Using saddle points for challenging optical design tasks

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ABSTRACT

The present research is part of an effort to develop tools that make the lens design process more systematic. In typical optical design tasks, the presence of many local minima in the optical merit function landscape makes design non-trivial. With the method of Saddle Point Construction (SPC) which was developed recently ([F. Bociort and M. van Turnhout, Opt. Engineering 48, 063001 (2009)]) new local minima are obtained efficiently from known ones by adding and removing lenses in a systematic way. To illustrate how SPC and special properties of the lens design landscape can be used, we will present the step-by-step design of a wide-angle pinhole lens and the automatic design of a 9-lens system which, after further development with traditional techniques, is capable of good performance. We also give an example that shows how to visualize the saddle point that can be constructed at any surface of any design of an imaging system that is a local minimum.

Keywords: saddle points, wide-angle lens

1. INTRODUCTION

In typical optical design tasks, the presence of many local minima in the optical merit function landscape makes design challenging. In traditional design, the result of local optimization critically depends on the choice of the starting system. Unfortunately, when the complexity of the design task increases, the designer gradually loses the ability to find a good starting system by an effort of intellect, and relies increasingly on trial and error. Methods such as genetic algorithms[1], simulated annealing[2], the Escape Function Method[3] and CODEV's Global Synthesis[4] have alleviated the difficulty, but the choice of an initial configuration that leads after optimization to a satisfactory solution still remains a significant challenge, especially when the number of design parameters is large.

Escaping from an unsatisfactory local minimum and exploring neighboring ones is non-trivial. Consider for simplicity the simple strategy of applying some random perturbation at the local minimum, in the hope that after reoptimization the result will be different. For a better understanding of the nature of the difficulty, we can think in terms of basins of attraction (see Fig. 1). The set of all starting configurations that lead to the same minimum after local optimization is called the basin of attraction of that minimum. The design space is then divided into basins of attraction corresponding to the different local minima present in the optimization problem. If the perturbation is too weak, the perturbed system will not reach the basin border and after local optimization will return to the known minimum; if the perturbation is too strong, it may jump over several basins, with the risk of missing good solutions. However, a neighboring minimum can be easily found if a point on the basin border can be identified, simply by "crossing" the border there. Saddle points (i.e. points for which the gradient of the merit function is zero, other than minima and maxima) are such points on basin borders.

In this paper we consider only saddle points (SPs) that are a maximum in only one direction and are therefore very similar to the well-known two-dimensional SPs. Finding new minima by detecting such SPs without a-priori knowledge

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about their location[5,6,7] can offer valuable insight in the topography of the design space, but may be too time-consuming for direct application for practical purposes. However, with a method that is very different from all other methods encountered in engineering or in mathematical global optimization called Saddle Point Construction (SPC)[8,9], SPs can be obtained in a computationally effective way by inserting a lens or by splitting an existing one in optical systems with one lens less that are local minima. In the next section, SPC will be discussed in more detail.
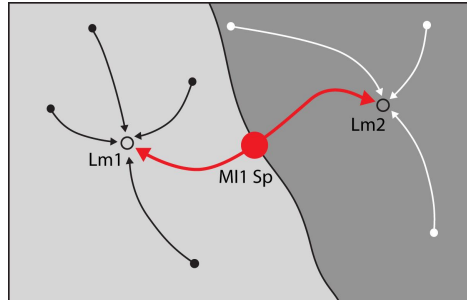

Figure 1. Basins of attraction and a SP on the basin border (the large dot). SPs considered in this paper are a maximum in one direction and are minima in all other directions in the variable space (i.e. they have a so-called Morse index(MI) of 1).

We can divide the local minima into stable and the unstable minima. The unstable ones typically have high values of the merit function and tend to disappear as minima when control parameters (e.g. aperture, field, system data that are not variables) are changed. These minima typically have a small basin of attraction (which reduces to a single point when the control parameters are such that the given minimum is just appearing in or disappearing from the landscape). A small perturbation is usually sufficient to kick the system out of them into a better minimum. Since they are less dangerous for optimization we will ignore them in what follows. The main danger for trapping the optimization comes from the stable local minima, which exist for larger ranges of the control parameters, which have larger basins of attraction and have in general lower values of the merit function than the unstable ones (although the merit function values can be significantly higher than that of the global minimum).

