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TNO-rapport

BI-87-112

Procedure for the determination of design resistance from tests

Background report to Eurocode 3 “Common unified rules for Steelstructures”

Datum	November 1987, update June 1988, update November 1988
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Exemplaarnummer	
Oplage	
Aantal pagina's	33
Aantal bijlagen	
Opdrachtgever	
Projectnaam	Eurocode 3
Projectnummer	63.5.5950

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0. NOTATIONS

r	= resistance function or strength function
$g_R(\underline{X})$	= resistance function with the relevant basic variables X_i before correction by comparison with test results
r_{ti}	= theoretical resistance determined form $g_R(\underline{X})$ with the measured parameters X_i for the test specimen i.
r_{ei}	= experimental resistance for the specimen i
$\bar{r}_t$	= mean value of the theoretical results r_{ti}
$\bar{r}_e$	= mean value of the experimental results r_{ei}
s_{rt}	= standard deviation of the theoretical results r_{ti}
s_{re}	= standard deviation of the experimental results r_{te}
ρ	= correlation coefficient for the comparison of theoretical and experimental values r_{ti} and r_{ei}
b_i	= correction term for the test specimen i
$\bar{b}$	= mean value correction for all test specimen i
$r_m(\underline{X}_m)$	= mean value corrected strength function calculated with the mean values $\underline{X}_m$ of the basic variables
δ_i	= error term for the test specimen i
$\bar{\delta}$	= mean value of the observed error terms δ_i
s_δ	= standard deviation of the observed error terms δ_i
V_δ	= coefficient of variation of the observed error terms δ_i
k_s, k_d	= fractile coefficients for standardized normal distribution for resp. the characteristic and design resistance
$\sigma_{\ell nr}$	= standard deviation of the natural logarithm of r
V_r	= coefficient of variation for the resistance r
Var(r)	= variance of the resistance r
S_{Xi}	= standard deviation of the basic variable X_i
r_n, r_k, r_d	= nominal, characteristic and design resistance
β	= safety index
γ_M	= model (partial safety) factor related to the 5% fractile
γ_M^*	= modified model factor $\gamma_M^* = \gamma_M \cdot \Delta K$
$r_n(\underline{X}_n)$	= resistance calculated from resistance funtion $g_R(\underline{X})$, before mean value correction, with one or more of the variables introduced as nominal (characteristic) value

f_{ui} = actual value of the tensile strength of the material of the test specimen i

f_{uk} = characteristic value of the tensile strength of the material

f_{um} = mean value of the tensile strength of the material

f_{un} = nominal value of the tensile strength of the material

$r_t(\underline{X}_m)$ = theoretical resistances calculated from $g_R(\underline{X}_m)$ with the mean value $\underline{X}_m$ for the variables

s_D = corrected standard deviation of the error term δ_i due to lack of measured values X_i but with preknowledge of X_{mi} and S_{Xi}

ΔK = ratio between nominal and characteristic resistance, $\Delta K = r_n/r_k$

1. INTRODUCTION

The rules in the Eurocodes are based on a limit state design format.

Effects of actions $\boxed{S_d \leq R_d}$ Design resistance

The following partial safety factors are used.

- Factors for loading
  - load combination ψ_o
  - accuracy of loading γ_f
- Gap factor (for model uncertainties)
  - structural γ_{Sd}
  - material γ_m
- Factors for resistance
  - accuracy of strength γ_{Rd}
  - manufacturing tolerances

γ_f, γ_{Sd} — γ_F

γ_m, γ_{Rd} — γ_M

In this note a standard procedure is described for the determination of characteristic values, design values and γ_M values for strength from tests that is in compliance with the basic safety assumptions outlined in chapter 2 of Eurocode 3.
Based on observation of actual behaviour in tests and on theoretical considerations, a "design model" is selected, leading to a strength funtion.

Then by statistical interpretation of all available test data, i.e. regression analysis, the efficiency of the model is checked. Eventually the design model has to be adapted until the correlation of the theoretical values and the test data is sufficient.
From the tests also the variation in the prediction of the design model can be determined (variation of the so called error term δ).
This variation is combined with variations of other variables in the strength function. These include:

- variation in material strength and stiffness;
- variation in geometrical properties.

The <u>characteristic strength</u> taking account of all variations of the variables can now be determined.
The note also describes a method how to derive <u>design values</u> from the given data and hence to deduct γ-factors, that may be applied to the characteristic strength functions.
For an easy understanding in chapter 2 the standard procedure is presented as a number of discrete steps under ideal assumptions for the test population and data. In later chapters modifications are given for situations deviating form the ideal assumptions.

2. STANDARD EVALUATION PROCEDURE

For the standard evaluation procedure the following ideal assumptions are made.

A. The strength function is a product function of independent variables.
As an example the procedure will be illustrated by using a linear strength function for bolts in bearing.

B. A large number of test results is available.

C. All actual geometrical and material properties are measured.

D. All variables have a log normal distribution. Adopting a log-normal distribution for all variables has the advantage that no negative values can occur for the geometrical and strength variables which is physically correct.

E. The design function is expressed in the mean values of the variables.

F. There is no correlation between the variables of the strength function.

The standard procedure comprises the following steps:

Step 1: Develop a "design model" for the strength of the member or the structural detail considered.

$$r = g_R(\underline{X}) \qquad ..... (1)$$

The strength function includes all relevant basic variables $\underline{X}$ which control the resistance in the limit state. All the basic parameters should be measured for each test specimen i and be available for the evaluation (assumption C).

Step 2: Compare experimental and theoretical values.

From the tests the experimental values r_{ei} are known. Using the relevant strength function and putting the actual properties into the formula, lead to the theoretical values r_{ti}.

