Problem-Matched Basis Functions for Microstrip Coupled Slot Arrays based on Transmission Line Green's Functions (TLGF)

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***Abstract*— A method is proposed for the analysis of arrays of linear printed antennas. After the formulation of pertinent set of integral equations, the appropriate equivalent currents of the Method of Moments are represented in terms of two sets of entire domain basis functions. These functions synthesize on one side the resonant behavior of the slot, microstrips or dipole and on the other side the field in proximity of the feeding source and of the discontinuities. In order to define these basis functions, canonical geometries are identified, whose Green's functions have been found in semi-analytical form. The accuracy and the effectiveness of the method in terms of convergence rate and number of unknowns is demonstrated by comparison with a standard fine meshing full-wave analysis. The method is extremely convenient for large arrays where the details of the feed are very small in terms of the wavelength. Since the proposed solution is independent on the dimensions of these details, it provides a dramatic reduction of the number of unknowns with respect to a standard fine mesh analysis.**

I. Introduction

Array antenna modelling is a challenging issue, since it involves large structures (in terms of the wavelength), as well as fine details that require discretizations much-smaller than wavelength, and that dominate the frequency response of input parameters. The Integral Equation (IE) approach is largely used to analyze these problems, through the Method of Moment (IE-MoM) discretization scheme. It is well known, however, that standard techniques are severely limited by the matrix size and condition number involved in the problems of interest.

A number of techniques have been presented in the past years to overcome the difficulties mentioned above. Let us just mention some of the methods which lead to a compression of the MoM size based on either physical or numerical schemes. Among the numerically based methods, we point out the Fast Multiple Method [1], the multilevel matrix decomposition algorithm [2] and the multi resolution method [3]. Physical based approaches which exhibit some common features are the Truncated Floquet Wave (TFW) method [4]–[6], the Synthetic basis Function expansion (SFX) method [7]–[9] and the Characteristic Basis Functions (CBF) method [10], [11]. These methods attempt to keep explicit information about the multi-scale nature of the solution directly into the representation of the unknown currents. These approaches can be applicable to quite general array geometries, with (possibly) different radiators. In any case, even if with different approaches, the complexity is reduced by compressing a (large) matrix. Despite the fact that these have been shown to be very successful they may possess a margin of improvement for certain typology of array problems.

In this paper we present a method framed in the same philosophy but based on the quasi-analytical form of Transmission Line Green's Function (TLGF). Even if limited to array elements constituted by pieces of slot-line, microstrip-line, dipoles and their relevant coupling, the present method allows the analytical derivation of the basis functions, thus simplifying the preprocessing phase and gaining physical insight. It is the extension and the systematization to multilayered arrays and relevant microstrip beam forming networks (BFN) of the technique presented in [12], that was originally developed for CPW fed slot problems alone.

The entire domain basis functions are generated by solving in quasi analytical form some specific transmission line Green's function (TLGF) problems with an appropriate chosen excitation. Each basis function is thus solutions of an integral equation which represents the boundary condition of a physical problem; consequently, they have the adequate content of local reactive energy, and include a good approximation of the right edge behavior. Indeed, as demonstrated here, the present method allows the reduction of the number of unknowns while preserving accuracy. Moreover, it implies a constitutional improvement of the condition number, due to the small MoM matrix size and to the fact that strong variations of the field are intrinsically described without the need of very small domain basis functions. Physical insight is also gained, since each one of the basis function can be associated with a specific electromagnetic GF-problem which is essentially regulated by well distinct wave contributions. These contributions are of travelling wave type and fringe type; the latter is associated to reactive energy stored close the feed-point and, due to their large spectral contents, dominate the short range coupling. The travelling wave basis functions dominate the large distance coupling; this allows to establish a robust criterion to select the essential large distance contributions of the MoM matrix, thus drastically improving the filling time. The procedure proposed in this paper can also be used as a matrix-compression technique when the basis function are viewed as ruling the grouping of the subdomain unknowns; this leads to the possibility to apply the present method in the framework of a conventional subdomain algorithm by reusing already available codes. This aspect also accomunates this approach to the TFW method or the SFX method.

