Proceedings of the Seventh European Conference on Underwater Acoustics, ECUA 2004
Delft, The Netherlands
5-8 July, 2004

TARGET LOCALISATION WITH MULTISTATIC SYSTEMS

Jean-Christophe Sindt[1] and Pascal A.M. de Theije[2]

[1] Thales Underwater Systems (TUS SAS), 525 Route des Dolines, 06903 Sophia-Antipolis Cedex, France, jean-christophe.sindt@fr.thalesgroup.com
[2] Underwater Acoustics Group, TNO-FEL, Oude Waalsdorperweg 63, 2509 JG, The Hague, The Netherlands, detheije@fel.tno.nl

This paper focuses on target localisation errors using a bistatic sonar. A list of parameters influencing the errors on the target range and bearing is analysed, evaluated and visualised. The theoretical results have been verified with sea-trials data.

1. INTRODUCTION

Submarines becoming more and more silent, the Navies have much interest in active-sonar applications for Anti-Submarine Warfare (ASW). The overt nature of low-frequency active sonar (LFAS) operations could give to an opposing submarine the potential initiative. This risk can be significantly reduced by the use of multistatic sonar techniques. Tactically, multistatic operations can lead to larger detection areas (e.g. [1]). Secondly, combining the results of more receivers, the target detection and classification may improve and provide useful information. To allow data fusion between multiple platforms, each system has to be able to localise targets accurate enough.

2. BISTATIC LOCALISATION AND RANGE ERRORS

Theory on bistatic range estimation, and the possible error sources on bistatic target localisation are discussed and analysed in this chapter.

2.1. Bistatic Target-Range Calculation

A typical bistatic geometry is shown in Figure 1. The following parameters are observable or known values:

$t_{str} = t_{st} + t_{rt} = (r_{st} + r_{rt}) / c$	Travel time of target echo to bistatic receiver
$t_{sr} = r_{sr} / c$	Direct-blast travel time to bistatic receiver

ϕ_{rt} Bearing of target relative to receiver ship

where c is the sound speed, and t_{str} and t_{sr} depend on the time synchronisation between the source and receiver, i.e. the exact time of transmission has to be known. Note that r_{sr}, the source-receiver distance, may be known directly if the source position is transferred from the source ship to the receiver ship.

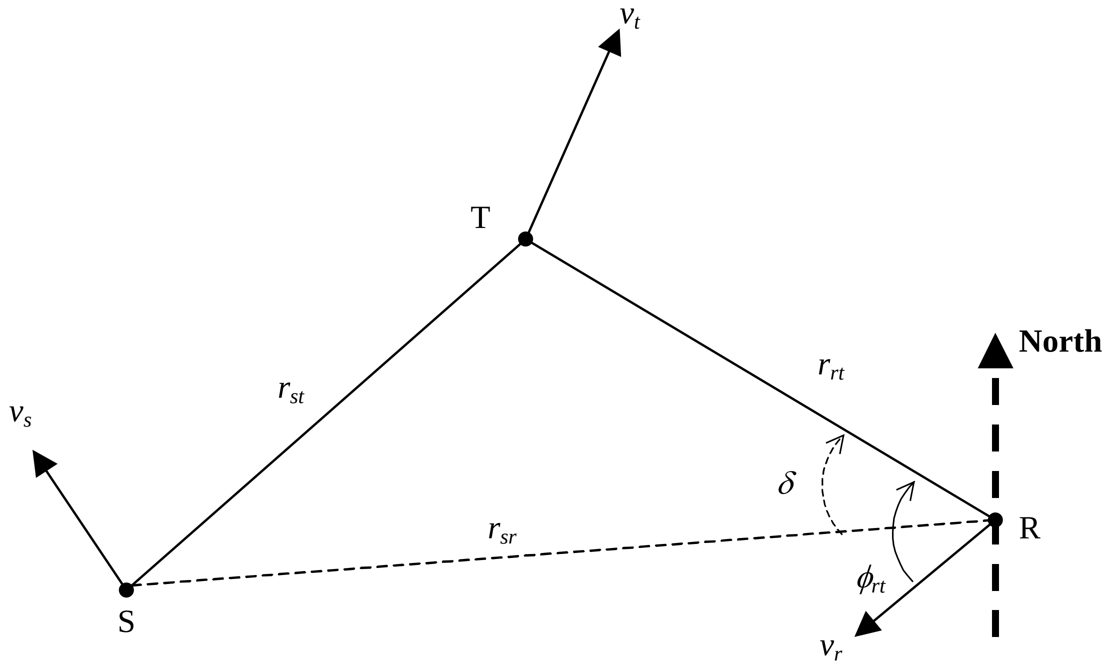


Figure 1: Typical bistatic geometry. v_S, v_r *and* v_t *are the velocities of the S(ource) ship, R(eceiver) ship, and T(arget) respectively.*

