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IMPROVED PERFORMANCE OF GLOBAL OPTIMISATION METHODS FOR INVERSION PROBLEMS IN UNDERWATER ACOUSTICS

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To invert for parameters of the seabed and the water column, global optimisation methods are used. These methods search for the best parameter combination, so as to make the difference between modelled and measured pressure fields as small as possible. Simulated annealing and genetic algorithms (GA) are commonly used as global optimisation algorithms for this inversion. Their efficiency depends on the tuning of the algorithm. For a bottom model with in the order of ten geo-acoustic parameters, a relatively new global optimisation method - Differential Evolution (DE) - is applied. We claim a ten times higher efficiency of DE, as it finds the optimal parameter combination with only one tenth of the number of model evaluations needed by GA. Apart from the tuning of the algorithms, the performance of DE in comparison with GA is investigated in terms of efficiency and robustness.

1. INTRODUCTION

In the last two decades global optimisation methods, such as simulated annealing (SA) and genetic algorithms (GA), have been applied to solve inversion problems in underwater acoustics. In this inversion a pressure field calculated by a model is compared with a measured pressure field, using a cost or energy function. The search is for that parameter combination that maximises similarity between modelled and measured field, i.e., giving the lowest energy value. In general, a fairly large amount of parameters are sought, typically 5 to 20. This is called 'matched field inversion'.

A purely random search would require too many forward model calculations. Search methods strongly reduce this number of function evaluations, but have the risk to fail by being trapped in a local minimum of the energy function. To overcome this problem, global optimisation methods, such as simulated annealing and genetic algorithms, have been designed.

Performance criteria for optimisation methods are efficiency, robustness and accurateness. Since the forward model calculations take most of the computation time during optimisation, efficiency is measured by the required amount of these calculations, typically a few tens of thousands. The stochastic nature of these methods causes that even a global search run can get stuck in a local minimum. Robustness is defined as the probability of an optimisation run to end in the global minimum, i.e., the probability of success of the optimisation method. Accurateness has to do with the distance between the outcome of a successful optimisation run and the precise location of the global minimum. In this article the attention is focused on robustness and efficiency.

Optimisation methods have a few setting parameters (e.g. crossover probability and mutation rate in GA), that have to be optimised themselves. It will be shown that these settings strongly influence the performance of the method. Although the optimal setting can be problem specific, we hope to find optimum settings that only need minor adjustments for other problems.

It is already well established that in matched field inversion GA outperforms SA. However, there is a need for methods even better than GA. A relatively new global search method, differential evolution (DE), is introduced. Its performance, in terms of efficiency and robustness, is shown to be better than that of genetic algorithms.

2. BASIC PRINCIPLES OF GENETIC ALGORITHMS AND DIFFERENTIAL EVOLUTION

In underwater acoustics the GA has been introduced by Gerstoft [1]. Its principle of operation is summarised as follows. To optimise an energy function E, GA improves populations of q parameter combinations during successive generations. A GA optimisation run starts by creating at random an initial population of q members. Each member represents a certain parameter value combination $\mathbf{m}$ and its energy E is calculated. A population member with a lower E (a higher fitness) has a higher probability to be selected in one (or more) of the pairs of parents to be formed. This results in $q/2$ pairs of parents with a larger proportion of fit members. From each pair of parents two children are created by a random exchange of parental values (steered by the crossover probability p_c), followed by random changes of individual parameter values (with mutation probability p_m). A set of q children vectors results. The next generation is established by taking at random $f_r q$ members of these children and selecting the $(1-f_r)q$ most fit members of the current population. This process is repeated over several hundreds of generations.

GA's have the following setting parameters: population size q, crossover probability p_c, mutation probability p_m, reproduction size f_r and the number of generations N_G.