The present research is part of an effort to develop tools that make the lens design process more systematic by using SPC. In general, SPC cannot produce all SPs that exist in the landscape. The question is therefore whether SPC can produce the necessary SPs for obtaining the good design solutions. Previous work shows that a certain degree of order exists in the design landscape and that this order is related to SPs that can be obtained with SPC. In a somewhat idealized lens design landscape[10] we have found a perfectly ordered structure that makes it possible to obtain any system shape in the landscape from any other system shape by successively using SPC (and its inverse), and also that there exists a SPC design route to any system shape starting from one singlet. (Local minima have the same system shape when the merit function values and the system drawings are similar, and the merit function barriers between them are low.)

The goal of this paper is twofold. First, for a problem that is realistic (a wide-angle pinhole lens landscape) but where the number of minima is sufficiently small so that the landscape can be studied in detail, we show that the same two desirable properties of the landscape and SPC exist if we consider only the stable minima. In Sec. 3, SPC will be used to obtain the best solution for the wide-angle pinhole design by rapidly switching between different stable local minima in the design landscape. In Section 4, SPC will be used in combination with traditional methods to design the same wide-angle pinhole lens by starting essentially from nothing. Finally, to show the potential of SPC for designing more complex lenses, in Sec. 5 an example of a 9-lens optical system generated automatically with SPC from a 4-lens system will be given.

2. SADDLE-POINT CONSTRUCTION

When a lens is inserted in an optical system in the traditional way, one system results after local optimization. If however a lens is inserted (or split) with SPC and a SP results, two local minima can be obtained. This is because SPs are points on the basin border, and minimum can be found on each side of the "saddle" (Fig. 2). SPC in reverse can be used to extract lenses from an existing system. SPC has two versions: a special version (SPCS)[8] and a general version (SPCG)[9].

In what follows we will briefly summarize SPCS, that will be used in the next two sections. For more details on SPCS see Ref.8.

We start with any usual merit function (e.g. one used in commercial lens design software) and a local minimum of this merit function. The first lens in Fig. 3 is part of the starting minimum. For spherical surfaces, we have proved mathematically that when the new lens (the second meniscus-shaped lens in Fig. 3) has the same glass as the first one and we have three consecutive surfaces with equal curvatures, and two zero distances between them, then the resulting system is a SP. This property can be used to "construct" SPs from known local minima (SPCS).

When we choose and optimize two points close to the SP but on opposite sides of the SP, the optimizations 'roll down' from the SP and arrive at two distinct local minima. The conditions of glass equality and zero axial distances are not severe limitations as it may seem, because in the resulting minima the distances between the surfaces and the glass of the new lens can be easily changed as desired. SPC can be used in reverse order to remove lenses in a systematic way. By removing and inserting lenses with SPC, one can start from minimum I in Fig. 2, "climb to the mountain pass" (i.e. the SP), and find a different, possibly better minimum (minimum II) on the other side. A modified version of SPCS was recently implemented in the commercial optical design software SYNOPSYS under the name SPBUILD[11].
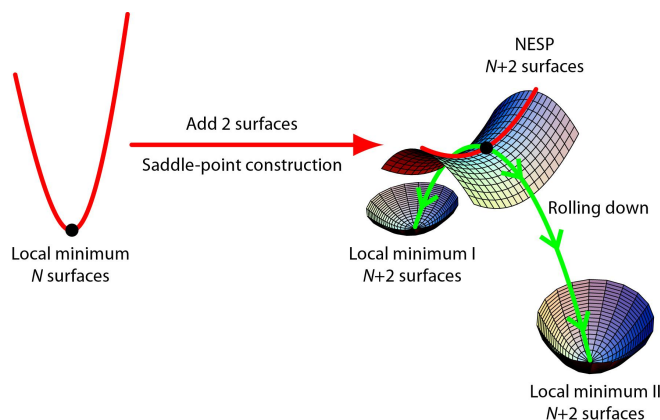