From Figure 1, and using the law of cosines, the range estimate between target and receiver (r_{rt}) can be obtained as (see also [2]):

$$r_{rt} = \frac{(ct_{str})^2 - r_{sr}^2}{2\left[ct_{str} - r_{sr}\cos(\delta)\right]} = \frac{c}{2}\frac{t_{str}^2 - t_{sr}^2}{\left[t_{str} - t_{sr}\cos(\delta)\right]} \tag{1}$$

where δ is the angle between the source-receiver line (r_{sr}) and the target-receiver line (r_{rt}).

2.2. Theoretical Errors on Bistatic Target-Range Estimate

The two alternative expressions in equation (1) imply that two cases can be distinguished:

- Using the *distance* r_{sr} between source and receiver (first equality in equation (1)).
- Using the *direct-blast travel time* t_{sr} between them (second equality in equation (1)).

Operationally, this difference means that either the source position is obtained from acoustic (t_{sr}, by using the direct blast) or non-acoustic (r_{sr}, by using GPS or radar) information. Assuming t_{sr} or r_{sr} can be known, the total error in r_{rt} can be derived as:

$$\begin{aligned}\Delta r_{rt} &= \left|\frac{\partial r_{rt}}{\partial c}\right|\Delta c + \left|\frac{\partial r_{rt}}{\partial t_{str}}\right|\Delta t_{str} + \left|\frac{\partial r_{rt}}{\partial t_{sr}}\right|\Delta t_{sr} + \left|\frac{\partial r_{rt}}{\partial \delta}\right|\Delta\delta \\ &= \left|\frac{\partial r_{rt}}{\partial c}\right|\Delta c + \left|\frac{\partial r_{rt}}{\partial t_{str}}\right|\Delta t_{str} + \left|\frac{\partial r_{rt}}{\partial r_{sr}}\right|\Delta r_{sr} + \left|\frac{\partial r_{rt}}{\partial \delta}\right|\Delta\delta\end{aligned} \qquad (2)$$

An error in c can be due to inaccurate knowledge of the environment, or large fluctuations of its conditions. The error in t_{str} is due to an inaccurate determination of the reception time of the target echo. Errors in the other two parameters, t_{sr} (or r_{sr}) and δ are due to inaccurate localisation of the source and/or receiver (t_{sr} and δ) and target (δ). More specifically, an error in the heading of the receiving array immediately translates into an error in δ.

2.3. Bistatic Target-Range Errors Using Acoustic Information

The exact position of the source ship is not always available from radar, e.g. when the bistatic platform is a submarine. In that case, the receiving ship may obtain the source position acoustically from the direct blast. Different error components can then be identified:

- Time synchronisation: a common time synchronisation has to be set for all platforms.
- Bearing accuracy: The Cramer-Rao formula estimates the lower bound for the standard deviation of the bearing as (see [3]): $\text{r.m.s. bearing resolution} = \frac{\text{beamwidth}}{\sqrt{SNR}}$, with $\text{beamwidth} = \frac{180}{\pi}\frac{\lambda}{L\sin\phi}$ (degrees), where λ is the wavelength of the pulse, L the acoustic aperture length, ϕ the bearing and SNR the detection signal-to-noise ratio (linear scale).
- Range accuracy (important for t_{str} and t_{sr}): the –3dB width of the pulse ambiguity function depends on the pulse type. For broadband pulses, the width is equal to $c/(2B)$ for a monostatic range calculation, where B is the pulse bandwidth. For CW pulses, it is equal to $cT / 2$, T being the pulse time-length.

