Measuring Spatial Correlation of Soil Properties

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The probability of geo-technical failure depends for a significant part on random system effects, i.e. parallel system effects due to averaging of fluctuations along failure surfaces and series system effects due to partial correlation among potential failure modes. Decisive for these effects is the structure of spatial correlation of soil properties or, equivalently, the so-called scales of fluctuation. Although much is known about the implications of scales of random fluctuation, very little is known about its magnitudes in practice. Classical geo-statistical methods to evaluate the scale of fluctuation of a soil property are only effective if a large range of sample values is available. As the determination of mechanical soil properties may be quite expensive, the number of sample points in routine soil investigation is usually very limited. Consequently, classical geo-statistical method may be of little help in the assessment of spatial correlation of mechanical soil properties. In this paper a new approach, based on Bayesian statistical inference, will be discussed. It will be demonstrated that this approach leads to fairly accurate estimates of scale of fluctuation in the case of limited number of measurements.

Key words: soil mechanics/ geo-statistics/spatial correlation/ Bayesian inference

1 Introduction

Properties of naturally deposited soils generally exhibit considerable spatial variation. Apart from possibly explainable or at least detectable average trend or drift, generic type of fluctuations may be part of it. Magnitudes of these fluctuations, though of zero mean, are often such that they may have significant effects on the design of geo-technical structures. In order to give due consideration to these fluctuations, two approaches may be considered. Either one attempts to establish a detailed map of the actual pattern of fluctuation, or one chooses to establish a statistical characterization of the fluctuation pattern, applying a probabilistic analysis of the geo-technical phenomenon in question.

The latter approach is indicated when detailed mapping of the fluctuation pattern is out of the question, considering the involved cost of soil investigation. An example of this is spatial fluctuation of soil shearing strength. Considerable effort has therefore been given to the development of

probabilistic procedures for the analysis of stability of earth slopes. Examples have been presented by, among others, Alonso [1] and Vanmarcke [2]. In these applications the fluctuation pattern of soil strength (and other properties) is modeled as a homogeneous random field, the characteristics of which are its probability density function (pdf) and auto-correlation function. Usually the Gaussian distribution type is adopted, which is described by a mean and a standard deviation. The auto-correlation function is often expressed in terms of an exponential decaying function, involving one or two parameters, the so-called scales of fluctuation (Vanmarcke [3]). In particular these scales of fluctuation are of vital influence in the probabilistic analysis.

Even when the first approach, detailed mapping, is pursued the question may arise what sample grid size should be adopted to ensure that the fluctuation pattern is represented up to some required degree of accuracy. To tackle this problem, theories of stochastic interpolation such as kriging (Journel and Huijbrechts [4], Delfiner [5]) or conditional estimating (Dagan [6], [7]) come into mind. These techniques rely on a similar random field modeling of the fluctuation pattern and again scales of fluctuation are important determining parameters.

Though the effects of decay of auto-correlation, or equivalently of scales of fluctuation, are known to be of vital interest, see a.o. Elkateb [21], actual values of scales of fluctuation of specific soil deposit properties are less known. Estimates of autocorrelation parameters for soil properties have been given in the literature (Alonso [2], Höeg and Tang [8], Catalan and Cornell [9]), however; these should be regarded as indicative. It is conceivable that correlation distances depend largely on deposition regime and loading history. Methods to determine scales of fluctuation from small sets of sample observations have been proposed by Vanmarcke [3], Tang [10], Tabba and Yong [19], and DeGroot and Baecher [22], among others, but relatively little attention has been given to robustness of these procedures. In geostatistical applications robust variogram fitting is often based on Least Squares, Weighted Least Squares or Generalized Least Squares techniques(see a.o. Cressie [25], Genton [26]), yet also based on availability of many data.

In this paper attention will be focussed on a Bayesian type of statistical inference, which leads to reliable estimates of the scales of fluctuations and yields, besides best guesses, also indications of its variances. The procedure is robust, in the sense that numerical instability hardly occurs, contrary to a direct fitting procedure, which has been investigated. It has a built-in property to detect whether a set of observed sample values intrinsically contains sufficient information about scales of fluctuation. The procedure has been tested on various generated random field realizations (pseudo measurements), giving excellent results. An application to the analysis of spatial correlation of average resistances, based on a series of cone penetration tests (CPT's), will be discussed.

