Non-linear Inversion Based on Contrast Source Gradients

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1. ABSTRACT

This paper describes a simple algorithm for reconstructing the complex index of refraction of a bounded object immersed in a known background from a knowledge of how the object scatters known incident radiation. The method described here is versatile accommodating both spatially and frequency varying incident fields and allowing *a priori* information about the scatterer to be introduced in a simple fashion. Numerical results show that this new algorithm outperforms the modified gradient approach which until now has been one of the most effective reconstruction algorithms available.

2. INTRODUCTION

The problem of reconstructing the complex index of refraction of a bounded object immersed in a known background medium, from a knowledge of how the object scatters known incident acoustic or electromagnetic radiation, has received a tremendous amount of attention in the past decade. Almost all reconstruction algorithms rely in some way upon the Lippmann-Schwinger equation or domain integral equation for the field inside the scattering object as well as the related integral representation for the field outside the object.

The present paper describes a simple algorithm for reconstructing unknown contrasts which is extremely versatile, accommodating both spatially and frequency varying incident fields and allowing for the introduction of *a priori* information, such as positivity constraints, in a simple fashion. The algorithm is a variant of the source type integral equation (STIE) method introduced by Habashy *et al.* [11] on one hand and the modified gradient approach used by the authors in [12], [13], [14] on the other. Numerical results show that, despite the simplicity of the algorithm, it outperforms the modified gradient approach which has been one of the most effective reconstruction algorithms available until now [25]. We present here the simplest version of the algorithm wherein we treat scalar waves in $\mathbb{R}^2$ for bodies immersed in a homogeneous background.

3. NOTATION AND PROBLEM STATEMENT

Denote by $\boldsymbol{p}$ and $\boldsymbol{q}$ position vectors in $\mathbb{R}^2$ and let B denote a bounded, not necessarily connected, scattering object (or objects) whose location and index of refraction or contrast is unknown but which is known to lie within another, larger, bounded simply connected domain D. If $u_j^{\text{inc}}(\boldsymbol{p}) = u^{\text{inc}}(\boldsymbol{p}, \boldsymbol{q}_j, k_j)$ denotes an incident wave with wavenumber k_j (assumed to be real) and source point $\boldsymbol{q}_j$ ($\boldsymbol{q}_j$ is replaced by the unit vector $\hat{\boldsymbol{q}}_j$ for plane waves) then for a large class of scattering problems the total field in D is known to satisfy the integral equation

$$u_j(\boldsymbol{p}) = u_j^{\text{inc}}(\boldsymbol{p}) + k_j^2 \int_D G_j(\boldsymbol{p}, \boldsymbol{q})\, \chi_j(\boldsymbol{q})\, u_j(\boldsymbol{q})\, \mathrm{dv}(\boldsymbol{q}) \,, \quad (1)$$

where $G_j(\boldsymbol{p}, \boldsymbol{q})$ denotes the Green function of the background medium,

$$G_j(\boldsymbol{p}, \boldsymbol{q}) = \frac{\mathrm{i}}{4} H_0^{(1)}(k_j|\boldsymbol{p} - \boldsymbol{q}|) \,, \quad (2)$$

and

$$\chi_j = \frac{k^2(\boldsymbol{q}, k_j)}{k_j^2} - 1 \,. \quad (3)$$

We assume that, while the spatial dependence of χ_j is unknown, the frequency dependence is known, so that, for example, in electromagnetics for a Maxwell medium,

$$k_j = \omega_j(\varepsilon_0 \mu_0)^{\frac{1}{2}} \quad (4)$$

and

$$\chi_j(\boldsymbol{q}) = \frac{\varepsilon(\boldsymbol{q}) - \varepsilon_0}{\varepsilon_0} + \mathrm{i}\frac{\sigma(\boldsymbol{q})}{\omega_j \varepsilon_0} \,, \quad (5)$$

where ε_0 and μ_0 are the permittivity and the permeability of the (lossless) background, while $\varepsilon(\boldsymbol{q})$ and $\sigma(\boldsymbol{q})$ are the permittivity and conductivity of the scatterer which is assumed to be nonmagnetic. Equation (5) may also be written as

$$\chi_j(\boldsymbol{q}) = \chi^r(\boldsymbol{q}) + \mathrm{i}\chi^i(\boldsymbol{q})\eta_j, \quad \eta_j = \frac{\omega_0}{\omega_j} = \frac{k_0}{k_j} \,, \quad (6)$$

where k_0 is the wavenumber for $j = 0$. This equation simplifies to

$$\chi_j(\boldsymbol{q}) := \chi(\boldsymbol{q}) = \chi^r(\boldsymbol{q}) + \mathrm{i}\chi^i(\boldsymbol{q}) \,, \quad (7)$$

