONS AND FUTURE DEVELOPMENTS

In this paper we have presented a weak formulation of the domain integral equation for solving three-dimensional scattering problems by strongly inhomogeneous dielectric objects. It has been shown that the iterative WCG-FFT method is an accurate and powerful tool to solve the frequency-domain integral equation and produces very accurate frequency-domain results. In order to obtain time-domain results, the WCG-FFT method could be extended in a fast marching-on-in-frequency procedure [2]. Subsequently, we use an inverse Fourier transform. This matter is under investigation at the moment. Further, it is anticipated that a considerable saving of CPU time could be obtained by using a hardware implementation of the FFT algorithm.

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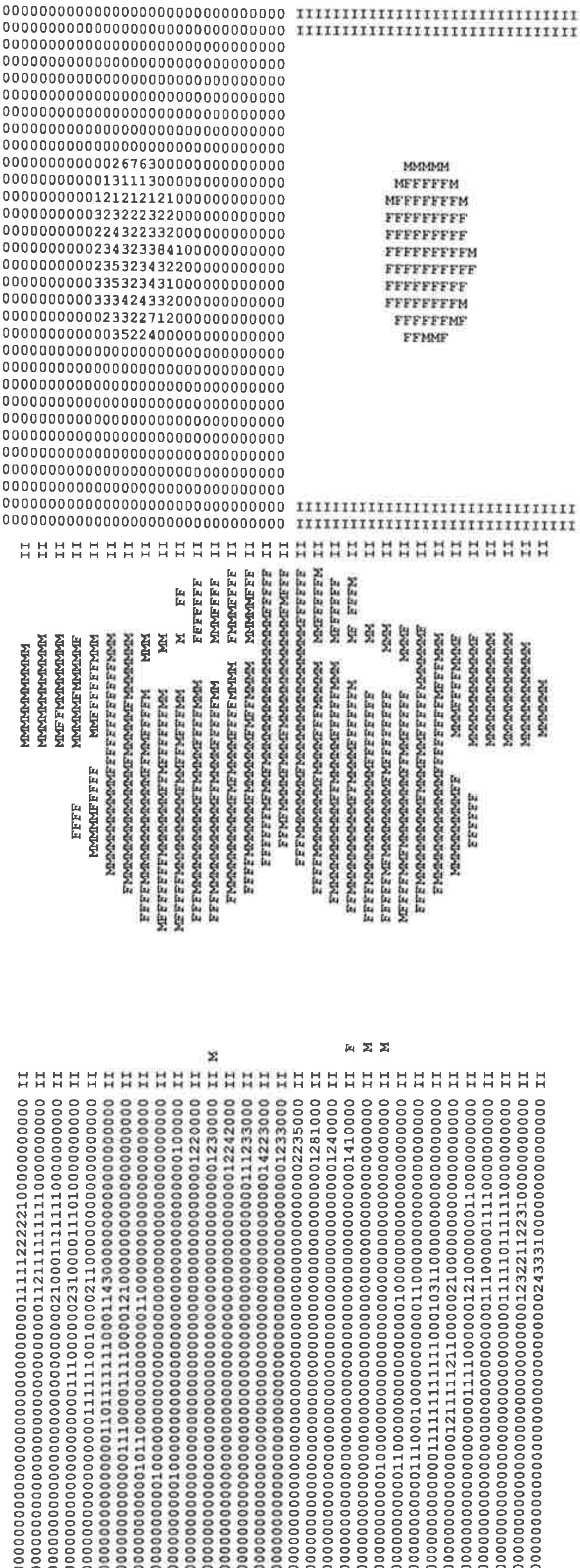

Figure 2: The normalized absorbed power density inside a human body.

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DISCUSSION

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1/ Did you also calculate the local and whole body SAR (Specific Absorption Rates) values in your model ?

2/ Did you compare your numerical results with those obtained by different numerical approaches for the same human-model ? Is there agreement ?

AUTHOR'S REPLY

1/ The model only calculates the total electric field. Once the field has been calculated one can calculate related quantities easily.

2/ At the moment we are comparing the numerical results of this model with outer schemes such as FD-TD. However, there is some agreement and disagreement between the results. Hence, there is still some research to be done in this field.