For bolts in bearing:

$$r_{ti} = 2.5\, d_{ni}\, t_i\, f_{ui} \qquad \text{with } \frac{e_1}{d} > 3$$

where:

e_1 = end distance
d = hole diameter
d_{ni} = bolt diameter
t_i = plate thickness
f_{ui} = actual value of the ultimate strength of the plate material

The combinations of corresponding values (r_{ti}, r_{ei}) form points in a diagram (see Fig. 1).

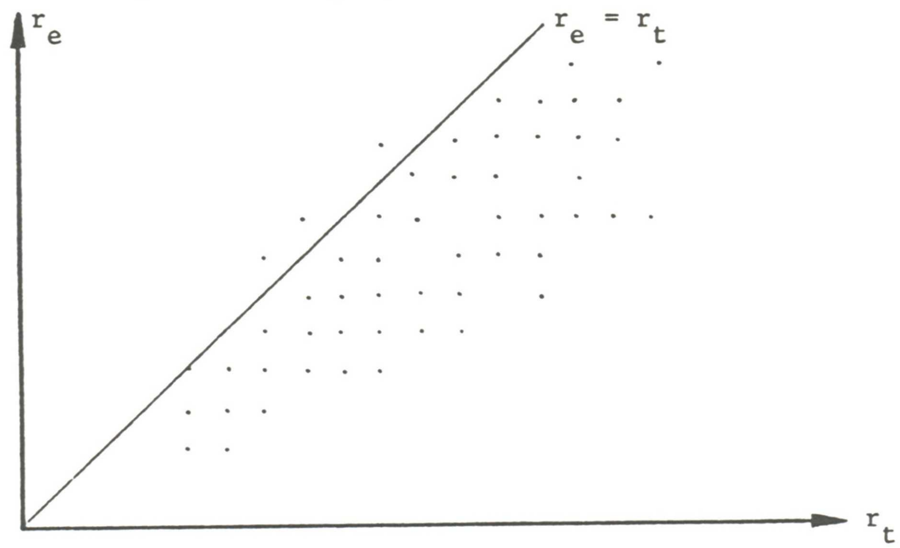


Figure 1: r_e - r_t diagram

If the strength function is exact and complete, all points (r_{ti}, r_{ei}) lie on the bisector of the angle between the axes of the diagram and the correlation coefficient $\rho = 1$.

<u>Step 3</u>: Check whether the correlation between experimental and theoretical values is sufficient.

In general the points (r_{ti}, r_{ei}) will scatter.
In that case the correlation coefficient ρ can be determined as follows.

a. Determine the mean values $\bar{r}_e$ en $\bar{r}_t$ of the experimental values r_{ei} and the theoretical values r_{ti} respectively and their standard deviations s_{re} en s_{rt}.

Experiments: $$\bar{r}_e = \frac{1}{n} \sum_{i=1}^{n} r_{ei} \qquad \text{..... (2)}$$

$$s_{re} = \sqrt{\frac{1}{n-1} \left(\sum_{i=1}^{n} r_{ei}^2 - n\, \bar{r}_e^2 \right)} \qquad \text{..... (3)}$$

Theory : $$\bar{r}_t = \frac{1}{n} \sum_{i=1}^{n} r_{ti} \qquad \text{..... (4)}$$

$$s_{rt} = \sqrt{\frac{1}{n-1} \left(\sum_{i=1}^{n} r_{ti}^2 - n\, \bar{r}_t^2 \right)} \qquad \text{..... (5)}$$

where n is the number of tests.

b. The correlation coefficient ρ then follows from:

$$\rho = \frac{\sum_{i=1}^{n} r_{ei}\, r_{ti} - n\, \bar{r}_e\, \bar{r}_t}{(n - 1)\, s_{re}\, s_{rt}} \qquad \text{..... (6)}$$

If the value

$$\rho \geq 0.9$$

than the correlation is considered to be sufficient.

Step 4: Determine mean value correction $\bar{b}$.

For each specimen i, comparison of the theoretical value r_{ti} with the corresponding experimental value r_{ei} renders a correction term b_i.

Correction terms : $b_i = \frac{r_{ei}}{r_{ti}}$ ..... (7)

Mean value correction: $\bar{b} = \frac{1}{n} \sum_{i=1}^{n} b_i$ ..... (8)

In the r_e - r_t diagram the mean value correction $\bar{b}$ is the direction coefficient of a straight line going through the origin of the diagram which represents the mean value of the test results via a correction of the theoretical values (see Fig. 2).

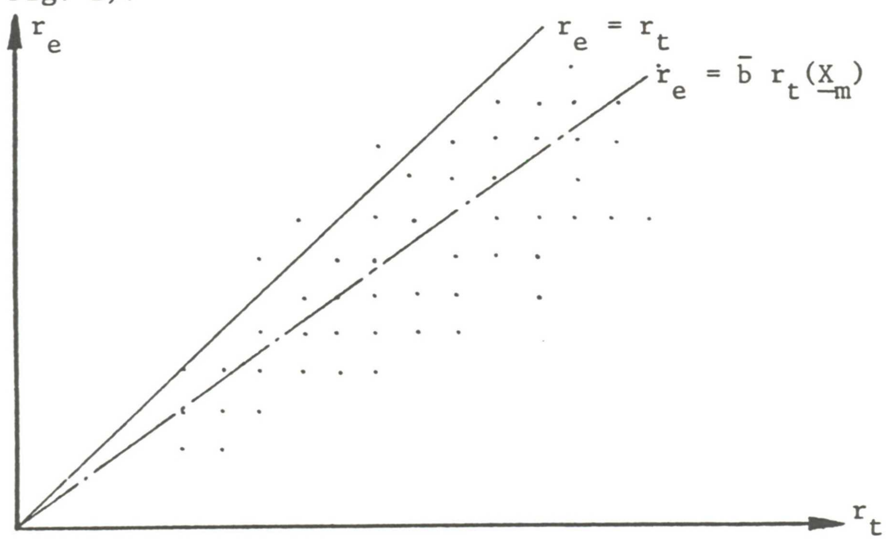


Figure 2: r_e - r_t diagram with the mean value correction line $r_e = \bar{b}\ r_t = r_m$.