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if all the measurements are made at the same frequency, say at $k_j = k_0$.
Observe that if p is not in B then χ_j vanishes, but if the location of B is unknown then it is not known *a priori* where χ_j vanishes. However with the assumption that $B \subset D$ it is known that χ_j vanishes for p outside D. In fact denoting by S a domain (or curve, or a discrete collection of points) outside of D where the scattered field is measured to be $f_j(p)$, (1) becomes

$$f_j(p) = k_j^2 \int_D G_j(p, q)\, \chi_j(q)\, u_j(q)\, \mathrm{dv}(q)\,, \qquad p \in S\,, \tag{8}$$

if there is no noise or error in the measurements. But error free data are extremely unlikely and we do not assume that (8) holds exactly. Rewriting (1) and (8) in symbolic form we have the object or state equations

$$u_j = u_j^{\text{inc}} + G_j^D \chi_j u_j\,, \quad p \in D\,, \tag{9}$$

and the data equations

$$f_j = G_j^S \chi_j u_j\,, \quad p \in S\,, \tag{10}$$

the superscripts D and S on the operators defined implicitly in (1) and (8) are added to accentuate the location of the point p, since the operators are identical in all other respects.

4. THEORETICAL BACKGROUND

In the absence of other *a priori* information, (9) and (10) are the only equations we have relating the unknown contrast χ_j (which, recall, consists of at most two unknown real valued functions) and the unknown fields u_j in D. The known data consist of the incident fields, u_j^{inc}, the wavenumbers, k_j, the measured data, f_j, and the test domain D. Equations (9) and (10) are linear in each of the unknowns χ_j and u_j, but since both are unknown the problem is in fact mildly nonlinear. Of course the dependence of the fields u_j on the contrast χ_j is highly non-linear. This may be seen by writing the formal inverse of (10) as

$$u_j = (I - G_j^D \chi_j)^{-1} u_j^{\text{inc}}\,. \tag{11}$$

This form has been utilized in a number of inversion methods. Introducing it into the data equations we obtain

$$f_j = G_j^S [\chi_j (I - G_j^D \chi_j)^{-1} u_j^{\text{inc}}]\,, \tag{12}$$

wherein the non-linearity of the inverse problem is clearly exposed. Approximating the inverse operator by

$$(I - G_j^D \chi_j)^{-1} \approx I\,, \tag{13}$$

leads to the Born approximation, while in iterative methods, where a sequence $\{\chi_{j,n}\}$ is constructed, the approximation

$$(I - G_j^D \chi_{j,n})^{-1} \approx (I - G_j^D \chi_{j,n-1})^{-1} \tag{14}$$

gives rise to the iterative Born method [20], while the linearization of

$$(I - G_j^D \chi_{j,n})^{-1} \approx [I - G_j^D \chi_{j,n-1} - G_j^D (\chi_{j,n} - \chi_{j,n-1})]^{-1} \tag{15}$$

in terms of $\Delta\chi_{j,n} = \chi_{j,n} - \chi_{j,n-1}$, namely,

$$(I - G_j^D \chi_{j,n})^{-1} \approx [I + (I - G_j^D \chi_{j,n-1})^{-1} G_j^D \Delta\chi_{j,n}](I - G_j^D \chi_{j,n-1})^{-1} \tag{16}$$

leads to the Newton-Kantorovich method [17], [19] which has been shown to be equivalent to the Distorted Born approach [6]. Observe that at each step in the iteration it is necessary to compute the action of the operator $[I - G_j^D \chi_{j,n-1}]^{-1}$ for known $\chi_{j,n-1}$ This means that forward problems, or direct scattering problems must be solved at each iterative step.
A method which avoids the necessity of solving forward problems completely was proposed by the authors [12] and was refined [13], [14], [25] and extended [24]. This modified gradient method involves the simultaneous construction of sequences $\{u_{j,n}\}$ and $\{\chi_{j,n}\}$ to minimize the error in a cost functional consisting of the normalized errors in both state equations (9) and data equations (10). It has proven to be very effective in a large number of numerical tests using both synthetic and experimental data.
Because the contrast and fields occur as a product, many workers have introduced the quantity