2.4. Source / Receiver Position Errors Using Non-Acoustic Information

- GPS positioning error: An accurate way to determine the position of each platform is to use its GPS co-ordinates that have an accuracy of about 3 meters nowadays (about 1 meter by using a Differential GPS).
- Tow-ship movement: During the pulse repetition time, the tow vessel sails with a certain (fixed) course and speed. Its position therefore changes during this time interval. To infer the true position a target echo originates from, this movement should be taken into account. For a pulse repetition time of 60 s, a receiver-ship speed of 10 knots, a source-ship at $r = 20$ nmi, $\phi = 90°$, the range difference can be as large as 150 – 200 m, and the bearing difference as large as 2°.
- Source / Receiver-array position w.r.t. tow-ship (offset): The offset of a towed source or receiver array relative to the GPS tow-ship consists of the layback (i.e., the horizontal distance behind the ship), the transverse distance, and the depth. Although not being small, the depth is neglected for 2D localisation purposes. The transverse distance can be obtained from the layback and the drift angle (i.e., the angle between ship course and tow cable). The drift angle can be estimated from the difference between the heading of the towed source or the receiving array (both equipped with

accurate heading sensors) and the ship course, assuming the tow cable and the sonar to have the same heading.

3. BISTATIC TARGET-RANGE CONSIDERING ALL ERROR SOURCES

This section discusses the expected errors on the bistatic target range considering the four error components Δc, Δt_{str}, Δt_{sr} and $\Delta\delta$ identified in equation (2). These components are independent of each other since Δt_{str} and Δt_{sr} are only due to the finite time resolution of the ambiguity function and to a possible time offset between source and receiver (non-synchronisation); not to an error in c. To visualise the results of equation (2), the errors have been assumed to be normally distributed with zero mean and standard deviations σ_c, σ_{t_str}, σ_{t_sr} and σ_δ, resp. (It implies that Δr_{rt} is also Gaussian distributed, with zero mean and standard deviation σ_{r_rt}, since Δr_{rt} is a linear combination of four Gaussian variables.)

3.1. Localisation Error Considering Δc, Δt_{str}, Δt_{sr} and $\Delta\delta$

Using section 2.3, for an FM pulse with B=500Hz, and a detetection SNR = 10dB, the errors assumed on the four components c, t_{str}, t_{sr}, and δ are: σ_c =7.5m/s (i.e. 0.5% of 1500m/s), σ_{t_str} = σ_{t_sr} = 0.63ms (=2s for a CW pulse with T=2s) and σ_δ =0.32°. Figure 2 presents the σ_{r_rt} map for all possible target positions for the FM (left-hand panel) and CW pulse (right-hand panel).

In both cases, a region exists where σ_{r_rt} is huge and is located in the blind ellipse. The minimum error in r_{rt} is higher for a CW pulse than for an FM pulse, and is due to pulse distance resolution, as explained in section 2.3. Simulating with different scales, it appears that σ_{r_rt} is homomorphic to the source-receiver distance for both FM and CW pulses.

Looking at the FM pulse case, it can be noticed that the closer the receiver is to the target, the more accurate the localisation will be. Thus, it would be interesting for the receiver, once the target is detected, to stay or move closer to get better target-position estimates.

3.2. Most Influent Error Components

It can be valuable to know which error component has the largest influence on σ_{r_rt} (for a given scenario) in order to identify where extra efforts should be made to improve the system. Figure 3 shows the contribution (in %) to the bistatic target-range error induced by the two most important error components: Δc (left-hand panel) and $\Delta\delta$ (right-hand panel).

As expected, the further away the target is, the bigger the influence of Δc is. Δc has also a major influence when the target is along the source-receiver line and not between them. $\Delta\delta$ has a major influence when the target is on broadside of the source-receiver line and not too far from them. Δt_{str} and Δt_{sr} have a very weak influence (only few %) on σ_{r_rt}, and thus can be neglected, except when the target is along the source-receiver line and in between them (influence reaches around 50% each). Considering a CW pulse, the two most influent error components are also Δc and $\Delta\delta$ and their respective influence areas are very similar to the ones presented for the FM pulse.