The corrected strength function is:

$$r_m(\underline{X}_m) = \bar{b}\ r_t(\underline{X}_m) = \bar{b}\ g_R(\underline{X}_m)$$

Step 5: Determine the coefficient of variation V_δ of the observed error terms.

The error terms δ_i of each experimental value r_{ei} with respect to each theoretical, mean value corrected, result $\bar{b}\, r_{ti}$ is determined as follows:

$$\delta_i = \frac{r_{ei}}{\bar{b}\, r_{ti}} \qquad \ldots\ldots (9)$$

From the δ_i-values the value for V_δ can be determined as follows:

$$\delta'_i = \ell n\, \delta_i \qquad \ldots\ldots (9')$$

Mean value : $\bar{\delta}' = \frac{1}{n} \sum_{i=1}^{n} \delta'_i$ .... (10')

Standard deviation : $s_{\delta'} = \sqrt{\frac{1}{n-1} \left(\sum_{i=1}^{n} \delta'^2_i - n\, \bar{\delta}'^2 \right)}$. (11')

Coefficient of variation: $V_{\delta'} = \frac{s_{\delta'}}{\bar{\delta}'} = s_{\delta'}$ .... (12')

In most cases $V_{\delta'}$ is small, so the transformation (9') can be omitted and the procedure will follow as:

Mean value : $\bar{\delta} = \frac{1}{n} \sum_{i=1}^{n} \delta_i$ ..... (10)

Standard deviation : $s_\delta = \sqrt{\frac{1}{n-1} \left(\sum_{i=1}^{n} \delta^2_i - n\, \bar{\delta}^2 \right)}$ ... (11)

Coefficient of variation: $V_\delta = \frac{s_\delta}{\bar{\delta}} = s_\delta \quad (\bar{\delta} = 1)$ ..... (12)

In the example of bolts in bearing $V_\delta = s_\delta = 0.08$

<u>Step 6</u>: Determine the coefficient of variation of the basic variables in the strength function (V_{xi}).

The coefficient of variation of all the basic variables may only be determined from the test-data if it may be assumed that the test population is fully representative for the variation in the actual situation.
This is normally not the case, so the coefficients of variation have to be determined from preknowledge.

For the strength function considered:

$$V_{dn} = 0.005$$
$$V_{t} = 0.05$$
$$V_{fu} = 0.07$$

<u>Step 7</u>: Determine the characteristic value of the strength.

For the log-normal distribution (assumption D) the characteristic strength follows from:

$$r_k = r_m(\underline{X}_m) \exp\left(-k_s\, \sigma_{\ell nr} - 0.5\, \sigma^2_{\ell nr}\right) \qquad \ldots\ldots (13)$$

where:

$$\sigma_{\ell nr} = \sqrt{\ell n\,(V_r^2 + 1)} \simeq V_r$$

is the equation for transferring the coefficient of variation determined for a normal (Gaussian) distribution to the logarithmic scale.

In case of 5% fractile $k_s = 1.64$ if a large number of tests is available:

$$V_{rt} = \frac{\sqrt{\text{VAR}\ g_R(\underline{X}_m)}}{g_R(\underline{X}_m)} \qquad \text{.....} \ (14)$$

$$\text{VAR}\ g_R(\underline{X}_m) =$$

$$\left(\frac{\partial\ g_R(\underline{X}_m)}{\partial\ X_1} S_{X1}\right)^2 + \left(\frac{\partial\ g_R(\underline{X}_m)}{\partial\ X_2} S_{X2}\right)^2 + \ldots + \left(\frac{\partial\ g_R(\underline{X}_m)}{\partial\ X_J} S_{XJ}\right)^2$$

In the case of bolts in bearing:

$$g_R(\underline{X}_m) = 2.5\ d_{nm}\ t_m\ f_{um} \quad \text{if} \quad \frac{e_1}{d} > 3$$

$$\text{VAR}\ g_R(\underline{X}_m) = g_R^2(\underline{X}_m)\ \left\{\left(\frac{S_{dnm}}{d_{nm}}\right)^2 + \left(\frac{S_t}{t_m}\right)^2 + \left(\frac{S_{fu}}{f_{um}}\right)^2\right\}$$