$$w_j = \chi_j u_j\,, \tag{17}$$

which is called a contrast source since u_j satisfies the equation

$$(\nabla^2 + k_j^2) u_j = -k_j^2 w_j \quad \text{in } B\,, \tag{18}$$

Then the data equations become

$$f_j = G_j^S w_j\,, \tag{19}$$

while the state equations become

$$u_j = u_j^{\text{inc}} + G_j^D w_j\,, \tag{20}$$

or, with (17),

$$\chi u_j^{\text{inc}} = w_j - \chi_j G_j^D w_j\,. \tag{21}$$

Equation (19) is called by some a source type integral equation and it has a long history. It is a classic ill-posed equation and for a time there was considerable attention paid to the question of uniqueness, e.g., [1], [8], [2]. It was shown that there exist non-trivial solutions of the homogeneous form of (19), although it was argued by some that uniqueness could be restored from physical considerations. A good summary of the debate is given by Devaney and Sherman [9] and the responses by Bojarski [3] and Stone [18]. It is not our intent to renew this controversy since it is now well accepted that non-trivial solutions of (19) exist. Moreover it has also been shown that the minimum norm solution of (19), the solution produced, for example, by the conjugate gradient method, is not the appropriate physical solution. Nonetheless this source type equation has served as an essential ingredient in many inversion procedures, e.g., [10], [4], [5], [7], [11]. Habashy *et al.* [11] present an inversion method wherein the minimum norm solution of (19) is found first and then a basis for the orthogonal complement of this solution is constructed in terms of which the physical solution is sought to satisfy (21). Van den Berg and Haak [22] proposed a variant of this technique wherein the full minimum norm solution is not found but rather it is sought iteratively, using conjugate gradient steps, with the contrast updated at each step to satisfy (21) and a new source defined through (17). This approach yielded promising numerical results however the error did not decrease monotonically.

In the present paper we propose a method which combines spirit of the approach of Van den Berg and Haak [22] using the source type integral equation with that of the modified gradient approach by seeking linear updates in the source in an error reducing method which does not require the solution of any forward problem.

5. CONTRAST SOURCE INVERSION METHOD

As in the modified gradient as well as the Van den Berg-Haak approach we simultaneously construct sequences of sources $w_{j,n}$, fields $u_{j,n}$ and contrasts $\chi_{j,n}$ to minimize a cost functional. Rather than choosing a cost functional consisting only of errors in the data equation, as Van den Berg and Haak did, we define the cost functional

$$F = \frac{\sum_j \|f_j - G_j^S w_j\|_S^2}{\sum_j \|f_j\|_S^2} + \frac{\sum_j \|\chi_j u_j^{inc} - w_j + \chi_j G_j^D w_j\|_D^2}{\sum_j \|\chi_j u_j^{inc}\|_D^2}, \quad (22)$$

where $\|\cdot\|_S^2$ and $\|\cdot\|_D^2$ denote the norms on $L_2(S)$ and $L_2(D)$, respectively. The normalization is chosen so that both terms are equal to one if $w_j = 0$. The first term measures the error in the data equations and the second term measures the error in the form of the state equations given in (21). This is a quadratic functional in w_j, but highly nonlinear in χ_j. We propose an iterative minimization of this cost functional using an alternating method which first updates w_j and then updates χ_j. Thus we construct sequences $\{w_{j,n}\}$ and $\{\chi_{j,n}\}$, for $n = 0, 1, 2, \cdots$, in the following manner.

Define the data error to be

$$\rho_{j,n} = f_{j,n} - G_j^S w_{j,n}, \quad (23)$$

and the state error to be

$$r_{j,n} = \chi_{j,n} u_{j,n} - w_{j,n}, \quad (24)$$

where

$$u_{j,n} = u_j^{inc} + G_j^D w_{j,n}. \quad (25)$$

Now suppose $w_{j,n-1}$ and $\chi_{j,n-1}$ are known. We update w_j by

$$w_{j,n} = w_{j,n-1} + \alpha_{j,n} v_{j,n}, \quad (26)$$

where $\alpha_{j,n}$ is constant and the update directions $v_{j,n}$ are functions of position.