The proposed procedure shows resemblance with the maximum likelihood estimation techniques, as discussed, among others, by Kitanidis [11], Friis Hansen et. al. [23], Pardo- Igúzquiza [24] related to different fields of application. The occurrence of uniformly distributed likelihood scores in our computations, due to non informative data, raises doubts concerning convergence and uniqueness of a maximum likelihood technique in such case. These complications, however, have not been

reported in the mentioned papers. A maximum likelihood technique, involving unknown pdf parameters, has been reported by Feinerman et al. [12]. They suggested extension of this procedure to involve trend coefficients. Based on our experience, however, we expect that such extension will suffer from heavy computational burden.

Outline of the Bayesian approach

In its simplest form the problem can be stated as follows. Consider a one-dimensional homogeneous Gaussian random field w(x), x being the spatial coordinate. Assume that the mean value μ and the standard deviation σ are known quantities and that the auto-correlation function is of a known type, for example:

$$\rho(\Delta x) = \exp\left(-\left(\frac{\Delta x}{D}\right)^2\right) \tag{1}$$

where D is the auto-correlation parameter. Suppose that the field has been sampled at locations x_i, i = 1 ..n, and that the observation results are $w(x_i) = \omega_i$. The problem is to determine the auto-correlation parameter D on the basis of these measurement results. The parameter D will be referred to as scale of fluctuation. It should be noted, however, that this does not correspond exactly to the definition given by Vanmarcke [16], [17]:

$$a = \int_{-\infty}^{\infty} \rho(\Delta x)\, d(\Delta x) \tag{2}$$

According to this definition it follows that a = 1.78 D. The method to determine D is a straightforward application of the standard procedure for Bayesian inference [13, 14]. Consider a set of values $\{D_k\}$, k = 1...m, which may be thought of as a representative discrete set of possible D values. Using Bayes' theorem, the posterior probability that D takes one of the D_k values, given the measurements $\{\omega_i\}$, can be expressed as:

$$P\{D = D_k \mid \underline{w} = \underline{\omega}\} = C\, f_{\underline{w}}(\underline{\omega} \mid D_k)\, P\{D = D_k\} \tag{3}$$

where $P\{D=D_k\}$ is the prior probability of D being equal to D_k and $P\{D=D_k \mid \underline{w}= \underline{\omega}\}$ the posterior probability, given the measurements. Further, $\underline{w}$ and $\underline{\omega}$ are vector representations of $\{w(x_i)\}$ and $\{\omega_i\}$ (i=1...n), $f_{\underline{w}}(\underline{\omega} \mid D=D_k)$ is the likelihood function and C a normalizing constant. When the joint probability distribution of $\underline{w}$ is assumed to be Gaussian then the likelihood function becomes:

$$f_{\underline{w}}(\underline{\omega} \mid D_k) = \frac{\exp\{-\frac{1}{2}(\underline{\omega}-\underline{\mu})^T\, \Sigma_k^{-1}\, (\underline{\omega}-\underline{\mu})\}}{(2\pi)^{\frac{n}{2}}\, \det(\Sigma_k)} \tag{4}$$

where Σk is an (nxn) covariance matrix with elements:

$$\Sigma_k(i,j) = [\sigma^2\ \rho(|\ x_i - x_j\ |)] = \sigma^2\ \exp(-\left(\frac{x_i - x_j}{D_k}\right)^2) \qquad (5)$$

If it is assumed that all values Dk are equally likely a priori the prior probabilities $P\{D = D_k\}$ in eq (3) can be merged with the normalizing constant C. The sequence of probabilities according to eq (3) constitutes the posterior pdf of the scale of fluctuation, provided that the range of D_k values has been chosen adequately.

Application to random generated fields

In order to investigate the performance of the outlined procedure, it has been applied to a series of Gaussian pseudo measurements (generated measurements). The following field statistics have been adopted in the simulations: expected mean value μ(w) = 0.0, standard deviation σ(w) = 1.0 and scale of fluctuation D = 1.0 with correlation decay according to eq. (1). Based on these parameters 11 equidistant realizations have been generated with sample distance Δx=0.5. Figure 1 shows the generated sample values. The resulting posterior probability distribution for D is presented in figure 2. The distribution has an average E[D] = 1.048 and a standard deviation σ(D)=0.083, which is a very close approximation of the value D=1.0 which was applied in the simulation.