$$= g_R^2(\underline{X}_m)\ \{V_{dn}^2 + V_t^2 + V_{fu}^2\}$$

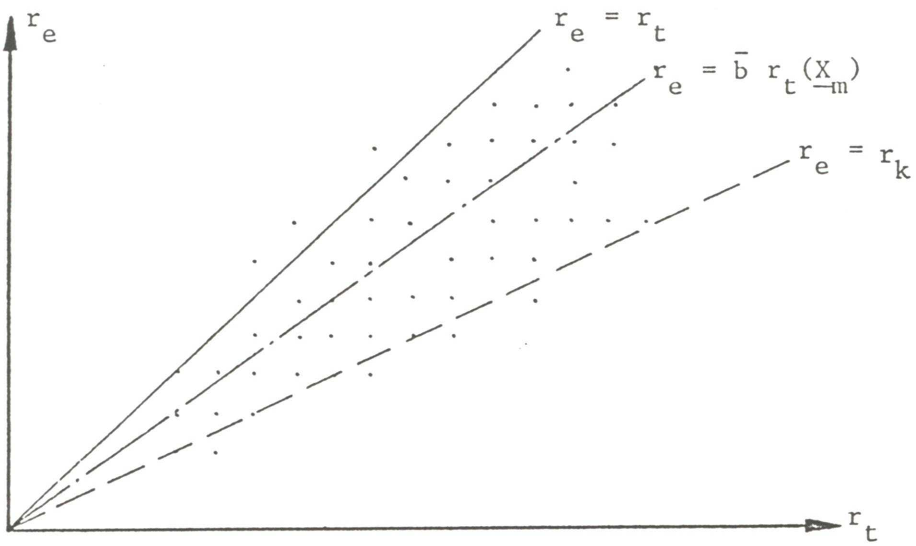
$$V_{rt} = \sqrt{V_{dn}^2 + V_t^2 + V_{fu}^2} = \sqrt{\sum_{i=1}^{J} V_{Xi}^2}$$

So in general:

$$V_r = \sqrt{V_{rt}^2 + V_\delta^2}$$

$$V_r = \sqrt{\sum_{i=1}^{J} V_{Xi}^2 + V_\delta^2} \qquad \text{.....} \ (15)$$

J = number of basic variables.



Figuur 3: r_e - r_t diagram with the characteristic line $r_e = r_k$

<u>Step 8</u>: Determine the design value of the strength and partial safety factor γ_M.

When the 5%-fractile of the strength function is determined, it is possible to extend the evaluation to obtain the design function r_d related to a given safety index β by replacing the fractile coefficient k_s for the 5%-fractile by k_d for the design fractile.

$$r_d = r_m(\underline{X}_m)\ \exp\ (-k_d\ \sigma_{\ell nr} - 0.5\ \sigma^2_{\ell nr})$$

The value of k_d in the equation above can be taken as $k_d = \alpha_R\ \beta$.

The sensitivity factor α_R on the resistance side (and α_S on the loading side) has to be determined under the assumption that the linearization of the ultimate limit state in the design point does not show large variations of the safety index β. Comparative studies with $\alpha_R = 0.8$ (and $\alpha_S = 0.7$) lead to an acceptable safety index $\beta = 3.8$.
So $k_d = \alpha_R\ \beta = 0.8 \times 3.8 = 3.04$ leads to a probability of failure $P_f = 10^{-3}$.

The design value of the strength is given by:

$$r_d = r_m(\underline{X}_m) \exp(-0.8\ \beta\ \sigma_{\ell nr} - 0.5\ \sigma^2_{\ell nr}) \quad \ldots\ldots (16)$$

and

$$r_d = \frac{r_k}{\gamma_M} = \frac{r_m(\underline{X}_m) \exp(-1.64\ \sigma_{\ell nr} - 0.5\ \sigma^2_{\ell nr})}{\gamma_M} \quad \ldots\ldots (17)$$

From (16) and (17) follows:

$$\beta = 3.8 \rightarrow \gamma_M = \frac{r_k}{r_d} = \frac{\exp(-1.64\ \sigma_{\ell nr} - 0,5\ \sigma^2_{\ell nr})}{\exp(-0.8\ \beta\ \sigma_{\ell nr} - 0.5\ \sigma^2_{\ell nr})}$$

$$= \exp\{(0.8\ \beta - 1.64)\ \sigma_{\ell nr}\}$$

$$= \exp(1.40\ \sigma_{\ell nr}) \quad \ldots\ldots (18)$$

In case of γ_M-values not being uniform over the full range of the strength function when sub-sets are used in the evaluation procedure, the average target value for the safety index is $\beta = 3.8$ and for the most unfavourable sub-set the target value for β may be decreased by $\Delta\beta = 0.5$.

So $k_{dmin} = 0.8 \times (3.8 - 0.5) = 2.64$, which corresponds to a probability of failure of $P_f = 4 \cdot 10^{-3}$.

The standard evaluation procedure is illustrated in the following diagram

Step 1	Design model $r = g_R(X)$	
Step 2	Compare theory <--> test r_t <--> r_e	
Step 3	Check correlation $\rho \geq 0.9$	
Step 4	Mean value correction $\bar{b}$ Mean value corrected strength $r_m = \bar{b}\, r_t$	$\rightarrow \bar{b}$
Step 5	Variation of strength function V_δ	$\rightarrow V_\delta$
Step 6	From preknowledge the coefficients of variation for the basic variables are determined V_{Xi}	$\rightarrow V_{Xi}$
Step 7	Characteristic strength $V_r = \sqrt{\Sigma V_{Xi}^2 + V_\delta^2}$ $\sigma_{\ell nr} = \sqrt{\ell n(V_r^2 + 1)} \simeq V_r$ $r_k = r_m(\underline{X}_m) \exp(-k_s\, \sigma_{\ell nr} - 0.5\, \sigma_{\ell nr}^2)$	$\rightarrow R_k = r_k/r_m$
Step 8	Design strength $r_d = r_m(\underline{X}_m) \exp(-0.8\, \beta\, \sigma_{\ell nr} - 0.5\, \sigma_{\ell nr}^2)$ $\gamma_M = r_k/r_d = \exp((0.8\, \beta - 1.64)\, \sigma_{\ell nr})$	$\rightarrow R_d = r_d/r_m$ $\rightarrow \gamma_M$

3. A VARIABLE IN THE STRENGTH FUNCTION IS DEFINED AS NOMINAL VALUE

Usually in codes, the design strength functions contain basic variables defined as nominal values.
For instance the nominal value for the material strength is defined as characteristic value and the nominal values for the geometrical variables are mostly defined as mean values.
The relationship between the characteristic strength function and the nominal strength function is as follows.

