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APPLICATION OF DIFFERENTIAL EVOLUTION AS AN OPTIMISATION METHOD FOR GEO-ACOUSTIC INVERSION

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In an accompanying paper ([1]) the global optimization method Differential Evolution (DE) is described and its performance assessed and optimized. DE is shown to be more efficient than a genetic algorithm (GA). Here, DE is applied for geo-acoustic inversion. The receiving system is a vertical array. The sound source was at a fixed position. From the received acoustic signals, spanning 8 hours, 41 snapshots are used for inverting for the geo-acoustic parameters. The variability in the resulting parameter estimates stems from the imperfect optimization method, variability in the water column and noise in the data. Both DE and GA were used for the optimization, again demonstrating superior performance of DE. Also DE is used for simulations quantifying the three contributions to the parameter uncertainty. These three origins are shown to almost completely account for the parameter uncertainty.

1. INTRODUCTION

The genetic algorithm (GA), a global optimization method, has been applied extensively to inversion problems in underwater acoustics. In an accompanying ([1]) paper an alternative global optimization method, viz. differential evolution (DE), is introduced. DE is found to be more (about 10 times) efficient than the GA, when searching for the global optimum of a test function. In this article we investigate whether this also holds for a real geo-acoustic inversion problem.

An important issue when dealing with parameter estimation problems is the accuracy of the estimates. For the situation considered we assume that there are three main causes for parameter uncertainty. These consist of the optimization method itself, which is designed to *find* the global optimum, but not necessarily to *accurately* locate it, oceanographic variability and noise in the data. In this paper we assess the accuracy of the parameter estimates quantitatively by estimating the contributions of each of the above-mentioned causes. Use is made of acoustic data that have been obtained during the ADVENT99 sea trial ([2],[3]).

2. THE ADVENT99 EXPERIMENT

A detailed description of the ADVENT99 experiments, jointly conducted by SACLANT Centre and TNO Physics and Electronics Laboratory, is given in [2] and [3]. A large part of the ADVENT99 sea trial consisted of acoustic experiments with both the source and the receiver at a fixed position. These experiments were conducted in a shallow water area (Adventure Bank, water depth 80 m) for source/receiver ranges of 2, 5 and 10 km. As in [2] we only consider data of the 2-km experiment. The acoustic source was mounted on a tower construction that was moored on the sea bottom for keeping it at a fixed position. The receiving system is a vertical array, containing 64 hydrophones and spanning 62 meters of the water column. As in [2] we consider the multi-tone signals transmitted in the band 200-700 Hz. 41 snapshots of 2 s data were selected from the received time series and were Fast Fourier Transformed into the frequency domain. We have selected the frequencies 200, 300, 400 and 600 Hz. The resulting complex pressures as a function of depth are referred to as 'pressure fields'. These 41 sets of pressure fields are used for the inversions, and correspond to data transmitted at 15 minutes interval, spanning the total duration of the 2-km experiment (about 8 hours). A CTD-chain was towed back and forth along the acoustic track providing information on the sound speed structure of the water column. These data are used for simulating the effect of oceanographic variability on the parameter uncertainty.

3. ACOUSTIC INVERSION METHOD

For the forward acoustic model we have used the standard normal mode technique. The sediment layer and the sub-bottom are treated as fluid layers and the high loss continuous eigenvalue spectrum is ignored.

Section 3.1 briefly describes the acoustic problem. In Section 3.2 the objective function to be minimized is described. Section 3.3 presents details on the optimization methods used.

3.1. The acoustic problem

The unknown parameters included in the inversion are extensively discussed in Ref. [2] and are briefly summarized below.

The bathymetry along the 2 km acoustic track was found to be fairly range-independent and therefore we assume a constant water depth H_w. The geo-acoustic model consists of a single sediment layer with thickness h_{sed}, overlying a homogeneous sub-bottom. The sediment compressional wave speed is assumed to vary linearly with depth from $c_{1,sed}$ at the top of the sediment to $c_{2,sed}$ at the bottom of the sediment and to have a constant value c_b in the sub-bottom. The attenuation constant α and the density ρ in the sea bottom are taken to be depth independent and are assumed to be equal in the sediment and the sub-bottom. The array configuration is defined by estimating h_1, the distance of the deepest hydrophone to the bottom. The source range, r_s, and the source depth, here defined by the distance Δ from the source to the bottom, have a large influence on the acoustic propagation and are not known to the required accuracy and are therefore inverted for too. The sound speed profile used for the inversions is the sound speed profile that corresponds to the CTD taken 17 minutes prior to the execution of the 8-hour experiment.