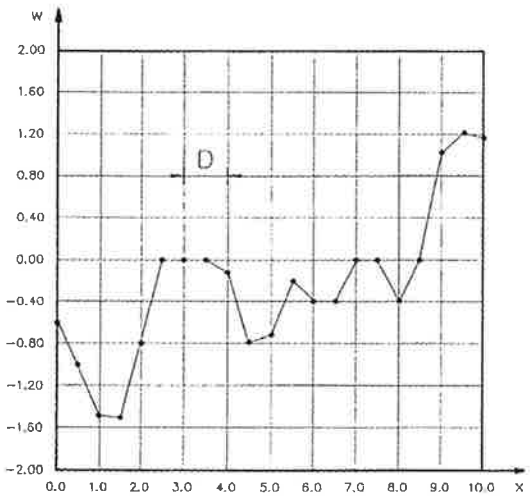

Figure 1: example of pseudo realization of homogeneous Gaussian random field; sample distance Δx = 0.5. Field parameters: expected mean value μ = 0, standard deviation σ =1 and correlation parameter D = 1. Correlation function type: eq. (1)

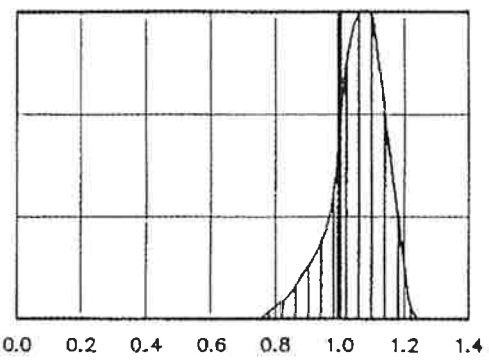

Figure 2: Posterior pdf for correlation parameter D, based on 11 pseudo observations (sample distance Δx = 0.5)

Similar calculations using a sample distance Δx = 0.2 gave an alike, however narrower, distribution. When increasing the sample distance to Δx= 1.0 a different probability distribution came out (figure 3). In this case the sample distance is relatively wide compared to the scale of fluctuation.

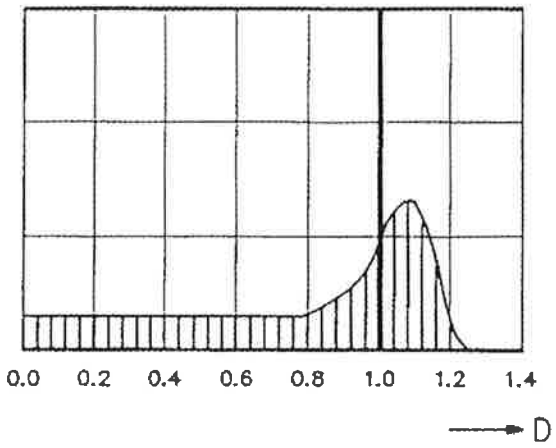

Figure 3: Posterior pdf for correlation parameter D, based on 11 pseudo observations; sample distance $\Delta x = 1.0$

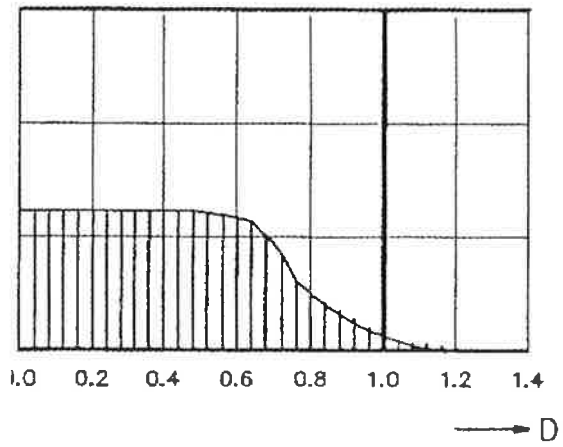

Figure 4: Posterior pdf for correlation parameter D, based on 11 pseudo observations; sample distance $\Delta x = 1.5$

Therefore the pseudo measurements contain insufficient information to be decisive on the rejection of low D values. This trend is even more pronounced when sample distances are further enlarged. The general picture then exhibits a uniformly distributed D between zero and the sample distance and a vanishing probability density for D values exceeding the sample distance (Figure 4). This result is independent of the number of sample points, as can easily be understood.