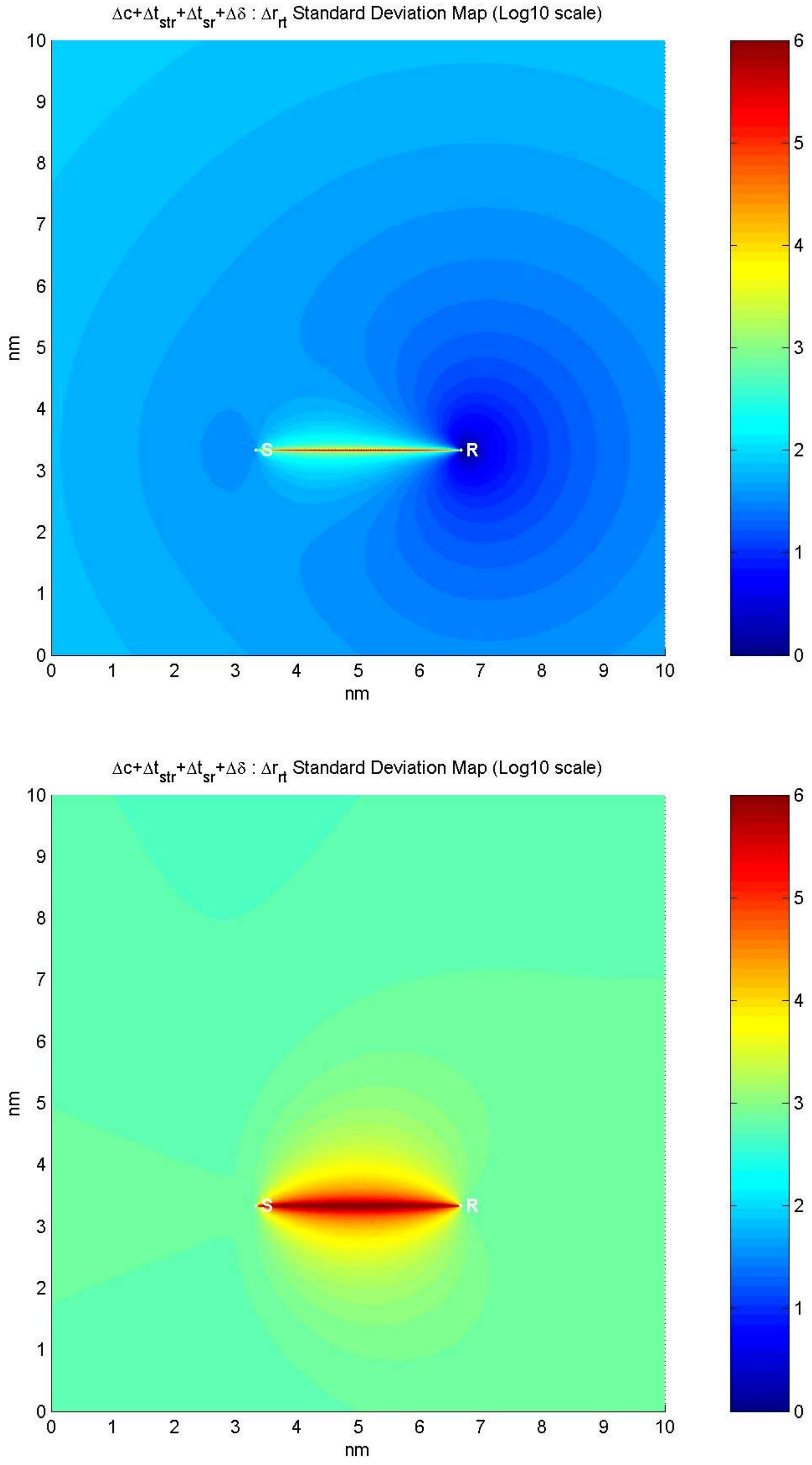


Figure 2: Standard deviation of the bistatic range error (^{10}log scale, in meters) for all target positions detection SNR=10dB, considering all four error components. Source(S)-Receiver(R) distance fixed at 3.33nmi. Top: FM pulse, B=500Hz. Bottom: CW pulse, T=2 s.