Define a factor ΔK:

$$\Delta K = \frac{r_n}{r_k} = \frac{r_t(\underline{X}_n)}{\bar{b}\ r_t(\underline{X}_m)\ .\ \exp\left(-\ k_s\ \sigma_{\ell nr} - 0.5\ \sigma^2_{\ell nr}\right)}$$

$$= \frac{r_t(\underline{X}_m)\ .\ \prod_{i=1}^{J} \{\exp\left(-\ k_{Xi}\ \sigma_{\ell nXi} - 0.5\ \sigma^2_{\ell nXi}\right)\}}{\bar{b}\ r_t(\underline{X}_m)\ .\ \exp\left(-\ k_s\ \sigma_{\ell nr} - 0.5\ \sigma^2_{\ell nr}\right)}$$

$$\Delta K = \frac{\prod_{i=1}^{J} \{\exp\left(-\ k_{Xi}\ \sigma_{\ell nXi} - 0.5\ \sigma^2_{\ell nXi}\right)\}}{\bar{b}\ .\ \exp\left(-\ k_s\ \sigma_{\ell nr} - 0.5\ \sigma^2_{\ell nr}\right)} \qquad \text{.... (19)}$$

with J is the number of basic variables

Furthermore define a modified model factor γ_M^* such that:

$$r_d = \frac{r_n}{\gamma_M^*} = \frac{r_k\ \Delta k}{\gamma_M^*} \qquad \text{.... (20)}$$

According to (17):

$$r_d = \frac{r_k}{\gamma_M}$$

So (17) and (20) lead to:

$$\gamma_M^* = \Delta K \; . \; \gamma_M \qquad \text{.... (21)}$$

In the case of bolts in bearing the resistance function $g_R(\underline{X})$ is:

$$g_R(\underline{X}) = 2.5 \, d_n \, t \, f_u$$

where:

d_n = bolt diameter
t = plate thickness
f_u = ultimate strength of plate material

From the evaluation of test results follows for instance:

$\bar{b}$ = 1.00 mean value correction
V_δ = 0.08 coefficient of variation of the error terms (model uncertainty)

From preknowledge:

V_{dn} = 0.005 coefficient of variation of the bolt diameter
V_t = 0.05 coefficient of variation of the plate thickness
V_{fu} = 0.07 coefficient of variation of the ultimate strength of the plate material

The mean value corrected strength function is:

$$r_m(\underline{X}_m) = 2.5 \, d_{nm} \, t_m \, f_{um} \, \bar{b}$$

The nominal strength function is:

$$r_n(\underline{X}_n) = 2.5 \, d_{nn} \, t_n \, f_{un}$$

The nominal value X_{in} of a basic variable is a characteristic value X_{ik} and is expressed in its mean value X_{im} via the corresponding fractile factor k_{Xi}:

$$X_{in} = X_{ik} = X_{im} \cdot \exp\left(- k_{Xi}\, \sigma_{\ell nXi} - 0.5\, \sigma^2_{\ell nXi}\right)$$

The nominal value of the bolt diameter is equal to the mean value, so the fractile factor $k_{dn} = - 0.5\, \sigma_{\ell ndn}$.
The nominal value of the plate thickness is equal to the mean value, so the fractile factor $k_t = - 0.5\, \sigma_{\ell nt}$.
The norminal value of the ultimate strength of the plate material is equal to a characteristic value, defined by the fractile factor $k_{fu} = 2$.

So:

$$X_{1n} = d_{nn} = d_{nm} \cdot \exp\left(0.5\, \sigma^2_{\ell ndn} - 0.5\, \sigma^2_{\ell ndn}\right) = d_{nm}$$

$$X_{2n} = t_n = t_m \cdot \exp\left(0.5\, \sigma^2_{\ell nt} - 0.5\, \sigma^2_{\ell nt}\right) = t_m$$

$$X_{3n} = f_{un} = f_{um} \cdot \exp\left(- 2 \times 0.07 - 0.5 \times 0.07^2\right) = f_{um} \times 0.867$$

$$V_r = \sqrt{\sum_{i=1}^{J} V^2_{Xi} + V^2_\delta}$$

$$= \sqrt{0.005^2 + 0.05^2 + 0.07^2 + 0.08^2} = 0.118$$

$$\sigma_{\ell nr} = \sqrt{\ell n\, (V^2_r + 1)} \approx V_r = 0.118$$

$$k_s = 1.64 \quad \text{(a large number of tests is assumed)}$$

$$\sigma_{\ell nfu} = \sqrt{\ell n\, (V^2_{fu} + 1)} \approx V_{fu} = 0.07$$

According to (19):

$$\Delta K = \frac{1 \times 1 \times \exp\left(- 2 \times 0.07 - 0.5 \times 0.07^2\right)}{1.00 \times \exp\left(- 1.64 \times 0.118 - 0.5 \times 0.118^2\right)} = \frac{0.867}{1.00 \times 0.818} =$$

$$= 1.06$$

According to (18):

$$\gamma_M = \exp\ (1.40\ \sigma_{\ell nr}) = \exp\ (1.40 \times 0.118) = 1.18$$

According to (21):

$$\gamma_M^* = \Delta K\ .\ \gamma_M = 1.06 \times 1.18 = 1.25$$

4. A LIMITED NUMBER OF TEST RESULTS IS AVAILABLE

Assumption B in chapter 2 is changed.