Figure 2. Although we typically have many variables, SPs obtained with SPC resemble 2D horse saddles, because they are maxima in one direction. Since the new lens has zero thickness and equal curvatures and therefore disappears physically ("null element"), the SP is called a "null element" SP (NESP).
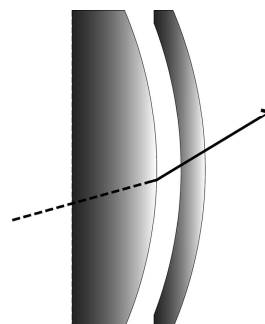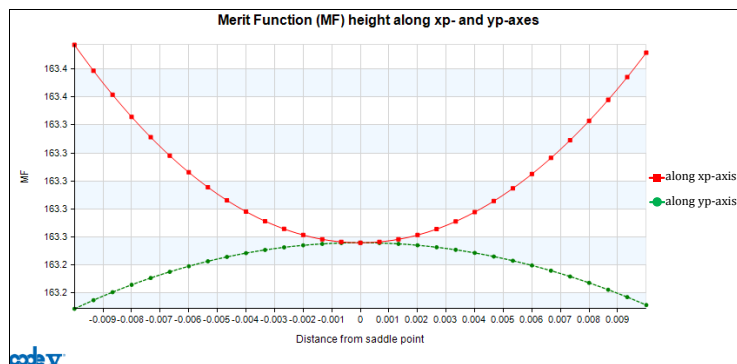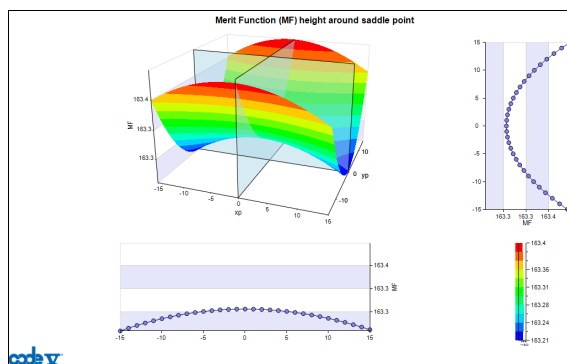

Figure 3. In SPCS the two "new" curvatures are identical with the neighboring "old" one. For clarity, the zero air and glass thicknesses are drawn as small widths. In reality, we have three overlapping surfaces.

Figure 4. 2D plot(a) and the equivalent 3D plot (b) showing that a system constructed with SPCS is indeed a saddle point. This plot has been obtained by applying SPCS in a doublet discussed in Sec. 4 but with the same macro we have obtained very similar plots even for highly complex systems such as lithographic objectives.

The mathematical proof of SPCS is given in Ref. 8, but we have written a CODEV macro that can be used to easily verify in each particular case that the system constructed with SPCS is indeed a SP. It is possible to find analytically two directions in the variable space (they are related to the newly added curvatures, see Eqs. 9-11 in Ref. 8) such that, when

the merit function is plotted along them, the system constructed with SPC (from any local minimum at any surface) is a maximum along one of the two directions and is a minimum along the other one (Fig. 4a). A 3D plot of the merit function shows indeed the familiar shape of a saddle surface (Fig.4b). There are other directions in the variable space in addition to the two directions used here, but because we have started from a local minimum the SP will be a minimum along all these other directions as well. The macro will be made available on our website[12].