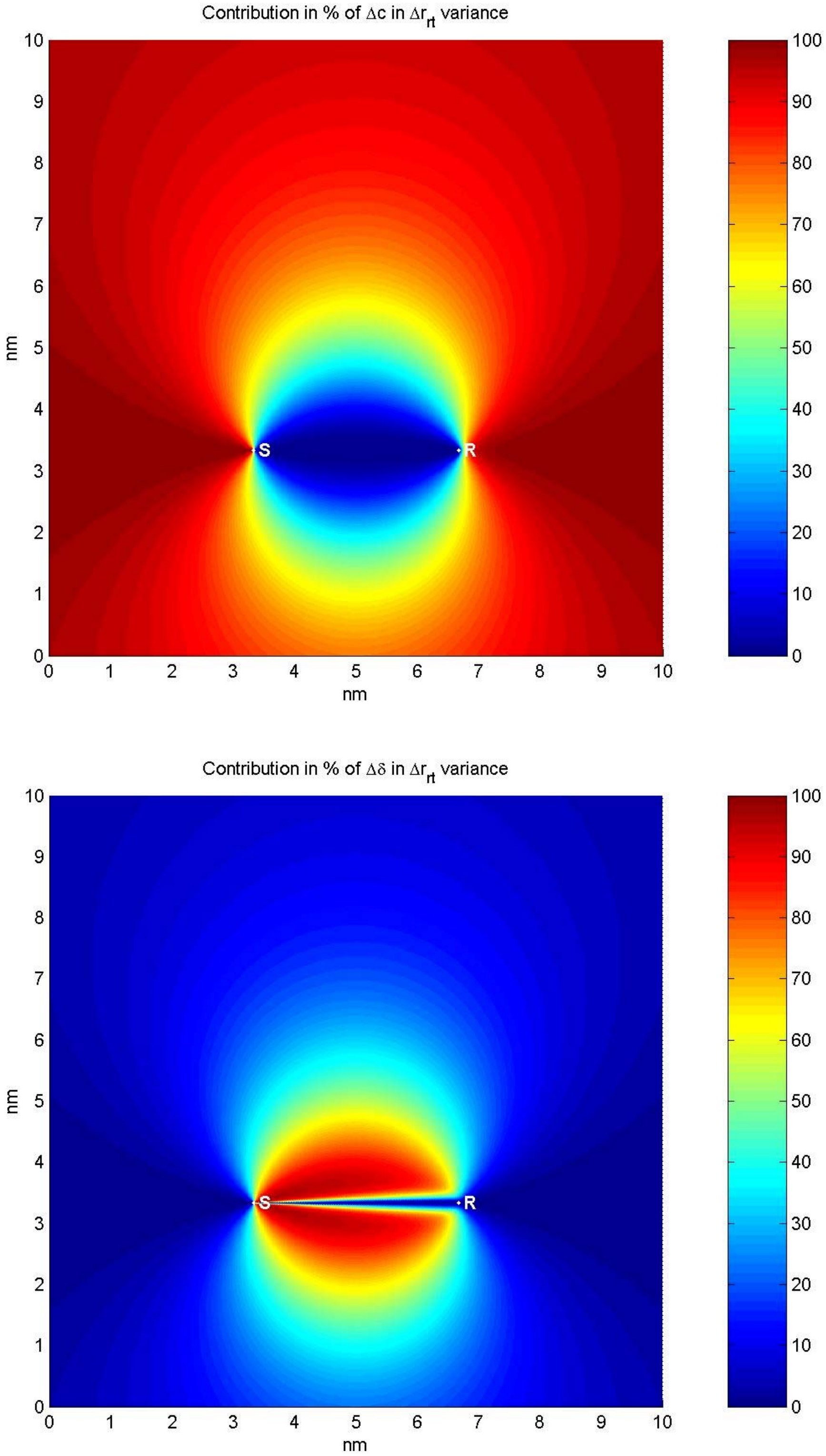
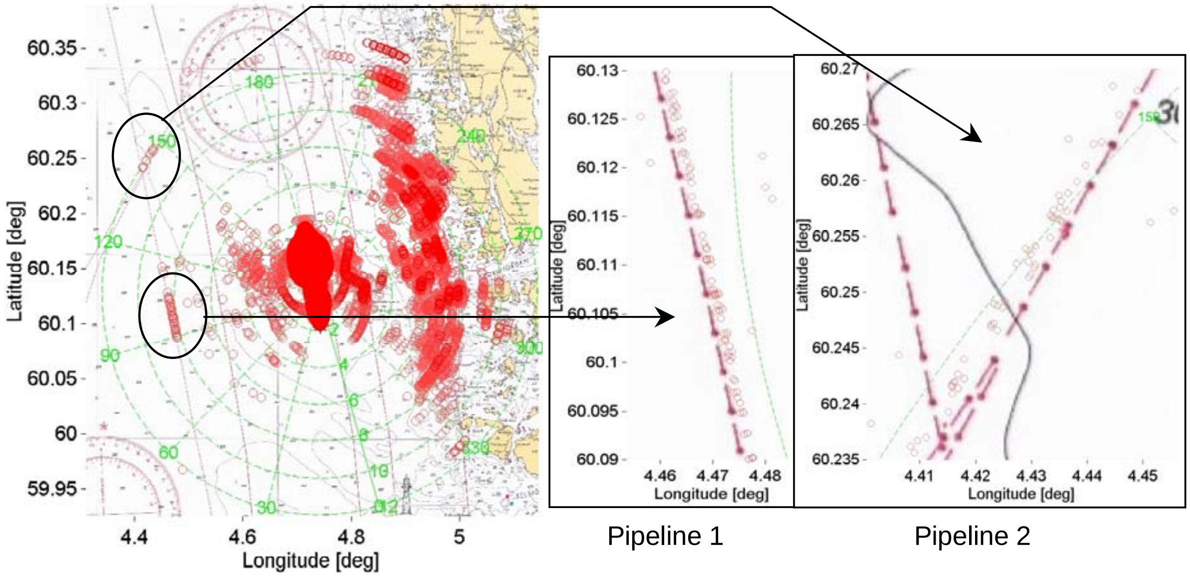


Figure 3: Contribution in % to localisation error for all target positions using a FM pulse with B=500 Hz and a detection SNR = 10 dB: top: Δc, bottom: $\Delta\delta$. The other two error components (Δt_{str} and Δt_{sr}) do not contribute significantly in this case.