In formula (13) $k_s = 1.64$ in case of a large number of test results available. If only a limited number of test results is available, k_s can be taken from table 1. The k_s-values are listed in table 1 as a function of $\nu = n - 1$.

Tabel 1: Fractile factors k_s for estimating 5%-fractiles for a level of confidence of 0.75.

ν	1	2	3	4	5	6	7	8	9	10	11	12
k_s	5.12	3.15	2.68	2.46	2.33	2.25	2.19	2.14	2.10	2.07	2.05	2.03
ν	13	14	15	16	17	18	19	20	21	22	23	24
k_s	2.00	1.99	1.98	1.96	1.95	1.94	1.93	1.92	1.92	1.91	1.90	1.90
ν	25	26	27	28	29	34	39	44	49	54	∞	
k_s	1.89	1.88	1.88	1.87	1.87	1.85	1.83	1.82	1.81	1.80	1.64	

For an infinite number of tests the value 0.8 β in formula (16) is adequate. In case of a limited number of test results, 0.8 β must be replaced by k_d and taken from tabel 2.

Tabel 2: Fractile-factor k_{dmin} and k_d for a 75% predicting probability.

ν	1	2	3	4	5	6	7	8	9	10
k_{dmin}	8.26	4.90	4.22	3.88	3.69	3.54	3.45	3.38	3.32	3.27
k_d	9.52	5.72	4.83	4.44	4.20	4.05	3.95	3.86	3.80	3.74
ν	11	12	13	14	15	16	17	18	19	20
k_{dmin}	3.23	3.20	3.17	3.14	3.12	3.10	3.09	3.07	3.06	3.05
k_d	3.70	3.66	3.63	3.60	3.58	3.55	3.54	3.52	3.51	3.49
ν	21	22	23	49	54	∞				
k_{dmin}	3.03	3.02	3.01	2.88	2.87	2.64				
k_d	3.47	3.46	3.45	3.30	3.29	3.04				

Formula (18) becomes:

$$\gamma_M = \exp\{(k_d - k_s)\,\sigma_{\ell nr}\}$$

Strictly speaking the correction for a limited number of tests should only be adopted for the variation of the strength function V_δ.
So the procedure given above is conservative.

This can however be adjusted in the following way:

Determine

$$\sigma'_{rt} = \sqrt{\ell n\,(V_{rt}^2 + 1)}$$

$$\sigma'_{\delta} = \sqrt{\ell n\,(V_{\delta}^2 + 1)}$$

$$\sigma'_{r} = \sqrt{\ell n\,(V_{r}^2 + 1)}$$

Determine the weighting factors

$$\alpha_{rt} = \frac{\sigma'_{rt}}{\sigma'_{r}}$$

$$\alpha_{\delta} = \frac{\sigma'_{\delta}}{\sigma'_{r}}$$

Determine the fractile factors $k_s(n)$ en $k_d(n)$ dependent on the relevant number n of tests.

$$\gamma_M = \frac{\exp\ (-1.64\ \alpha_{rt}\ \sigma'_{rt} - k_s\ \alpha_{\delta}\ \sigma'_{\delta} - 0.5\ \sigma_r^2)}{\exp\ (-3.04\ \alpha_{rt}\ \sigma'_{rt} - k_d\ \alpha_{\delta}\ \sigma_{\delta}' - 0.5\ \sigma_r^2)}$$

5. NOT ALL ACTUAL PROPERTIES OF THE TEST SPECIMEN ARE MEASURED

Assumption C in chapter 2 is changed.

5.1 Mean values and standard deviation of variables available

When there are no measured values for the parameters available to be used in calculating r_{ti}, but only the mean values m_j and the standard deviations s_j are known for those parameters (J = number of parameters) the procedure has to be adjusted.

. The r_{ti} values have to be determined with the mean values instead of the measured values of the parameters.
  This gives a series of r_{tmi}-values.

. The original procedure can be followed from formula (1) up to and including formula (10) but using r_{tmi} instead of r_{ti}.

. The standard deviation s_δ of the error terms δ_i as given in formula (11) has to be adjusted:

$$s_D = \sqrt{s_\delta^2 + \frac{n-1}{n-2} \sum_{j=1}^{J} \left(\frac{s_j}{m_j}\right)^2} \qquad \text{... (11-a)}$$

with s_δ according to formula (11) and J is the number of parameters involved.

. The rest of the original procedure can be followed, however in formula (12) the factor s_δ has to be changed into s_D.

$$V_D = \frac{s_D}{\bar{\delta}} = s_D \qquad \text{... (12-a)}$$

Note: This procedure can be concervative in the case where the test population is fully representative for the variation of the variables in the actual situation. In that case the variation of the variables is taken into account in calculating s_δ, it is added in formula (11-a) for the calculation of s_D and it is added in the

formula (15) where the coefficient of variation for the resistance is calculated.
However, the test population is most times not fully representative for the variation of the variables.
So, to reach safe results, this procedure has to be followed.

5.2 Characteristic values of parameters available

When only information about the characteristic values of the parameters is available, the procedure has to be adjusted as follows.

. Instead of the values r_{ti} derived from the measured values of the parameters, the $r_t(\underline{X}_m)$ values, calculated with the mean values X_{mj} of the parameters, are used.
The mean values X_{mj} of the parameters are determined using the estimated variation coefficient V_{xj} and using k = 2 for the 97.7% fractile.

$$X_{mj} = \frac{X_{kj}}{\exp\left(-2\,\sigma_{\ell nXj} - 0.5\,\sigma^2_{\ell nXj}\right)}$$

6. THE STRENGTH FUNTION IS NONLINEAR WITH RESPECT TO THE VARIABLES AND CONTAINS ADDITIONS OF THE VARIABLES

Assumption A in chapter 2 is changed.