Table 1 lists the unknown parameters and their search bounds.

3.2. The energy function

The energy function gives a quantitative measure for the agreement between the calculated and measured pressure fields. We have selected the following energy function E, which is based on the incoherent multi-frequency Bartlett processor ([4])

$$E(\mathbf{m}) = 1 - \frac{1}{K}\sum_{k=1}^{K}\left|\mathbf{p}(f_k)\cdot\mathbf{p}_c{}^*(f_k,\mathbf{m})\right|^2 \qquad (1)$$

with **m** the vector containing the unknown parameters, see previous section; the * denotes the complex conjugate transposed and $\cdot$ is the inner product of vectors $\mathbf{p}(f_k)$, the measured pressure field, and $\mathbf{p}_c(f_k,\mathbf{m})$, the pressure field calculated for parameter set **m**, both at frequency f_k. The pressure vectors are normalized ($\|\mathbf{p}\|=\|\mathbf{p}_c\|=1$). The number of frequencies K is 4, see previous section.

3.3. The optimization methods

For the optimization both DE and the GA are applied. In [1] a description of the GA and DE is given. For DE a population of 32 elements was improved during 150 generations, with a crossover probability of 0.8 and a multiplication factor of 0.7. The GA was run with the following settings: 5 independent runs per snapshot, 400 generations, a population size of 64, a reproduction size of 0.5, a crossover rate of 0.8 and a mutation rate of 0.05. Simulations in [2] indicated that the GA always locates the global optimum, but that there is a significant variation in the parameter estimates due to the fact that the GA is designed to *find* the global optimum, but not to *accurately* locate it. To reduce this uncertainty a local method is applied after the GA optimization. For this we have selected downhill simplex (DHS). DHS is not efficient in the amount of function evaluations that it requires, but has the important advantage that it does not require the calculation of derivatives. For further details we refer to [5]. DHS was also applied on the DE outcomes. However, due to the smaller uncertainty of the DE results compared to the GA results the number of DHS runs is much smaller.

4. COMPARISON OF DE AND GA PERFORMANCE

Both optimization approaches of section 3.3 were applied for estimating the unknown parameters through inversion of the 41 snapshots of acoustic data. Since the experimental configuration is stationary, the unknown parameters should be constant with time. Therefore, the inversion results can be used for determining the mean ($\overline{m}$) and the standard deviation (σ) for each parameter, out of the 41 observations. Assuming statistically independent observations, the uncertainty (or error) on the mean ($\sigma_{\overline{m}}$) and on the standard deviation (σ_σ) is given by $\sigma_{\overline{m}} = \frac{\sigma}{\sqrt{N}}$ and $\sigma_\sigma = \frac{\sigma}{\sqrt{2N}}$.

Table 1 presents the two resulting sets of means of the parameter estimates with their uncertainties. Both optimization approaches are seen to give equal (within their uncertainties) estimates for all unknown parameters. Fig. 1 graphically presents the standard deviations of the parameter estimates (σ) together with their uncertainties ($2\sigma_\sigma$). This figure indicates that also the accuracy of the parameter estimates is the same for both optimization approaches.