The update directions are chosen to be the Polak-Ribière conjugate gradient directions

$$\begin{aligned} v_{j,0} &= 0, \\ v_{j,n} &= g_{j,n} + \frac{\langle g_{j,n}, g_{j,n} - g_{j,n-1}\rangle_D}{\langle g_{j,n-1}, g_{j,n-1}\rangle_D} v_{j,n-1}, \quad (27) \\ & \qquad n \geq 1, \end{aligned}$$

where $g_{j,n}$ is the gradient (Frechet derivative) of the cost functional with respect to w_j evaluated at $w_{j,n-1}$, $\chi_{j,n-1}$, while $\langle\cdot,\cdot\rangle_D$ denotes the inner product on $L_2(D)$. Explicitly this found to be

$$g_{j,n} = -\frac{G_j^{S^*} \rho_{j,n-1}}{\sum_k \|f_k\|_S^2} - \frac{r_{j,n-1} - G_j^{S^*}(\overline{\chi}_{j,n-1} r_{j,n-1})}{\sum_k \|\chi_{k,n-1} u_k^{inc}\|_D^2}, \quad (28)$$

where $G_j^{S^*}$ and $G_j^{D^*}$ are the adjoints of G_j^S and G_j^D mapping $L_2(S)$ into $L_2(D)$ and $L_2(D)$ into $L_2(D)$, respectively. Further the overbar denotes complex conjugate.

With the update directions completely specified the constant $\alpha_{j,n}$ is determined to minimize the cost functional

$$\begin{aligned} F = & \frac{\sum_j \|f_j - G_j^S w_{j,n}\|_S^2}{\sum_j \|f_j\|_S^2} + \frac{\sum_j \|\chi_{j,n-1} u_{j,n} - w_{j,n}\|_D^2}{\sum_j \|\chi_{j,n-1} u_j^{inc}\|_D^2} \\ = & \frac{\sum_j \|\rho_{j,n-1} - \alpha_{j,n} G_j^S v_{j,n}\|_S^2}{\sum_j \|f_j\|_S^2} \\ & + \frac{\sum_j \|r_{j,n-1} - \alpha_{j,n}(v_{j,n} - \chi_{j,n-1} G_j^D v_{j,n})\|_D^2}{\sum_j \|\chi_{j,n-1} u_j^{inc}\|_D^2} \quad (29) \end{aligned}$$

and is found explicitly to be

$$\begin{aligned} \alpha_{j,n} &= \frac{a+b}{c+d}, \quad (30) \\ a &= \frac{\langle \rho_{j,n-1}, G_j^S v_{j,n}\rangle_S}{\sum_k \|f_k\|_S^2}, \\ b &= \frac{\langle r_{j,n-1}, v_{j,n} - \chi_{j,n-1} G_j^D v_{j,n}\rangle_D}{\sum_k \|\chi_{k,n-1} u_k^{inc}\|_D^2}, \\ c &= \frac{\|G_j^S v_{j,n}\|_S^2}{\sum_k \|f_k\|_S^2}, \\ d &= \frac{\|v_{j,n} - \chi_{j,n-1} G_j^D v_{j,n}\|_D^2}{\sum_k \|\chi_{k,n-1} u_k^{inc}\|_D^2}, \end{aligned}$$

where $\langle\cdot,\cdot\rangle_S$ denotes the inner product on $L_2(S)$.

Once $w_{j,n}$ is determined, $u_{j,n}$ is obtained via (25) and (26) as

$$u_{j,n} = u_{j,n-1} + \alpha_{j,n} G_j^D v_{j,n}, \quad (31)$$

and we then seek χ_j to minimize the cost functional

$$F_D = \frac{\sum_j \|\chi_j u_{j,n} - w_{j,n}\|_D^2}{\sum_j \|\chi_j u_{j,n}\|_D^2}. \quad (32)$$

Since this minimization is not so easy especially in the case of *a priori* information, we use a minimization in two steps. First we minimize the much simpler cost functional

$$F_D' = \sum_j \|\chi_j u_{j,n} - w_{j,n}\|_D^2, \quad (33)$$

and then we use the found contrast as an optimization direction in a line minimization to minimize (32). This two-step minimization technique guarantees that the process is always error reducing and allows for easy implementation of *a priori* information or constraints on χ_j. Since we finally need to minimize the cost functional of (32), we define the contrast function that minimizes (33), $\tilde{\chi}_j$, through

$$F_D'(\tilde{\chi}_j) = \min(F_D') \quad (34)$$

Restricting attention to inhomogeneities complying with the Maxwell model, (6), and in the absence of any a priori information on χ, we find that, [23], F_D' is minimized by choosing

$$\begin{aligned} \tilde{\chi}_n^r &= \frac{\sum_j \mathrm{Re}(w_{j,n}\overline{u}_{j,n})}{\sum_j |u_{j,n}|^2}, \\ \tilde{\chi}_n^i &= \frac{\sum_j \eta_j \mathrm{Im}(w_{j,n}\overline{u}_{j,n})}{\sum_j \eta_j^2 |u_{j,n}|^2}. \quad (35) \end{aligned}$$