The strength function is nonlinear with respect to the variables

In case of nonlinear strength function:

$$r = g_R(X_1, X_2, \ldots. X_J)$$

the variation can be determined by:

$$VAR\ [r] = (\frac{\partial g_R}{\partial X_1}\ S_{X1})^2 + (\frac{\partial g_R}{\partial X_2}\ S_{X2})^2 + \ldots + (\frac{\partial g_R}{\partial X_J}\ S_{XJ})^2$$

From this follows the variation coefficient:

$$V_{rt} = \frac{\sqrt{VAR\ [r]}}{r_m(\underline{X}_m)} \quad \text{and} \quad V_r = \sqrt{V_{rt}^2 + V_\delta^2}$$

and as in the standard procedure, the characteristic strength follows from:

$$r_k = r_m(\underline{X}_m)\ \exp\ (-k_s\ \sigma_{\ell nr} - 0.5\ \sigma_{\ell nr}^2)$$

where $\sigma_{\ell nr} = \sqrt{\ell n\ (V_r^2 + 1)}$

The method is illustrated for the following fictitious strength function:

$$r = b_o^{0.5}\ t_o^{1.5}\ f_u$$

Assume:

$$V_{bo} = 0.005$$

$$V_{to} = 0.05$$

$V_{fu} = 0.07$

$V_{\delta} = 0.09$ from the test evaluation

$$VAR\ [r] = (\frac{\partial r}{\partial b_o} S_{bo})^2 + (\frac{\partial r}{\partial t_o} S_{to})^2 + (\frac{\partial r}{\partial f_u} S_{fu})^2$$

$$= r^2 \{(0.5 \frac{S_{bo}}{b_o})^2 + (1,5 \frac{S_{to}}{t_o})^2 + (\frac{S_{fu}}{f_u})^2\}$$

Substitute mean values for the variables in calculating VAR [r] and calculate:

$$V_{rt}^2 = \frac{VAR\ [r]}{r_m^2} = \frac{r_m^2 \{(0.5 \frac{S_{bo}}{b_{om}})^2 + (1.5 \frac{S_{to}}{t_{om}})^2 + (\frac{S_{fu}}{f_{um}})^2\}}{r_m^2}$$

$$V_{rt}^2 = 0.25\ V_{bo}^2 + 2.25\ V_{to}^2 + V_{fu}^2$$

$$V_r^2 = V_{rt}^2 + V_{\delta}^2 = 0.25\ V_{bo}^2 + 2.25\ V_{to}^2 + V_{fu}^2 + V_{\delta}^2$$

$$V_r^2 = V_{rt}^2 + V_{\delta}^2 = 0.25 \times (0.005)^2 + 2.25 \times (0.05)^2 + (0.07)^2 + (0.09)^2$$

$$V_r^2 = 0.019$$

$$V_r = 0.14$$

$$\sigma_{\ell nr} = \sqrt{\ell n\ (0.14^2 + 1)} = 0.139$$

$$r_k = r_m(\underline{X}_m) \exp\ (-1.64 \times 0.139 - 0.5 \times 0.139^2) = r_m(\underline{X}_m) \times 0.789$$

for a large number of tests available.

The strength function contains an addition of the variables

In case of a strength function containing an addition of the variables:

$$r = g_R\ (X_1,\ X_2,\ \ldots\ldots\ X_J)$$

the variation and the characteristic strength are still as described before.

However, the calculation of VAR [r] is different and the method is illustrated for the following fictitious strength function:

$$r = b_o^{0.5}\ t_o^{1.5}\ f_u + b_1^{0.5}\ t_1^{1.5}\ f_u = r_1 + r_2$$

$$\mathrm{VAR}\ [r] = \left(\frac{\partial r}{\partial b_o}\ s_{bo}\right)^2 + \left(\frac{\partial r}{\partial t_o}\ s_{to}\right)^2 + \left(\frac{\partial r}{\partial b_1}\ s_{b1}\right)^2 + \left(\frac{\partial r}{\partial t_1}\ s_{t1}\right)^2 + \left(\frac{\partial r}{\partial f_u}\ s_{fu}\right)^2$$

$$= (b_o^{0.5}\ t_o^{1.5}\ f_u)^2\ \left\{\left(0.5\ \frac{s_{bo}}{b_o}\right)^2 + \left(1.5\ \frac{s_{to}}{t_o}\right)^2 + \left(\frac{s_{fu}}{f_u}\right)^2\right\} +$$

$$(b_1^{0.5}\ t_1^{1.5}\ f_u)^2\ \left\{\left(0.5\ \frac{s_{b1}}{b_1}\right)^2 + \left(1.5\ \frac{s_{t1}}{t_1}\right)^2 + \left(\frac{s_{fu}}{f_u}\right)^2\right\}$$

Substitute mean values for the variables in calculation VAR [r] and calculate:

$$V_{rt}^2 = \frac{\mathrm{VAR}\ [r]}{r_m^2} =$$

$$= \frac{r_{1m}^2\ \left\{\left(0.5\ \frac{s_{bo}}{b_{om}}\right)^2 + \left(1.5\ \frac{s_{to}}{t_{om}}\right)^2 + \left(\frac{s_{fu}}{f_{um}}\right)^2\right\} + r_{2m}\ \left\{\left(0.5\ \frac{s_{b1}}{b_{1m}}\right)^2 + \left(1.5\ \frac{s_{t1}}{t_{1m}}\right)^2 + \left(\frac{s_{fu}}{f_{um}}\right)^2\right\}}{r_m^2}$$

$$V_{rt}^2 = \frac{r_{1m}^2\ \{0.25\ V_{bo}^2 + 2.25\ V_{to}^2 + V_{fu}^2\} + r_{2m}^2\ \{0.25\ V_{b1}^2 + 2.25\ V_{t1}^2 + V_{fu}^2\}}{r_m^2}$$

$V_r = \sqrt{V_{rt}^2 + V_\delta^2}$ Calculating this value V_r for every test, using mean values for the variables, lead to a range of values for V_r from which the largest value is to be taken to continue the procedure.