Actually the calculations mentioned above have been carried out for a large number (100 for each combination of n and Δx) of independent generated fields. Table I gives an impression of the numerical results of these calculations. From this table we can draw the following conclusions:

(1) the resulting estimates E(D) for D are always close to the value 1.0, used in the generation, unless Δx is too large relative to D, as discussed previously;

(2) the calculated variances correspond very well to the observed differences {E(D) -1.0} in the various simulations. The simulations confirm the small standard deviations for D, as predicted by the Bayesian procedure.

Table I: Results of Bayesian inference on the correlation parameter D, based on pseudo measurements of a one dimensional Gaussian field with known; expected mean value and standard deviation are known.

n	Δx	Run with (out of 100)	E[D]	s(D)
11	0.2	Greatest mean	1.095	0.032
		Smallest mean	0.906	0.030
		Greatest standard deviation	1.062	0.052
		Smallest standard deviation	0.997	0.018
11	0.5	Greatest mean	1.176	0.090
		Smallest mean	0.821	0.105
		Greatest standard deviation	1.020	0.138
		Smallest standard deviation	1.020	0.070
11	1.0	Greatest mean	1.335	0.183
		Smallest mean	0.587	0.229
		Greatest standard deviation	0.994	0.343
		Smallest standard deviation	1.335	0.183
Characteristics of generated field: $\mu(w)=0.0$, $\sigma(w)=1.0$, $D=1.0$, correlation function type eq. (1) n is number of (pseudo) sample points, Δx is sample distance Number of simulations for each combination (n,Δx): 100				

A similar exercise has been performed for a two dimensional field, having one unknown correlation parameter D. Also in this case the results were very satisfactory, as long as the sample distance was smaller than the scale of fluctuation (table II).

Table II: Results of Bayesian inference on the correlation parameter D, based on pseudo measurements of an isotropic two dimensional Gaussian field; expected mean value and standard deviation are known

n x n	Δx = Δy	Run	E[D]	s(D)
6 x 6	0.5	1	0.999	0.010
		2	0.997	0.017
6 x 6	1.0	1	0.735	0.228
		2	0.684	0.218
Characteristics of generated field: m(w) = 0.0, s(w) = 1.0, D = 1.0, correlation function type eq. (1), with Δx replaced by $\Delta r = \sqrt{(\Delta x^2+\Delta y^2)}$; N is number of sample point in x and y direction Δx and Δy are sample distances				

In practice the mean and standard deviation of the field are not known and must be estimated from the available measured data. Basically the extension of the procedure to unknown pdf parameters is straightforward. In table III results are given for unknown mean and unknown standard deviation based on pseudo measurements for a one-dimensional field. Comparison with previous results shows that the uncertainty in the estimate of D hardly is increased because of the additional unknowns. Also this result has been verified by doing 100 simulations. What did increase substantially, however, was the amount of computation time. This gives the idea that exorbitant computational burden may be expected when adding for instance some more unknown trend parameters. The reason for this is obvious: the covariance matrix Σ has to be inverted for all combinations of unknown parameters. One way out might be to develop some First Order Second Moment type of approximations. Another idea is to combine methods: use the Bayesian procedure for the correlation parameters and use different types of estimators for mean, standard deviation and trend. An efficient method of this type is presented in (Calle and Van Heteren [20]).

Table III: Results of Bayesian inference on expected mean value m, standard deviation s and correlation parameter D, based on pseudo measurements of a Gaussian field (i.e. expected mean value and standard deviation were supposed to be unknown)

n	Δx	$E[\mu]$	$\sigma(\mu)$	$E[\sigma]$	$\sigma(\sigma)$	E[D]	$\sigma(d)$
11	0.2	0.43	0.33	0.92	0.28	1.05	0.05
simulations		0.0	0.33	1.0	0.26	1.0	0.06
11	0.5	-0.62	0.35	0.73	0.23	0.99	0.09
21	0.5	0.07	0.27	0.74	0.16	0.86	0.08

Characteristics of generated field: see table I
n is number of sample points and Δx is sample distance
simulations: results based on 100 simulations for n=11 and Δx=0.2

It could be argued that the selected type of auto-correlation function itself is basically uncertain when considering real field measurements. However, it can be demonstrated that slightly different types will not lead to significant changes in performance of stochastic interpolation techniques (Calle and Van Heteren [15]), nor to essential changes in performance of probabilistic failure surface analysis (Vanmarcke [2]). Therefore it is assumed that the selection of the auto-correlation type is not as critical as might be expected at a first look.