3. SWITCHING BETWEEN LOCAL MINIMA

In this section we consider a design landscape that is simple enough to be studied in detail and show how the global minimum can be rapidly obtained with SPC, if local optimization is trapped in any of the other stable local minima in the landscape.
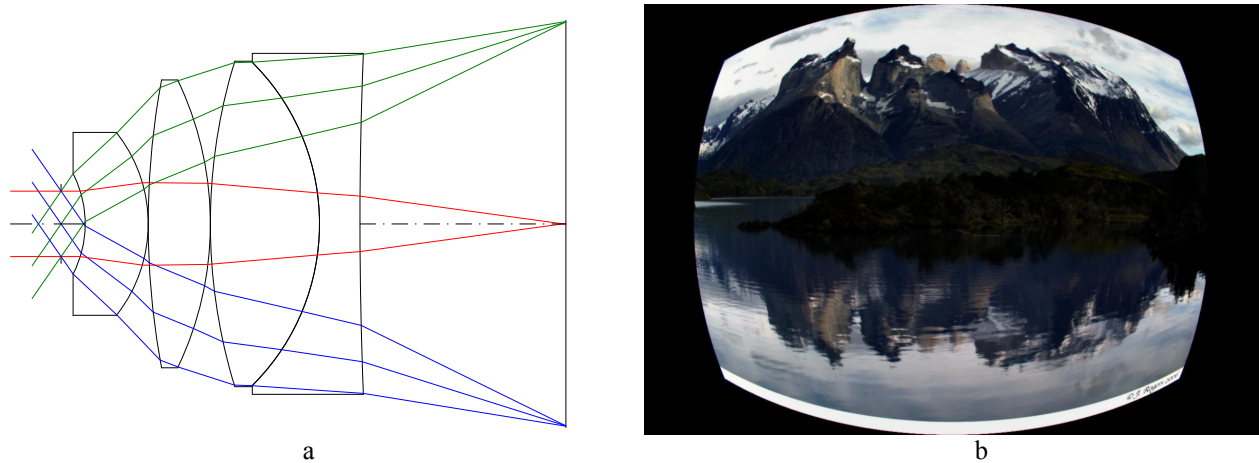

Figure 5. (a) wide-angle pinhole lens for security purposes, with an effective focal length of 3mm, (b) 2D image simulation of its imaging quality.
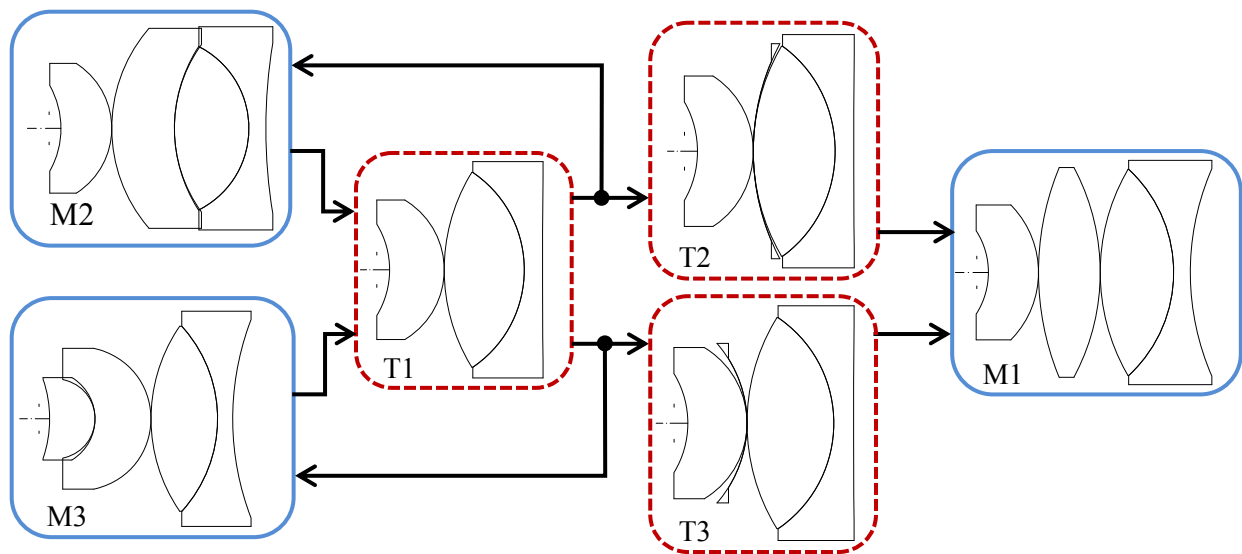

Figure 6. Obtaining the global minimum M_1 with SPCS from the two poorer local minima M_2 and M_3, if optimization is trapped in them.