4. BISTATIC LOCALISATION ACCURACY DURING SEA-TRIALS

The left-hand panel of Figure 4 plots on a sea chart all the bistatic detections above a 5dB threshold get during an experiment carried out in 2001 near the Norwegian coast, emitting HFM pulses with B = 800Hz. Inside the ellipses, two sets of pipelines detections are observed and magnified in the middle and right-hand panels. The source-receiver separation has been computed with the direct blast travel time (see section 2.3). Two other methods can be used: with the two tow-ships GPS positions only ('GPS' method), or with the GPS positions and the estimated offset of the source and receiver array ('GPS+offset' method), see section 2.4.



Pipeline 1 Pipeline 2

Figure 4: Left: Bistatic detections above SNR=5dB during a sea-trial experiment. Detections inside the black ellipses are pipeline highlights. Middle and right: Overlaying of hydrographic map and bistatic detections of pipelines (○).

Method ↓	Pipeline 1 [m]	Pipeline 2 [m]
GPS	191 ± 86 (*130 samples*)	220 ± 79 (*57 samples*)
GPS + offset	190 ± 86 (*130 samples*)	236 ± 86 (*58 samples*)
Acoustic	203 ± 99 (*134 samples*)	155 ± 91 (*60 samples*)
Theoretical standard deviation	± 86	± 77

Table 1: Mean and standard deviation of range difference, in meters, between the position of the pipeline detections and their corresponding true positions from the bathymetric map. The number of samples used to get the following statistics is between parentheses. The lower row gives the theoretical standard deviation of the target-receiver range (see chapter 3), using an error on sound speed of 1%, c=1500 m/s, beamwidth=2°, SNR=5dB and an FM pulse with B=800Hz.

Table 1 shows statistics of the range differences between each bistatic pipeline detection and its corresponding true position on the bathymeric map. The mean range differences are all of the order of 200 m, and the standard deviations are all 80 – 100 m (or 0.5 – 1.0% of the distance between the pipelines and the bistatic receiver). These numbers are very similar to those expected theoretically (see lower row of Table 1). From sections 3.1 and 3.2, the mean range difference is expected to be mostly due to an error in the sound speed. (Note that the 1% error in distance agrees well with the assumed error in c in the simulations).

For pipeline 2, the acoustic source positioning gives the smallest mean range difference, 155 m, but this fact is not observed for pipeline 1.

5. CONCLUSION

This paper discusses the target localisation error using a bistatic receiver. Firstly, the theory of bistatic localisation is approached and the error on r_{rt}, the target-receiver distance, is derived. The error on r_{rt} is shown to depend on the sound speed, the bistatic echo travel-time, the direct-blast travel-time, and the bearing accuracy. Secondly, the error dependence on r_{rt} is analysed. The target position plays a major role in the final result and makes an error component to be less or more influential; the source-receiver distance only scales the error map.

Data recorded during a multistatic sea-trials are analysed. Positions of bistatic detections of two pipelines are compared to the true positions on a hydrographic map. The measured standard deviations of the position difference agree very well with the theoretical results. The most likely cause of error in localising bistatically these two targets is an error in the sound speed.

6. ACKNOWLEDGEMENTS

The present research has been carried out in the framework of the New-Array Technology programme. This programme is carried out by TUS (Sophia, France), TNO-FEL (The Hague, The Netherlands), and FFI (Horten, Norway), and is sponsored by the Royal Netherlands Navy and the French Ministry of Defence.

REFERENCES

[1] Multistatics in airborne ASW*; Requirements for tactical support systems*, Naval Forces, 5, 1999

[2] **S. Sutherland-Piertrzak**; *Bistatic Reverberation effect, SACLANTCEN Conf. Proc*, CP-40 ocean, 1993

[3] **W. Burdic**; *Underwater acoustic system analysis*; Prentice-Hall.; 1991