II. Printed Transmission Line Green's Functions

In this section the TLGF of a generic source in the presence of an infinite linear printed structure in TL multilayered stratifications is described. In order to derive the unknown current, the width of the TL (w_t) is considered small in terms of the wavelength so that the separation of variables can be invoked for the space dependence of the current. This procedure was introduced in [15] for the microstrip case. The transverse function is taken to verify the edge singular conditions and it possesses a closed-form Fourier transform. After solving the pertinent integral equation in quasi-analytical form, i.e. either the CMFIE (continuity of magnetic field integral equation) or the EFIE (electric field integral equation), the longitudinal dependence of the currents can be expressed as:

$$c_l(l) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{N(k_l)}{D(k_l)} e^{-jk_l l} dk_l \tag{1}$$

where $N(k_l)$ and $1/D(k_l)$ represent the Fourier Transform of the excitation field and the Green's function of the TL respectively. Since the higher spectral components are associated to reactive energy, it is significant to separate the higher and lower spectral components. While the reactive energy is stored in the surrounding of the discontinuity that the source represents, the lower spectral components are instead associated to dynamic propagation and radiation phenomena. As such they can be accounted responsible for mutual coupling between different elements of a structure under analysis. In particular the lower portion of the spectra in all cases that we encountered is mostly associated to spectral poles arising from the zeros of $D(k_l)$. Accordingly the overall unknown currents can be systematically expressed as summation of two contributions:

$$c_l(l) = c_{fr}(l) + c_{tw}(l) \tag{2}$$

In (2) the fringe current $c_{fr}(l)$ is the remaining portion of the total current once the dynamic contribution $c_{tw}(l)$ has been extracted. The fringe current component, in general, can only be evaluated numerically. The dynamic current, instead, can always be evaluated analytically once the poles oh the TLGF are found.

III. Entire Domain GF-based Basis Functions

Consider the geometry in Fig. 1. The microstrip is excited via a δ-gap generator over a zone of length δ and width w_μ. The slot is electromagnetically coupled by the microstrip and a scattered current is induced on the microstrip by the slot. The first step is to divide the structure in portions free of discontinuities. With reference to Fig. 1, the microstrip can be divided in three regions by the two sections localized at the δ-gap generator and at the slot cross over. The slot can instead be divided in two regions by the cross-over with the microstrip. In each discontinuity-free region, the bulk of the unknown currents can be represented via travelling wave interference, while fringe functions are needed close to the section boundary between two regions.

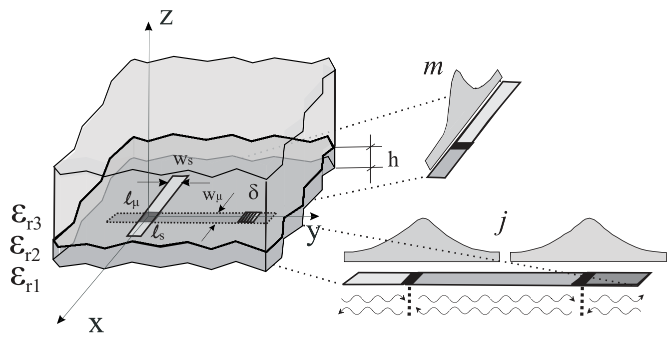

Fig. 1. Example of finite slot fed by a microstrip line. The discontinuity sections at the source and at the slot-microstrip crossing identify the boundary of the regions on which the unknown currents are described by travelling wave. Around the discontinuities additional fringe basis functions are defined, which incorporate the quasi-static field behavior.

The first type of basis functions we consider are the travelling waves. These functions may be parameterized by the propagation constant k_{lp} that coincides with the poles of the denominator $D(k_l)$. Both a forward and a backward travelling wave are defined on each block. In practice the travelling waves account for the actual linear dimensions of the structure investigated. It is interesting to consider that since their spatial expression is so simple their Fourier Transforms are known analytically and can be directly used in the evaluation of mutual couplings in the spectral domain.

A second type of basis functions is needed to represent the local field behavior around the boundary between uniform regions. They are shaped as the fringe current associated to each canonical problem. The present test case involves the following fringe basis functions

- J_{fr}: it is the fringe electric current on the microstrip for a magnetic source located on the ground plane.
- $\widetilde{J}_{fr}$: it is the fringe electric current on the microstrip associated to the δ-gap discontinuity.
- M_{fr}: it is the fringe magnetic current on the slot associated to the electric source located at the microstrip level.