A wide-angle pinhole lens for security purposes with a full field of 110 degrees has been designed earlier[13] by one of us (IL). Because it is used in covert observation cameras (i.e. the lens should not be visible), the aperture stop is situated in front of the lens and has a very small diameter of only 0.8 mm. A very similar design with the same aperture and field is

shown in Fig. 5a. Only spherical surfaces are used. In Fig. 5b, a 2D image simulation made with CODEV shows that the system has a imaging quality that is adequate for its purpose.

We analyze the design landscape of this problem by using the default error (i.e. merit) function of CODEV, for fixed values of the thicknesses, but with variable curvatures. Minor edge thickness violations (that can be easily corrected in a later stage) are acceptable in this analysis. Global Synthesis of CODEV[4] produces three stable solutions, M_1, M_2 and M_3 (see Fig.6). M_1 is the global minimum, and when thicknesses are optimized as well (a non-critical thickness is not optimized, but chosen to be reasonably small) it leads to the system in Fig. 5a. M_2 has an error function about twice that of M_1, M_3 about three times larger. Our own software NETMIN[7] gives, in addition to these three solutions, a few other unstable solutions, with large error functions, which will be ignored in what follows. When the focal length is different, the same three stable solutions exist, but the unstable ones are replaced by other unstable minima.

If optimization is trapped in the poorer minima M_2 or M_3, the global minimum M_1 can be easily obtained from them with SPCS. We consider first M_3. First we remove from M_3 the 2nd lens, and after reoptimization we obtain the system T_1. Candidates for removal are often weaker-power lenses or airspaces that seem to have no function. Then we perform SPCS by inserting a zero-thickness meniscus at the position where the previous lens was extracted. We take the 2nd surface as reference, and make the new meniscus curvatures equal to that of the 2nd surface. The meniscus glass is the same glass as for the previous lens. Since we have two zero thicknesses (for the meniscus and for the preceding airspace), the lens drawing of the resulting SP shows three overlapping surfaces and therefore looks the same as for T_1. We then perturb the SP on one side by making the two meniscus curvatures slightly weaker than the curvature of the 2nd surface (system T_3). Optimizing T_3 (only with curvatures as variables) leads to a local minimum where the new lens still has zero thickness. However, this minimum can be easily reoptimized with the glass type before removal and desired thickness for the new lens, and the result turns then out to be the global minimum M_1. If we perturb the SP on the other side and reoptimize (first only with curvatures as variables, then with thickness as well and the desired glass type) we obtain the starting system M_3. Since one side of the SP obtained at the 2nd surface of T_1 leads to M_3, the removal of the 2nd lens that leads from M_3 to T_1 is in fact SPCS in reverse.

If in T_1 after inserting the zero-thickness meniscus we take as reference the surface following the meniscus, from this second SP we obtain on one side M_1 (T_2 is the perturbed saddle point on that side), and on the other side M_2. If on the other hand we start from M_2, the removal of the 2nd lens in M_2 leads again to the same T_1, and then we can obtain the global minimum M_1 as shown above. Note in Fig. 6 the perfect symmetry of the routes from M_2 and M_3 to the other two minima.

This way of switching between different local minima is typical for SPC. Note that, once the position in the system for performing SPC is chosen, there is no randomness involved any more in switching to a new minimum.

4. COMBINING SPC WITH TRADITIONAL TECHNIQUES

The design in Fig. 5a can also be obtained in a different way: by starting from nothing, by addressing first the monochromatic aberrations with SPC, and then by controlling the chromatic aberrations with traditional techniques. Monochromatic aberrations are more nonlinear than the chromatic ones. Therefore, although both types can affect the performance significantly, the monochromatic ones are the main cause for the presence of multiple local minima.

To avoid ray failure (the risk for it is high because of the large field) we start first with a longer focal length (3.5 mm) and adjust it to the required value (3mm) in a later stage.