Comparison with direct estimates

Most of the classical methods for the estimation of correlation functions or, equivalently, spectral parameters are based on the assumption that there is a continuous record of sufficient length and the availability of a large number of sample data. In practical geo-technical survey this limits the

application to those properties which can be measured almost continuously by geo-electric, seismic or radar devices.

A very simple method, for instance, is the estimation of the parameter D from the observed distances between up-crossings of the mean value of record:

$$D = \frac{\mu(\ell_o)\sqrt{2}}{\pi} \tag{6}$$

where $\mu(\lambda_o)$ is the average distance between up-crossings of the mean value. It can be shown that this is an unbiased estimator, however, the coefficient of variation equals $1/\sqrt{n}$, n being the number of up-crossings [16]. In order to obtain a coefficient of variation equal to 0.2 we need about 25 up-crossings and as a consequence a record length of 50 to 60 D. The method clearly does not work in the case of the 11 data points of figure 1.

A direct estimate of the correlation is possible by the estimator

$$r_k = \frac{1}{n-k}\sum_{i=1}^{n-k}\omega_i\ \omega_{i+k} \tag{7}$$

This statistic is an unbiased estimator for $\rho(k\Delta x)$, from which D can be calculated. The confidence limits for such an estimator are however very poor, as can be concluded from figure 5. For example, a correlation coefficient estimate of 0.6, based on 50 sample values, may range between 0.35 and 0.75 with probability 0.90. A further disadvantage of this estimator is that it only can be used in case of an equidistant grid.

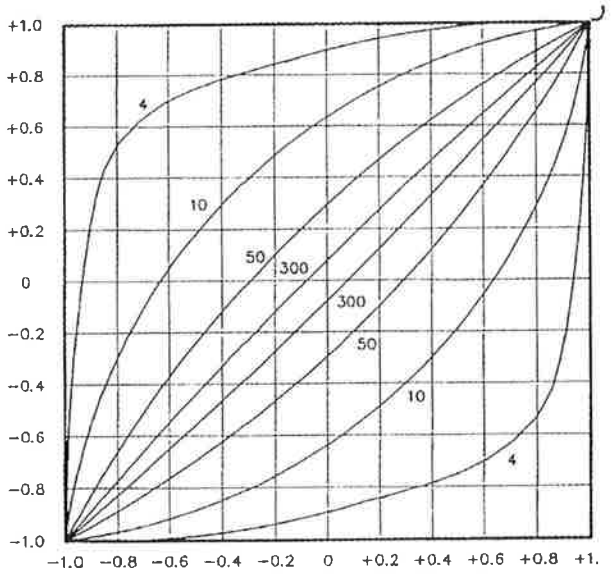

Figure 5: Confidence bounds (95 pct) of estimates of correlation coefficient (Dixon [18])

In [19] Tabba and Yong have presented a method which enables to estimate the parameters of a correlation function from a non-equidistant grid. We will reproduce here the original method, as well as the modification, which has proven to give better results. It should be noted that this method originally was intended for a polynomial type of correlation function type instead of the exponential type adopted in this paper. Define the function R(D):

$$R(D) = \sum_{j=1}^{n}(\omega_i - r(\Delta x_{ij}, D)\omega_j)^2 \qquad (j \neq i) \tag{8}$$

for some specific choice of i. In this equation r( ) is some chosen correlation function type and $\Delta x_{ij} = |x_i - x_j|$. It is then possible to derive an unbiased estimate for D by requiring that R(D) is a minimum:

$$\frac{\partial R(D)}{\partial D} = 0 \tag{9}$$

The modification works quite similar, but is based on:

$$R(D) = \sum_{i=1}^{n} \sum_{j=1}^{n} (\omega_i - r(\Delta x_{ij}, D)\omega_j)^2 \tag{10}$$

There is no theoretical way by which we can establish effectiveness of this estimator. Therefore this has been investigated by applying the method to a large number of generated fields, in the same way as has been done to validate the Bayesian technique. Note that in the Bayesian case the theory provides the standard deviation from only one record.