However if a priori information is available then it is relatively simple to incorporate it in choosing $\bar{\chi}_n$. If either χ^r or χ^i is known, then we merely use this known value in place of either the first equality or second equality of (35). Thus, for example, if $\chi^i = 0$, we limit our reconstruction procedure to $\bar{\chi}_n^r$ from the outset. If we have a priori information that χ^r and χ^i are positive, we use, [23]

$$\bar{\chi}_n^r = \left\{ \frac{\sum_j \left(\frac{\mathrm{Re}(w_{j,n}\bar{u}_{j,n})}{|u_{j,n}|} \right)^2}{\sum_j |u_{j,n}|^2} \right\}^{\frac{1}{2}}, \qquad (36)$$

$$\bar{\chi}_n^i = \left\{ \frac{\sum_j \left(\frac{\mathrm{Im}(w_{j,n}\bar{u}_{j,n})}{|u_{j,n}|} \right)^2}{\sum_j \eta_j^2 |u_{j,n}|^2} \right\}^{\frac{1}{2}}. \qquad (37)$$

These choices of $\bar{\chi}_n^r$ and $\bar{\chi}_n^i$ coincide with those obtained by Kohn and McKenney [15] for an optimization problem with a positivity constraint and employed in the modified gradient algorithm [14] with good results.

Next a line minimization is used to make the cost functional of equation (32) error reducing. We introduce a contrast update direction as

$$d_n = \bar{\chi}_n - \chi_{n-1} \qquad (38)$$

and we write χ_n as

$$\chi_n = \chi_{n-1} + \theta d_n \qquad (39)$$

Then θ is chosen to minimize the cost functional of equation (32)

$$\begin{aligned} \frac{\sum_j \|\chi_n u_{j,n} - w_{j,n}\|_D^2}{\sum_j \|\chi_n u_j^{\mathrm{inc}}\|_D^2} &= \frac{\sum_j \|\chi_{n-1} u_{j,n} - w_{j,n} + \theta(\bar{\chi}_n - \chi_{n-1})u_{j,n}\|_D^2}{\sum_j \|\chi_{n-1} u_j^{\mathrm{inc}} + \theta(\bar{\chi}_n - \chi_{n-1})u_j^{\mathrm{inc}}\|_D^2} \\ &= \frac{a\,\theta^2 + 2b\,\theta + c}{A\,\theta^2 + 2B\,\theta + C}, \end{aligned} \qquad (40)$$

where

$$\begin{aligned} a &= \sum_j \|(\bar{\chi}_n - \chi_{n-1})u_{j,n}\|_D^2, \\ b &= \mathrm{Re}\sum_j \langle \chi_{n-1}u_{j,n} - w_{j,n}, (\bar{\chi}_n - \chi_{n-1})u_{j,n}\rangle_D, \\ c &= \sum_j \|\chi_{n-1}u_{j,n} - w_{j,n}\|_D^2, \\ A &= \sum_j \|(\bar{\chi}_n - \chi_{n-1})u_j^{\mathrm{inc}}\|_D^2, \\ B &= \mathrm{Re}\sum_j \langle \chi_{n-1}u_j^{\mathrm{inc}}, (\bar{\chi}_n - \chi_{n-1})u_j^{\mathrm{inc}}\rangle_D, \\ C &= \sum_j \|\chi_{n-1}u_j^{\mathrm{inc}}\|_D^2, \end{aligned} \qquad (41)$$

This is the quotient of two quadratics which, using elementary analysis, may be shown to attain its minimum when

$$\theta = \frac{-(aC - Ac)}{2(aB - Ab)} + \frac{\sqrt{(aC - Ac)^2 - 4(aB - Ab)(bC - Bc)}}{2(aB - Ab)}. \qquad (42)$$

This completes the description of the algorithm except for designating the starting values $w_{j,0}$. Observe that we cannot start with $w_{j,0} = 0$ since then $\chi_0^r = \chi_0^i = 0$ and the cost functional (29) is undefined for $n = 1$. Therefore we choose as starting values either the constant values that minimize the data error,

$$w_{j,0}^c = \frac{\langle f_j, G_j^S 1\rangle_S}{\|G_j^S 1\|_S^2}, \qquad (43)$$

or the values obtained by backpropagation,

$$w_{j,0}^{\mathrm{bp}} = \frac{\|G_j^{S^*} f_j\|_D^2}{\|G_j^S G_j^{S^*} f_j\|_S^2} G_j^{S^*} f_j. \qquad (44)$$

This completes the description of the algorithm.