We first use only one glass type and obtain a doublet T_1' (see Fig.7) which is similar with the system T_1 in Fig. 6 (except that, as noted, it has only one glass type). We choose for both lenses the same thickness (fixed at this stage) and use the curvatures as variables. For the values of aperture and field mentioned above, Global Synthesis and NETMIN runs show that only one doublet minimum exists in the landscape. Therefore T_1' can be obtained from any ray-traceable starting point (e.g. from plane-parallel plates with the focal length requirement). Then, in the same way as in the previous section, by inserting a meniscus after the first lens we obtain with SPC two SPs, one SP where the meniscus curvatures are the same as the one of the preceding surface, the other SP where the meniscus curvatures are the same as the one of

the following surface. As for the systems T_2 and T_3, we obtain the systems T_2' and T_3' by perturbing these two saddle points on one side (we choose weaker curvatures for the menisci). On that side, both SPs lead to the same minimum. After we increase the thickness of the new zero-thickness lens and reoptimize we obtain the lens M_1' in which all three lenses have the same glass and axial thickness.

At this stage, we continue the design with traditional methods and address the chromatic aberrations. We replace the 3rd lens in M_1' with a cemented doublet, and also change the glass of the 2nd lens. We then reoptimize the system by varying thicknesses as well and by changing the focal length to 3mm and obtain the system M_1''. It turns out that in this system the last glass thickness can be reduced to a smaller value without significant loss in performance. After reoptimization it turns out that other glass thicknesses decrease as well, and the final result is the design shown in Fig. 5a.

If for one of the two SPs mentioned in this section we choose to go down on the other side, and then repeat in the traditional design part the same steps as above, we obtain a solution that closely resembles the system M_3 in Fig. 6.
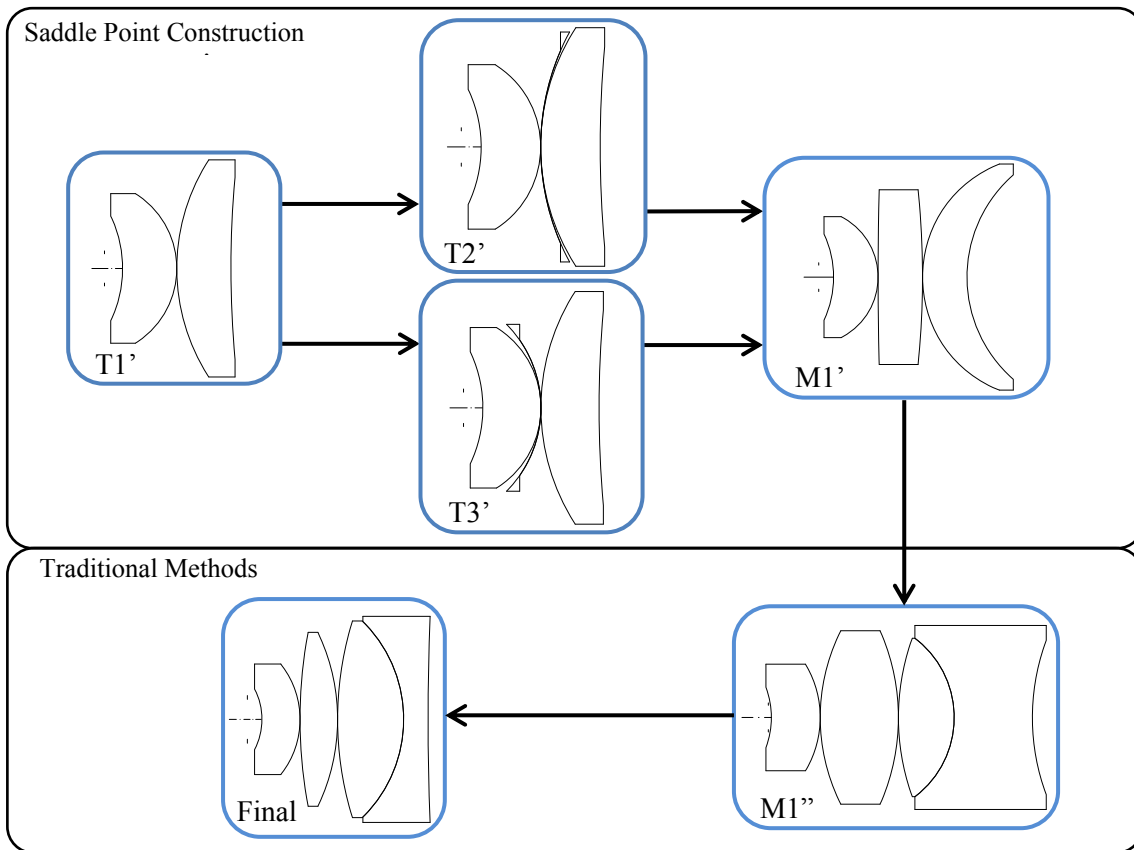

Figure 7. Obtaining the system shown in Fig. 5a by combining SPC with traditional methods.