Differential evolution, just like GA, starts with an initial population of q randomly chosen parameter value combinations $\mathbf{m}$. These $\mathbf{m}$'s are improved during successive generations of constant size q, in the sense that a descendant replaces an $\mathbf{m}$, becoming its successor, if it has a lower energy. The distinctive feature of DE is the way in which these descendants are created. In [2] various ways to generate new $\mathbf{m}$'s are described. Here only the following procedure is considered. At the start of generation k the parameter vectors $\mathbf{m}_{k,1},\ldots,\mathbf{m}_{k,q}$ are given and for each of them a descendant is created, being a potential successor. To create this descendant $\mathbf{d}_{k,i}$ of $\mathbf{m}_{k,i}$ first a partner $\mathbf{p}_{k,i}$ is constructed according to

$$\mathbf{p}_{k,i} = \mathbf{m}_{k,i_1} + F(\mathbf{m}_{k,i_2} - \mathbf{m}_{k,i_3})$$

with three different vectors $\mathbf{m}_{k,i_1}, \mathbf{m}_{k,i_2}$ and $\mathbf{m}_{k,i_3}$ chosen at random from the population and F a scalar multiplication factor between 0 and 1. The descendant $\mathbf{d}_{k,i}$ of $\mathbf{m}_{k,i}$ results from applying crossover to $\mathbf{m}_{k,i}$ and $\mathbf{p}_{k,i}$ with crossover probability p_c. A higher value of p_c leads (on the average) to more dimensions of $\mathbf{p}_{k,i}$ to be copied into $\mathbf{d}_{k,i}$, while the values for the remaining dimensions are taken from $\mathbf{m}_{k,i}$. $\mathbf{d}_{k,i}$ only replaces $\mathbf{m}_{k,i}$, becoming its successor, if its energy is lower.

The setting parameters of DE are population size q, multiplication factor F, crossover probability p_c and the number of generations N_G.

Comparing GA and DE, both show improvement of a fixed sized population of parameter combinations. However, in each generation of GA a fixed fraction of the population is replaced by children, whether they are better or not. In DE each member of the population will be replaced by its descendant only if this descendant has lower energy. On the other hand, the fitness of a member increases the probability of that member to play a role in the creation of children in GA, where in DE this fitness does not influence the probability of a member to be selected. Another difference is the way a partner $\mathbf{p}_{k,i}$ is found for $\mathbf{m}_{k,i}$. While in GA the partner is an element of the given generation, in DE it is constructed from the population and thus not an element of this generation. The partner creation process in DE is steered by the multiplication factor F and replaces the mutation mechanism of GA.

3. PERFORMANCE OF GENETIC ALGORITHMS AND DIFFERENTIAL EVOLUTION

A test function given in [3] is known to have characteristics that correspond to those of the geo-acoustic inversion problem. Its features are: number of parameters $N_p = 6$, various local minima, parameter coupling and a varying sensitivity to the parameters m_i. The global minimum is at the origin $m_i = 0$. The research on the performance of GA and DE presented here, is restricted to minimisation of this energy function called E_1.

To determine the probability of success multiple optimisation runs are needed, each run starting with a different randomly chosen population of q $\mathbf{m}$'s. The number of independent runs per setting is denoted N_r. N_c is the amount of these runs that have converged to the global minimum. The fraction of converged runs serves as an estimate for the probability of success p_s according to

$$p_s = \frac{N_c}{N_r} \quad \text{with} \quad \sigma_{p_s} = \frac{\sigma_{N_c}}{N_r} \quad \text{and} \quad \sigma_{N_c} = \sqrt{p_s(1-p_s)N_r}$$

When comparing the performance of different settings of the optimisation algorithms, also the statistical uncertainty on p_s, being σ_{p_s}, should be taken into account (where N_c has a binomial distribution). The efficiency is measured by the number of function evaluations or forward model calculations N, given by $N = f_r\, q\, N_G$ for GA and $N = qN_G$ for DE.

3.1. Settings for the Genetic Algorithm

GA is applied to E_1 with the following settings: (q = 64, f_r = 0.8) and (q = 128, f_r = 0.4), both with N_G = 500 (therefore both with N = 25600). p_c was varied in between 0.1 and 0.9 in steps of 0.2, whereas p_m was varied in between 0.025 and 0.25 in steps of 0.025. The number of independent runs N_r equals 50 for each combination of settings. The estimates of the success probabilities for the considered settings are presented in the figures 1 and 2.