Parameter	*GA/DE search bound*	*GA/DHS result* $(\overline{m} \pm 2\sigma_{\overline{m}})$	*DE/DHS result* $(\overline{m} \pm 2\sigma_{\overline{m}})$
$c_{1,sed}$ (m/s)	[1475 1700]	1549 ± 3	1548 ± 2
h_{sed} (m)	[1 25]	18 ± 2	18 ± 1
$c_{2,sed}$ (m/s)	[1475 1800]	1678 ± 22	1687 ± 14
c_b (m/s)	[1515 1900]	1823 ± 81	1781 ± 22
ρ (g/cm^3)	[1 2.3]	1.4 ± 0.02	1.4 ± 0.02
α (dB/λ)	[0 1]	0.34 ± 0.02	0.34 ± 0.02
r_s (m)	[1700 2500]	2186 ± 14	2181 ± 13
Δ (m)	[0 10]	3.7 ± 0.06	3.7 ± 0.05
H_w (m)	[75 85]	79.7 ± 0.2	79.6 ± 0.2
h_1 (m)	[7.5 12.5]	9.9 ± 0.2	9.8 ± 0.2

Table 1: GA and DE parameter search bounds (second column). Parameter estimates (mean and uncertainty) obtained by inversion of the experimental data: by GA/DHS (third column) and by DE/DHS (fourth column).

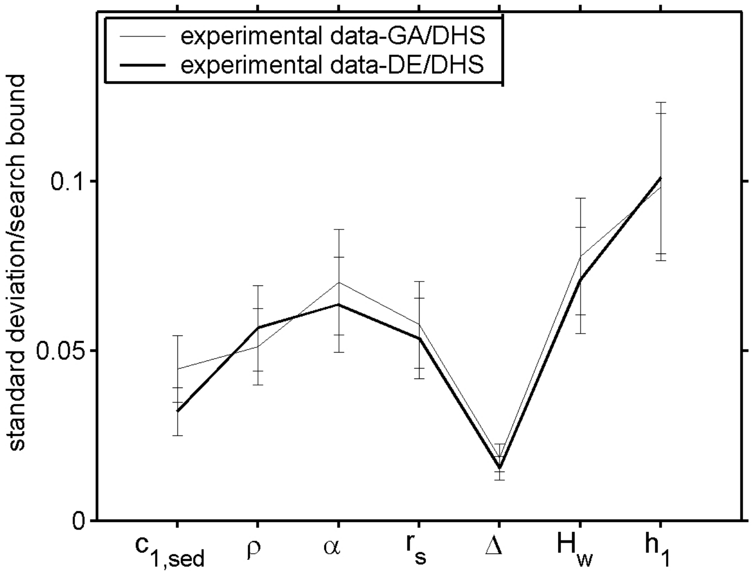

Fig.1: Standard deviations σ, normalized by the search bounds, of the parameter estimates that are obtained by the inversion of the experimental data using the GA (thin line) and DE (thick line) (both followed by DHS). The error bars indicate the statistical errors ($\pm 2\sigma_\sigma$) on the standard deviations.

The amount of forward model calculations, however, differs significantly. For the GA 64000 model runs are required per snapshot (5 independent runs x 400 generations x a population of 64 elements x a reproduction size of 0.5). The subsequent DHS runs require on the average an additional 17000 forward calculations per snapshot. Each GA/DHS result, therefore, requires about 81000 model calculations. For the DE/DHS runs, this number is significantly less, amounting to about 6000, indicating a superior efficiency of DE.

5. ASSESSMENT OF THE UNCERTAINTIES OF THE PARAMETER ESTIMATES

The uncertainty on the parameter estimates as observed in the previous section are attributed to the following causes: imperfectness of the optimization method, temporal variability of the water column and noise on the data. These three processes are assumed to be independent.

In [2] the effect of the first two origins has been assessed through simulations, denoted by SIM1 and SIM2 respectively. The SIM1 simulations consisted of inverting 41 times for the same synthetic pressure field. The variation in the resulting parameter estimates reveals the contribution of the imperfect optimization method. The SIM2 simulations consisted of inverting 41 pressure fields that were generated using 41 different measured (see Section 2) sound speed profiles. In this way the contribution of oceanographic variability to the uncertainty of the parameter estimates is assessed. In [2] these two simulations were done using the GA only, resulting in a high contribution of the first origin, i.e., the optimization method.