5. AUTOMATING SPC

In order to test whether good designs can be obtained with SPC (in its general version SPCG[9]) for more complex systems, one of us (PvG) has written the computer program SCADO (Saddle-point Construction Algorithm for Designing Optics) that applies SPC in a systematic way, and uses CODE V for the subsequent local optimization (the communication is via the COM interface). Despite of the fact that SPCG has not yet reached full maturity, when used in SCADO to insert (or extract) lenses in a structured way, SPCG is already capable to produce interesting results. In SCADO (and also with the SPBUILD of SYNOPSYS[11]) SPC can be performed at different places and the results are then evaluated and the best of them are used for further design.
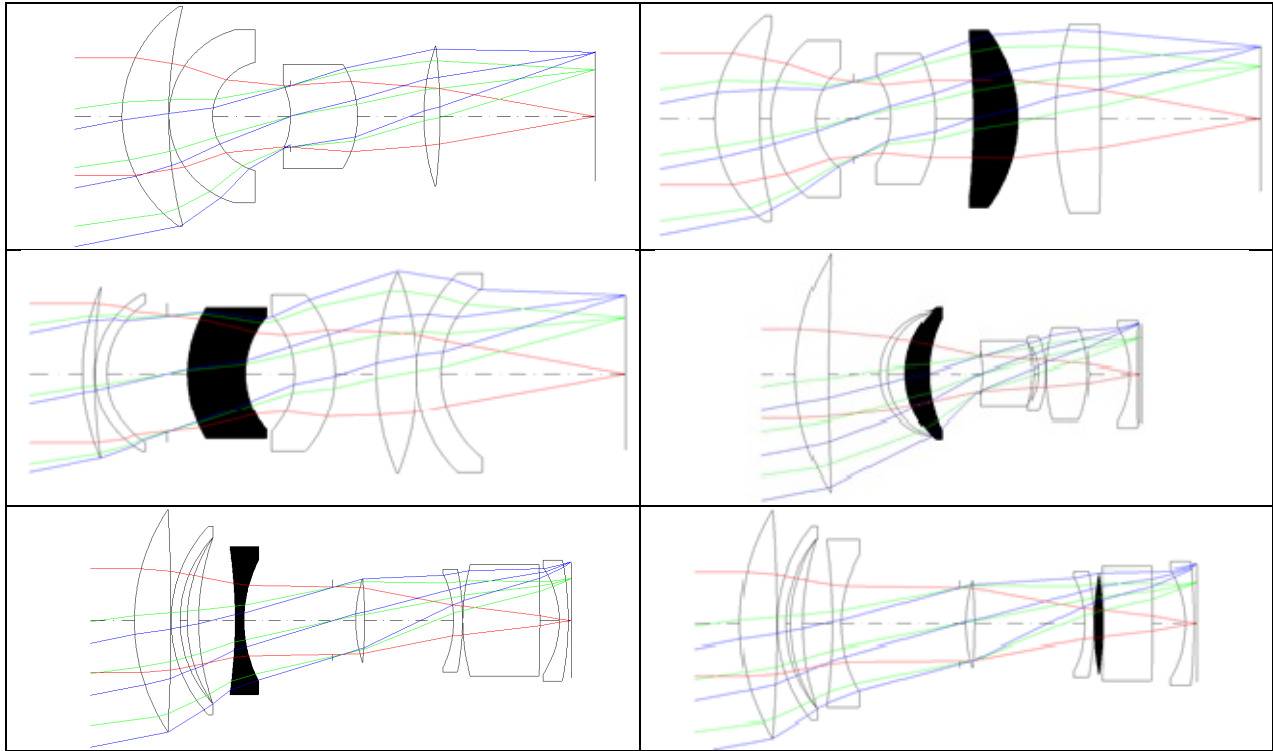
Figure 8. The route to the best 9-lens system (bottom right) obtained from a 4-lens system (top left) by SCADO, using SPCG. At each stage, the black lens is the new lens that resulted from performing SPCG in the previous stage at that position. For generating the systems with 5,6 and 7 lenses, the new lens resulted from a glass meniscus in air. In the 8 and 9-lens systems, the new lens resulted from an air meniscus in glass (i.e. a lens was split).
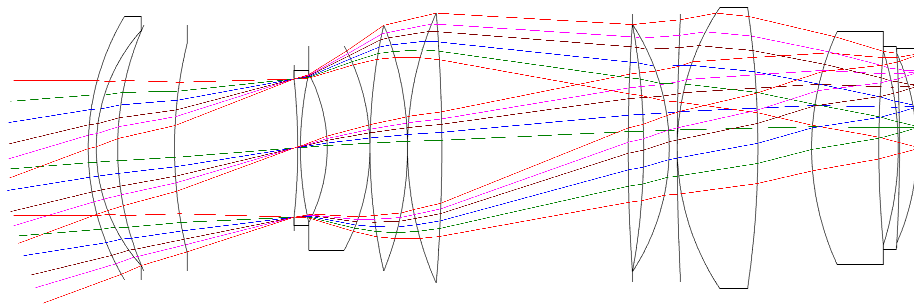