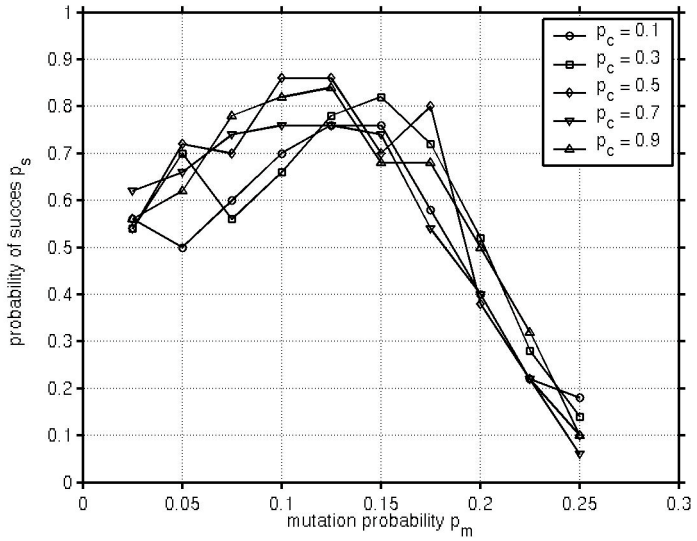

Figure 1: GA with q = 64, f_r = 0.8; statistical error on p_s is about 0.06

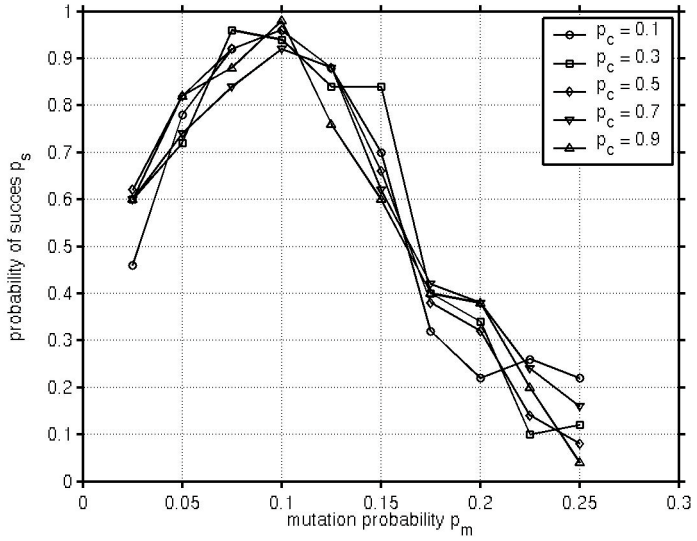

Figure 2: GA with q = 128, f_r = 0.4; statistical error on p_s is about 0.06

It is observed that the attainable level of p_s hardly depends on p_c, while the choice of p_m is of major importance. There is a rapid fall off of p_s versus p_m for p_m > 0.15. The optimum setting of p_m is slightly larger than 0.1. The setting q = 128 and f_r = 0.4 (also with N=25600)

leads to higher levels of p_s, with the possibility of nearly 100 % success. Notice the need of taking the (statistical) error on p_s into account! Finally, Gerstoft's default setting [1] (q = 64, N_G = 500, f_r = 0.5, p_c = 0.8, p_m = 0.05) is not robust, leading to p_s = 0.69 (with N only 16000).

3.2. Settings for Differential Evolution

Applying DE to E_1, two combinations of q and N_G are considered: (q = 16, N_G = 150) and (q = 32, N_G = 75), both with N = 2400 (instead of the 25600 for GA). To provide more accurate estimates for p_s, the number of independent runs per setting is taken N_r = 500. p_s for different combinations of F and p_c is given in figures 3 and 4.

It is seen that the smaller number of generations N_G = 75 requires a lower setting of the contraction factor F. DE seems to be more sensitive to the choice of F than to that of the crossover probability p_c.