6. COMPUTATIONAL TESTS

A number of tests have been done with the algorithm including stability tests, resolution tests and test for various kinds of contrast profiles [23]. Here some tests are presented for using the method for reconstructing AP-mines. We used one frequency of 500 MHz, a relative background permittivity of $\varepsilon_r = 5$ and a background conductivity of $\sigma = 0$ S/m. Further we used a computational domain of 29×29 cm. The AP-mines were given a relative permittivity of $\varepsilon_r = 7$ and a conductivity of $\sigma = 0$ S/m. Their diameter varied between 5 cm and 10 cm. The measurement curve, S, is a circle of radius 1 m and center at the center of D. The discrete form of the algorithm is obtained by dividing D into 29×29 subsquares, assuming the contrast, sources and fields are piecewise constant and the integrals over subsquares were approximated by integrals over circles of equal area which were calculated analytically [16]. The discrete spatial convolution of the operators G^D and G^{D^*} were computed using FFT routines [21]. The incident fields were chosen to be excited by line sources parallel to the axis of the scatterer. These sources were taken to be equally spaced on the measurement circle, and the source locations were also chosen as discretization points on the circle. All integrals on S were approximated by point collocation at the discretization points, that is, the rectangular rule with the integrand evaluated at the mid-points. The measured data were simulated by solving the direct scattering problem with a conjugate gradient method [21]. The circle S was subdivided into 30 equally spaced arcs. Each mid-point served as the location of a line source and all the mid-points served as receiver. In all test backpropagation has been used for the initial guess.

In figure 1, we show the original contrast profile of a circular mine with a 7 cm diameter.

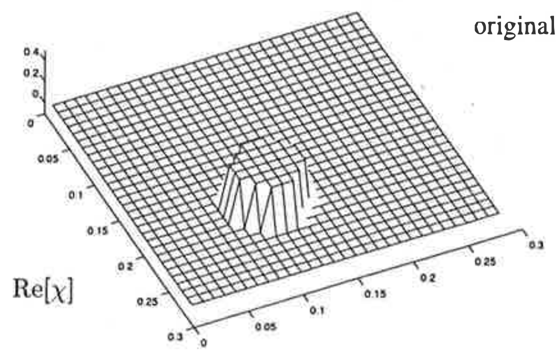

Fig 1: The original profile

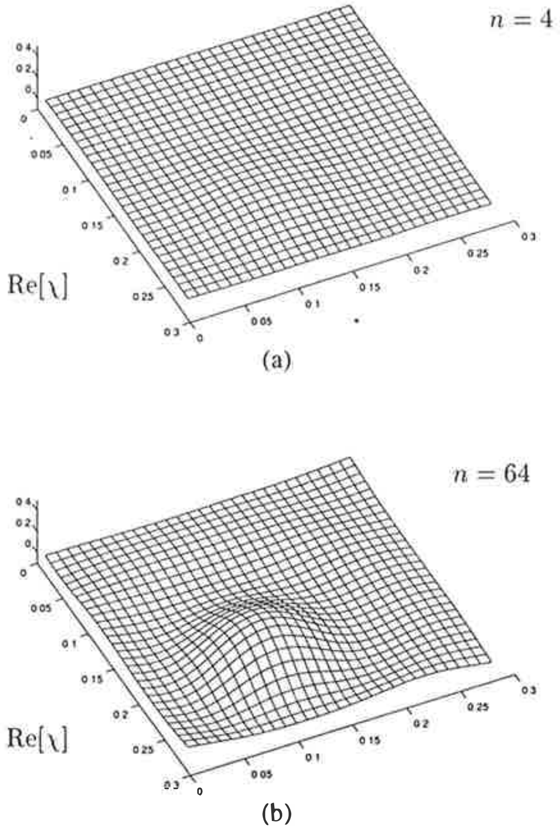

Fig 2: The reconstructions after 4 iterations (a), and after 64 iterations (b), using CSI without *a priori* information.