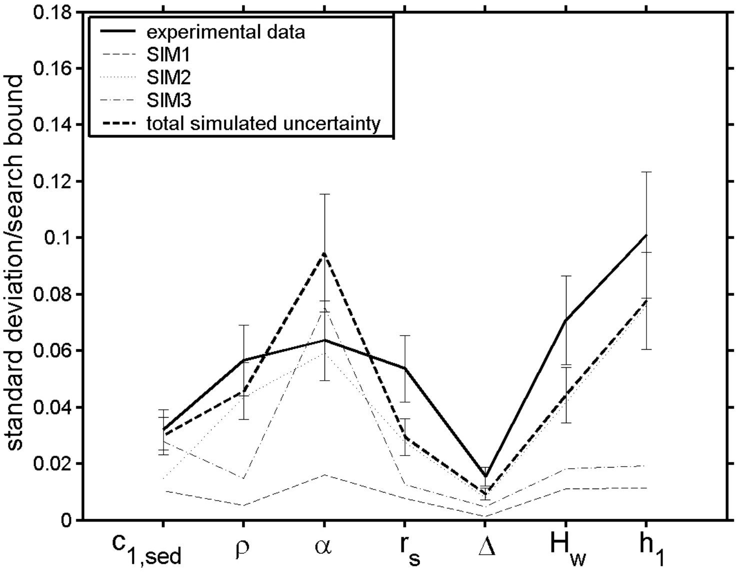

Fig.2: Standard deviations (normalized by the search bounds) of the parameter estimates corresponding to the SIM1, SIM2, SIM3, and experimental data inversions (using DE/DHS). Also shown is the total simulated uncertainty (thick dashed line). The error bars indicate the statistical errors (2σ) on the standard deviations.

Here we have used the DE/DHS combination for redoing the simulations. (Alternatively DHS could have been applied to the GA results of [2], but this is less efficient than repeating the simulations with the DE/DHS approach, see previous section). The decreased contribution of the optimization method compared to the results in [2], where use was made of a GA only, allows for a further investigation of the build-up of the uncertainty observed in the parameter estimates obtained by inversion of the experimental acoustic data. In [2] the effect of noise on the data has not been simulated. For this, as with SIM2, a set of 41 pressure fields has been generated, consisting of synthetic pressure fields with measured noise added to it such that the measured signal-to-noise ratios are reproduced. The corresponding simulations will be denoted by SIM3.

Fig. 2 presents the results of these simulations. The optimization method (SIM1) is seen to now contribute to a minor extent only. From the simulations also the total simulated uncertainty is estimated, assuming the three contributions to be independent. The SIM1 uncertainty is accounted for only once, since it is also present in the SIM2 and SIM3 simulations.

6. CONCLUSIONS

By applying both GA and DE (both followed by DHS) to the same geo-acoustic problem, DE/DHS is demonstrated to outperform the GA/DHS combination in terms of efficiency by almost a factor of 15. The accuracy of both optimization approaches is equal.

The improved efficiency allows for assessing, through simulations, a series of possible contributions to the parameter uncertainty, being the optimization method itself, oceanographic variability and noise in the data. The simulations indicate that these three origins almost completely explain the observed parameter uncertainty. The accuracy as obtained with the DE/DHS combination is such that the contribution of the optimization method to the total uncertainty is the smallest of the three simulated contributions.

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REFERENCES

[1] **C. van Moll and D.G. Simons**, Improved Performance of Global Optimisation Methods for Inversion Problems in Underwater Acoustics, Proceedings ECUA 2004, Delft, 5-8 July, 2004

[2] **M. Snellen, D.G. Simons, M. Siderius, J. Sellschopp & P.L. Nielsen**, An evaluation of the accuracy of shallow water Matched Field Inversion results, *Journal of the Acoustical Society of America*, 109 (2), 514 – 527, 2001.

[3] **M. Siderius, P.L. Nielsen, J. Sellschopp, M. Snellen & D.G. Simons**, Experimental study of geo-acoustic inversion uncertainty due to ocean sound-speed fluctuations, *Journal of the Acoustical Society of America*, 110 (2), 769 – 781, 2001.

[4] **D.G. Simons and M. Snellen**, Multi-frequency matched-field inversion of benchmark data using a genetic algorithm, *J. Comp. Acoust.*, 6 (1&2), 135-150, 1998.

[5] **J. A. Nelder and R. Mead**, A simplex method for function minimization, *The computer journal*, **7**, 308-313, 1965.