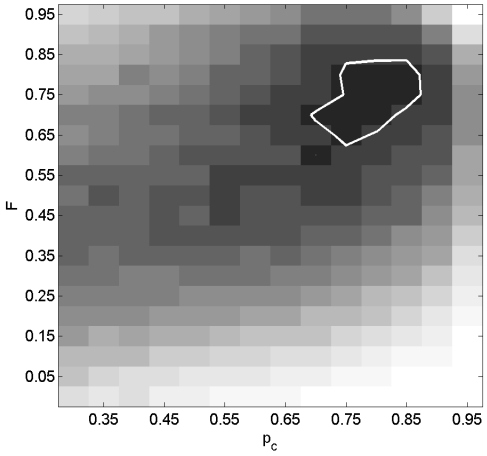

Figure 3: DE with q = 16, N_G = 150
90 % p_s contour

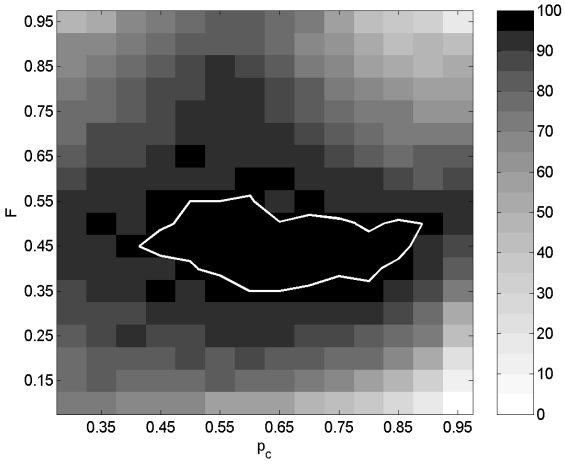

Figure 4: DE with q = 32, N_G = 75
97 % p_s contour

3.3. Comparing the performance of GA and DE

Table 1 gives the Gerstoft GA setting [1] and the optimum settings found for GA and DE.

	q	N_G	p_c	p_m	f_r	F	N	p_s
GA Gerstoft	64	500	0.8	0.05	0.5		16000	0.69 ± 0.05
GA best	128	500	0.9	0.1	0.4		25600	0.98 ± 0.02
DE	16	150	0.85			0.8	2400	0.91 ± 0.013
DE	32	75	0.75			0.5	2400	0.98 ± 0.007
DE	32	150	0.8			0.6	4800	0.99 ± 0.005

Table1: GA and DE best settings and corresponding performances.

With comparable robustness (p_s = 0.98) DE requires only 2400 function evaluations instead of the 25600 of GA, an order of magnitude improved efficiency.

How to explain this better performance? We suspect that the partner creation process of DE is a better way to escape from a local minimum. Therefore, the contraction factor F is an important setting parameter of DE. At the same time this partner creation process offers the

possibility to better explore the parameter search space, provided the mutual distances of the population members are sufficiently large.

4. SUMMARY AND CONCLUSIONS

In this paper a relatively new global optimisation method called differential evolution (DE) has been tested. This method can be applied for solving inversion problems in underwater acoustics. Its performance in terms of efficiency and robustness was compared with that of a genetic algorithm (GA). Efficiency is measured by the number of function evaluations. Robustness is defined as the probability of success. The fraction of runs that converge to the global minimum, out of a number of independent repetitions of the algorithm for a given setting (typically 50), estimates this probability.

The optimisation problem comprises the minimisation of an energy function depending on 6 parameters, featuring various local minima, parameter coupling and varying sensitivity to the parameters. Minimising this function with GA requires several tens of thousands of function evaluations. The performance of GA hardly depends on crossover probability, whereas it depends critically on mutation probability. Using 25600 function evaluations almost 100 % success probability was realised, provided a mutation probability of 10 % is taken. With DE the same success rate is obtained with 10 times less function evaluations. The performance of DE depends mainly on the multiplication factor and less on crossover probability.

The following question remains. Do the best settings found for the optimisation algorithms depend on the optimisation problem at hand? To this end we introduce another optimisation problem, i.e., geo-acoustic parameter estimation using matched field inversion [Snellen, 2004].

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