Table IV: Estimation of correlation parameter D based on pseudo measurements of a one dimensional Gaussian field, using the original and modified procedure of Tabba and Yong. Mean and standard deviation were supposed to be known.

n	Δx	Procedure Tabba & Yong	k	E[D]	σ(D)
11	0.2	Original	73	6.04	2.74
11	0.2	Modified	98	1.36	0.76
11	0.5	Original	44	3.14	0.73
11	0.5	Modified	98	1.13	0.48
11	1.0	Original	40	1.63	0.32
11	1.0	Modified	89	0.95	0.35
21	0.2	Original	31	5.62	2.56
21	0.2	Modified	99	1.34	0.66
21	0.5	Original	21	2.72	0.63
21	0.5	Modified	57	1.27	0.74
21	1.0	Original	25	1.59	0.27
21	1.0	Modified	30	1.11	0.63

Characteristics of the generated field: see table I
Each case is based on 100 simulations; k is number of simulations where convergence is achieved
Estimations of expected mean E[D] and standard deviation σ(D) are based on the k successful simulations

The results are presented in the tables IV and V for the one and two-dimensional fields respectively. The results clearly are disappointing, especially for the one-dimensional field. In the case where D had to be determined from 11 points with $\Delta x = 0.2$, the coefficient of variation equals about 50% for the modified method, which should be compared with 4% for the Bayesian analysis. In many cases the method did not even converge (see column 4). Knowing, however the results of the Bayesian analysis we have reason to believe that the Δx is too large and, consequently, the record not informative. On the other hand it is interesting to observe that, if the method manages to converge for large Δx, the results are better than for small Δx. This is surprising if we compare it with the trend in the Bayesian analysis.

Table V: Results of the modified Tabba and Yong procedure for the estimation of the correlation parameter D based pseudo measurements of an isotropic 2D Gaussian field; mean and standard deviation are supposed to be known

n x n	$\Delta x = \Delta y$	E[D]	s(D)
5 x 5	1.0	1.05	0.32
5 x 5	0.5	1.20	0.50
7 x 7	1.0	1.00	0.21
7 x 7	0.5	1.09	0.29

Characteristics of generated field: see table II
n is number of sample points in x and y direction,
Δx and Δy are the sample distances
Number of simulations for each case: 50

From these analyses we conclude that it is very difficult to find a simple and efficient estimator for D. It is absolutely necessary to use all the information which is contained in the data. The Bayesian method clearly does better, its only disadvantage being the large amount of computational effort.

Application in a practical case

In order to obtain some experience with the Bayesian method and to get some first indications of correlation distances in real soil, a practical case has been analyzed. Use was made of available cone penetration tests on a site in Leidschendam in the Netherlands, originally conducted to investigate the effects of different CTP-devices on test results. It should be kept in mind that the site was chosen because of an expected low variability.

The cone penetration tests have been performed on 18 locations at the test site, with two tests at every location with distance of 2 to 5 m. The distances between the locations varied from 40 to 250 m (see Figure 6). The variable of interest was the average penetration resistance over a depth of

2.8 m in the deep sand layer, approximately 8 meters below terrain level. The measured values and the corresponding x and y coordinates are presented in Table VI for the mechanical cone and for the electrical cone.

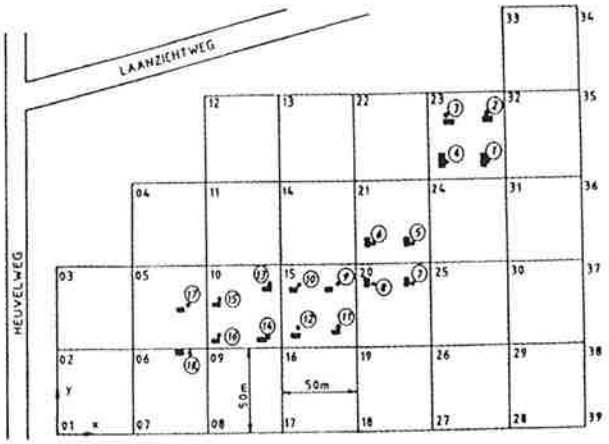

Figure 6: Map of Leidschendam site showing the test locations

The correlation function for this analysis was assumed to be of the exponential type:

$$\rho(\Delta x) = a \exp\left(-\left(\frac{\Delta r}{D}\right)^2\right) \tag{11}$$

where r is the distance between measurement locations. The factor a has been added to account for possible direct fall of the correlation among penetration resistances for $\Delta x > 0$, due to a lack of reproducibility of the penetration test (random measurement errors and very small-scale fluctuations).