7. IMPROVEMENT OF THE STRENGTH FUNCTION BY CONSIDERING SUBSETS OF THE TEST POPULATION

If the scatter of the r_{ei} - r_{ti}-values is regarded too high as to give economic characteristic strength functions, the scatter may be reduced by correcting the strength functions, such that additional parameters not sufficiently contained in the strength functions are taken into account.

To make clear what parameters influence the scatter, the test results can be splitted up into subsets with respect to those parameters.

As an illustration in fig. 4 the results of shear tests on bolts are given, splitted in subsets with respect to the bolt grade.
Obviously the strength function in this case can be improved if the factor 0.7 in the strength function is modified and expressed as a function of the bolt grade (f_{ub}).

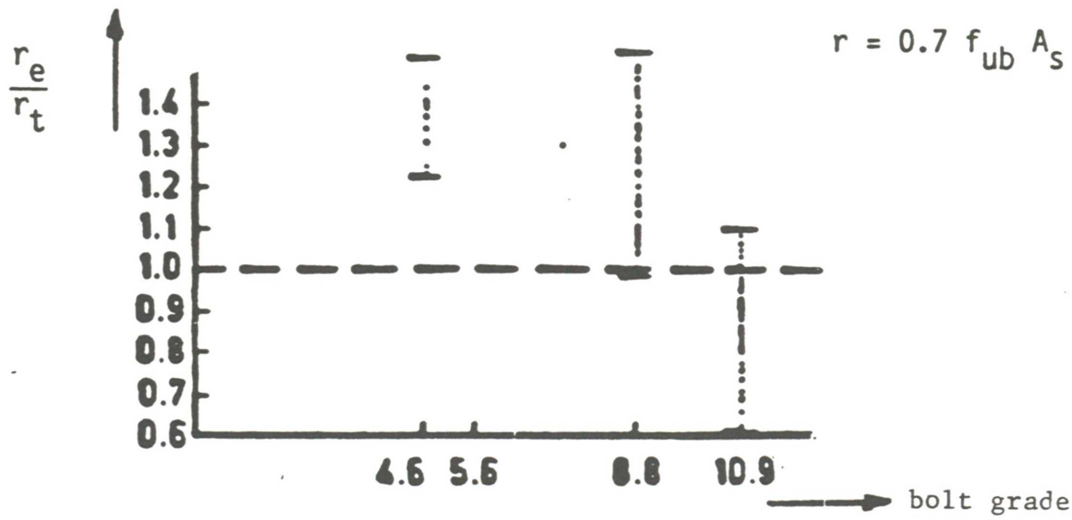


Figure 4: Shear failure of bolts of different grades with the shear plane through the threaded portion.

So it is suggested to improve the strength function per subset by analysing the subset with the standard procedure.

The disadvantage of splitting up into subsets is that the number of test results per subset can become rather small.

In determining the fractile factors k_s it is suggested to determine the k_s-value for the subsets on the total number of all the tests of the original series. This can be justified by the fact that via the first evaluation it was shown that if $\rho \geq 0.9$ there was already a sufficient correlation between the experimental values and the theoretical values using the original strength functions.
In this way an improved strength function is obtained consisting of the original strength function multiplied by a factor dependent on the variation of a few important parameters.

8. ACKNOWLEDGEMENT

This paper is the result of observing Note No. 11, CEB/ECCS Draft September 1985 and discussions between:

a) Prof.Dr. G. Schulz
Dr. M. Kersken-Bradley
Dipl.-Ing. E. Hemmerich
Prof.Dr.-Ing. G. Sedlacek
Ir. F.S.K. Bijlaard
during the Munich-Meeting, 10 February 1987

b) Prof.Dr.-Ing. G. Sedlacek
Prof.Ir. J.W.B. Stark
Ir. F.S.K. Bijlaard
during the Delft-Meeting, 1 April 1987

c) the first mentioned group together with Dr.-Ing. W. Maier and Dr.-Ing. Ch. Stutzki during the Munich-Meeting, 13 April 1987

d) Prof.Dr.-Ing. G. Sedlacek
Prof.Ir. J.W.B. Stark
Ir. F.S.K. Bijlaard
Prof.Dr.Ir. J. Wardenier
during the Cachan-Meeting, 25 May 1987

e) Prof. R. Maqoi
Ir. J. Janss
Prof.Dr.-Ing. G. Sedlacek
Dr.-Ing. C. Stutzki
Dr.-Ing. A. Hemmert-Halswick
in Aachen, 2 July 1987

f) Prof.Dr.-Ing. G. Sedlacek
Prof.Ir. J.W.B. Stark
Ir. F.S.K. Bijlaard
Dr.-Ing. A. Hemmert-Halswick
in Aachen, 13 July 1987

g) Revision proposed by Prof.Dr.-Ing. G. Sedlacek d.d. 13 August, 1987

h) Final draft by Prof.Dr.-Ing. G. Sedlacek, Prof.Ir. J.W.B. Stark and Ir. F.S.K. Bijlaard in Aachen, 21 August 1987.

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