Figure 9. 12-lens system obtained with traditional techniques (by hand) from the 9-lens system (bottom right in Fig. 8). The system has an effective focal length of 100mm, an F/number of 2 and a half field of 20 deg. The RMS wavefront error is lower than 0.07 waves for all field points.

As an example, Figures 8 and 9 show the design route of SCADO and the final design of a 12-lens system that was designed to meet the requirements of the 2014 IODC Lens Design Competition, respectively. With SPC, lenses are only added in this example (not removed). The input for SCADO is a 4-lens system (top left in Fig.8). At each stage, lenses are inserted with SPC at different positions in the system and the aperture and field are increased according to a predefined rule. Then, from the stable systems which have survived (or from the best 20 of them if their number is too large) the next generation is obtained by inserting a new lens with SPC.

Figure 8 shows the evolution from a 4-lens system to a 9-lens system. Obtaining the 9-lens system automatically by SCADO took about 4 hours on a 4-core computer. From this system, the 12-lens system having good performance shown

in Fig. 9 has been obtained with traditional methods. Other interesting 9-lens systems, such as e.g. the one shown in Fig. 10, have also been automatically obtained with SCADO in the same run.
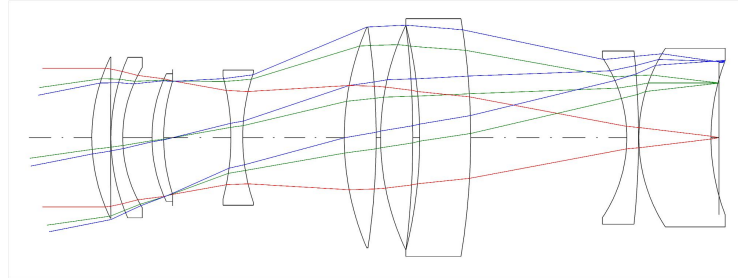

Figure 10. Another 9-lens system obtained automatically from the 4-lens system shown in Fig.8. After further design by hand, it also leads to a design that has a very good imaging quality.

CONCLUSION

The present research is part of an effort to develop tools that make the lens design process more systematic. Earlier work shows ideal cases where relationships between saddle points and minima with one lens less can be used to find the best design solutions. The analyses in Sec.3 and 4 show that the same is true for the realistic design landscape considered there, that of a wide-angle pinhole lens design where the number of minima is small enough so that the landscape can be studied in detail. By inserting and extracting lenses with SPC, or by combining SPC with traditional design the best local minimum in the landscape can be obtained. Together with design examples of more complex systems such as that shown in Sec.5, this increases our confidence that SPC is a useful new tool that, if used together with traditional design tools, can increase the productivity of the design process.

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