Table VI: Observed average CPT-values at Leidschendam site

	Mechanical CPT			Electrical CPT		
NR of test	Location x-coordinate [m]	Location y-coordinate [m]	CPT value [0.1 MPa]	Location x-coordinate [m]	Location y-coordinate [m]	CPT value [0.1 MPa]
1	285.25	158.80	105	285.25	163.60	112
2	286.75	158.80	102	289.75	158.80	123
3	283.90	188.25	74	286.30	189.75	109
4	283.90	186.75	60	291.10	188.25	105
5	258.90	160.75	99	263.70	160.75	131
6	266.10	159.25	85	258.90	157.75	125
7	261.75	191.10	79	264.75	191.10	117
8	260.25	188.70	68	263.25	183.90	101
9	238.25	116.10	89	239.75	111.30	107
10	238.25	111.30	82	238.25	108.90	110
11	210.25	116.10	91	213.25	111.30	131
12	214.75	108.90	105	213.25	108.90	125
13	238.25	91.10	86	210,25	86.30	122
14	235.25	83.90	61	214.75	86.30	123
15	213.25	88.70	98	186.30	86.75	91
16	211.25	83.90	85	188.70	86.75	81
17	191.10	88.25	64	166.10	89.75	117
18	191.10	85.25	63	163.70	85.25	122
19	163.70	88.25	91	191.10	64.75	81
20	158.90	86.75	108	191.10	60.25	83
21	191.10	63.25	57	163.70	63.25	81
22	186.30	60.25	75	166.10	63.25	92
23	158.90	64.75	76	140.80	85.25	102
24	161.30	64.74	71	145.60	85.25	109
25	143.20	88.25	89	140.80	55.25	113
26	143.20	86.75	88	143.20	53.75	108
27	138.40	56.75	81	104.40	87.75	67
28	140.80	56.75	65	106.80	87.75	86
29	104.40	92.25	73	104.40	60.25	137
30	109.20	92.25	82	104.40	58.75	119
31	104.40	57.25	98	88.70	75.75	59
32	106.80	55.25	105	91.10	75.75	69
33	83.90	77.25	60	88.70	47.25	87
34	91.10	72.25	39	83.90	45.75	74
35	83.90	50.25	70			
36	88.7	50.25	95			
Average value:			81.1			103.5
Standard deviation:			16.5			20.7

The results have been summarized in table VII. In the case of the mechanical cone, the expectation of the parameter a equals 0.73, which implies that 27 percent of the variance of measured values is due to irreproducibility of the CPT-tests. Similarly, for the electrical cone the percentage of variance due to test irreproducibility is 14, which is a significant better result.

For the correlation distance D values of 12.6 m (mechanical cone) and 19.0 m (electric cone) were found. The standard deviations were 5.0 m and 4.3 m respectively. From the likelihood scores a negative correlation between a and D could be derived, implying that over estimation of reproducibility combines with underestimation of scale of fluctuation and vice versa. The point is probably that the distances of 2 to 5 meters at the test locations do not allow distinguishing between these possibilities, while the 50 meters and up between the test locations is too large to be informative. For the moment we only may conclude that a cone penetration test only gives location specific information for a radius of about 10 m. Beyond that limit the information is of statistical character.

Table VII: Results of Bayesian inference on the correlation parameters a and D for the average CPT values of the Leidschendam site.

Cone type	E[a]	σ(a)	E[D] [m]	σ(D) [m]	Correlation ρ(a,D)
Mechanical	0.73	0.15	12.6	5.0	-0.44
Electrical	0.86	0.07	19.0	4.3	0.04

Summary and conclusions

In order to describe the fluctuation patterns of soil properties, use can be made of random field models. A common assumption is to model the pattern of spatial variability as the superposition of some deterministic trend function and a homogeneous Gaussian process. The problem that bas been discussed in this paper is how to find the parameters of the auto-correlation function of the Gaussian process, i.e. the scale of fluctuation. It has been demonstrated that classical ways of direct estimation are not very effective. A Bayesian procedure proved to work very well. In the first place because of the small variance of the posterior distribution and secondly because of the built in warning system: if the data do not contain sufficient information to determine the correlation parameters, the method gives a clear indication. The method worked well for a number of artificially generated fields as well as in a practical example

A disadvantage of the method is its computational burden, which makes it unrealistic to extend the method to the general case of a large number of trend and correlation parameters for the time being.

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