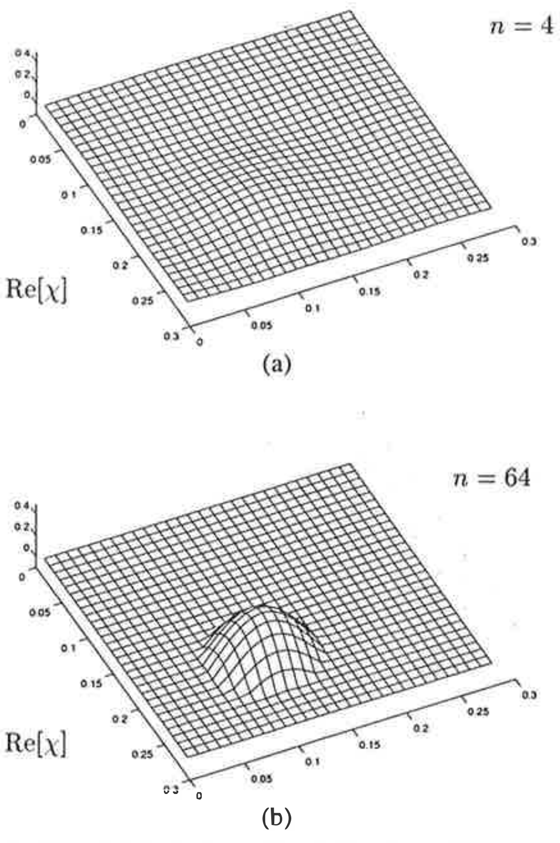

Fig 3: The reconstructions after 4 iterations (a), and after 64 iterations (b), using CSI with *a priori* information.

In figure 2, we show the reconstruction of the contrast profile of the mine after 4 and after 64 iterations. Continued iteration provided no noticeable improvement.
The conductivity of the mines is zero. We can therefore set the imaginary part of the contrast equal to zero. Since tests indicated that this restriction does not improve the algorithm, this restriction has been left out. Furthermore, since we used a permittivity of the mines which is higher than that of the background, we could use positivity for the mine permittivity. In figure 3, we show the reconstructed contrast profile after 4 iterations using CSI with *a priori* information, (a), and after 64 iterations with *a priori* information, (b). Each iteration took approximately 5 seconds on a Pentium PC computer.
Next we have put two mines in the configuration, one circular mine having a diameter of 5 cm and one having a

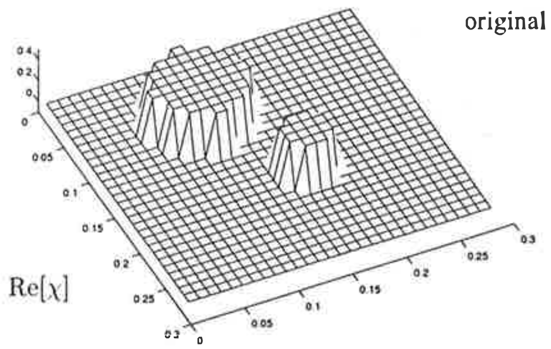

Fig 4: The original profile with two mines

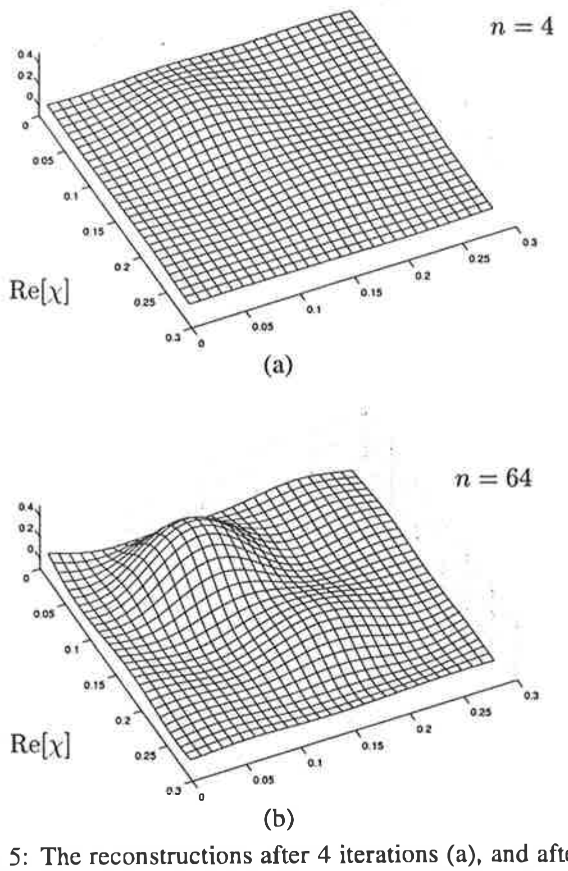

Fig 5: The reconstructions after 4 iterations (a), and after 64 iterations (b), using CSI without *a priori* information.

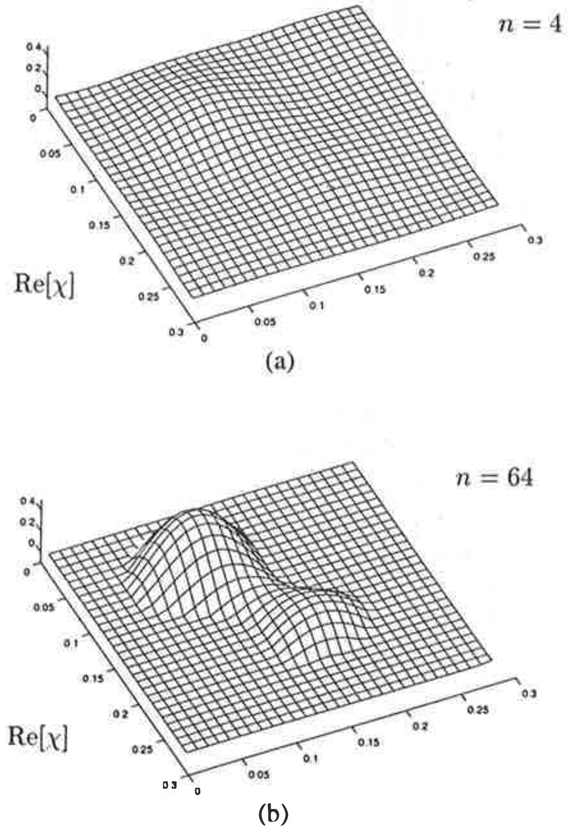

Fig 6: The reconstructions after 4 iterations (a), and after 64 iterations (b), using CSI with *a priori* information.

diameter of 10 cm. We have done the same tests. The results are in figures 4, 5 and 6. It is observed that the resolution in these figures is not as good as the resolution in the figures of the original paper. This is due to the size of the domain which is about $\frac{1}{2}\lambda \times \frac{1}{2}\lambda$. Higher frequencies, thus shorter wavelengths, will lead to an improved resolution but these waves have a lower penetration depth.

7. CONCLUSION

In this paper we have presented a new inversion algorithm (CSI) for profile reconstruction in acoustics and electromagnetics. The algorithm is based on the source type integral equation which relates measured data to a source distribution in the scattering object. The algorithm is akin to what Kohn and McKenney [15] call an alternating direction implicit (ADI) method wherein two sequences, one of sources and one of contrasts, are reconstructed iteratively by alternately updating the sources and the contrasts. Unlike the Kohn-McKenney method and other approaches based on the source type integral equation, e.g., Habashy *et al.* [11], the CSI algorithm does not involve completely solving the source type equation for each updated contrast. Similar to the modified gradient method, in each iteration there is no full inversion of the state equations involved.

A cost functional is defined consisting of errors in the source type equations and the state equations and the updates in the sources are found as a conjugate gradient step after which the contrast is updated by minimizing the error in the state equations which can be done very simply. The source updates are similar in spirit to those used in the modified gradient method while the contrast updates are found in a simple fashion in which *a priori* information is easily included. A number of numerical tests indicate that this new algorithm exhibits the best features of the modified gradient algorithm, successfully reconstructing a variety of contrasts and fairly insensitive to noise. However, the new algorithm exhibits additional properties which surpass the modified gradient approach. It is faster, requires less memory as well as less data and more easily accommodates *a priori* information.

To give some idea of the computational complexity, if J denotes the number of excitations and N denotes the number of subdomains in the test domain, then the time required for each iteration is roughly $2J$ times the time for one step in the conjugate gradient solution of the forward problem for one excitation, with a memory requirement of approximately $5JN \times 16$ bytes (complex double precision). The time required for each iteration is roughly one third that needed in the modified gradient algorithm with no *a priori* information on the contrast and is an order of magnitude faster if positivity is included for both real and imaginary parts.

No tests have yet been carried out on the effect of additional regularizers such as total variation which proved effective for the modified gradient algorithm [24]. This is one of the subjects for future work. The simplicity, speed and reduced memory requirements offer hope that this technique will provide a feasible approach to three-dimensional inversion problems.

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