The functions J_{fr} and $\widetilde{J}_{fr}$ result pure imaginary while M_{fr} presents also a real part.

IV. MoM Analysis

Once the basis functions to represent the problem are introduced, the reaction integrals are evaluated as in any conventional MoM procedure. To clarify the procedure we will now present some results. The results are compared with those obtained by a conventional small domain full-wave code developed in house and by Ansoft Designer.

A. Resonant Microstrip-Coupled Slot

The solution for the test case of Fig. 1 is presented in Fig. 2a and Fig.2b, relevant to the electric currents on the microstrip and the magnetic currents on the slot, respectively. The different blocks in which the structure is partitioned are indicated below the horizontal scale. The results from our method are successfully compared with those from a conventional subdomain method which make use of rectangular small domain basis functions.

It is clear that the advantage of the process in this case is not so important in terms of calculation time; the actual

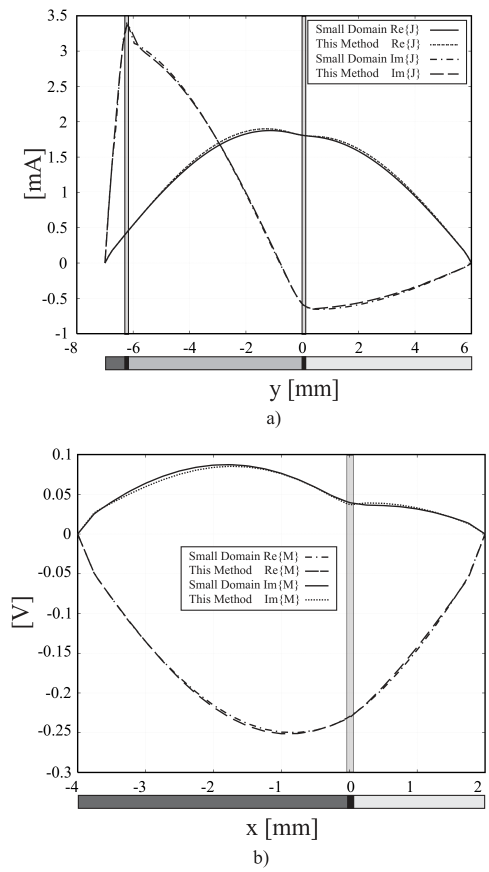

Fig. 2. Electric and magnetic currents for the geometry shown in Fig. 1 ($f = 9\ Ghz$, $w_\mu = 0.5\ mm$, $l_\mu = 6\ mm$, $w_s = 0.4\ mm$, $l_s = 2\ mm$, $h = 0.275\ mm$, $\epsilon_{r1} = 1$, $\epsilon_{r2} = 2.55$, $\epsilon_{r3} = 11.7$). a) Real and imaginary part of the microstrip electric current. b) Real and imaginary part of the slot magnetic current.

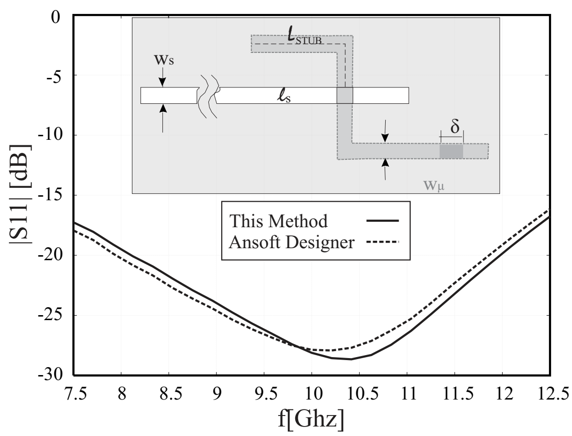

Fig. 3. S parameter for microstrip fed slot ($l_s = 90\ mm$, $w_s = 0.4\ mm$, $l_{stub} = 5.6\ mm$, $w_\mu = 0.7\ mm$, $\delta = 2.1\ mm$, $h = 0.275\ mm$, $\epsilon_{r1} = 1$, $\epsilon_{r2} = 2.55$, $\epsilon_{r3} = 11.7$).

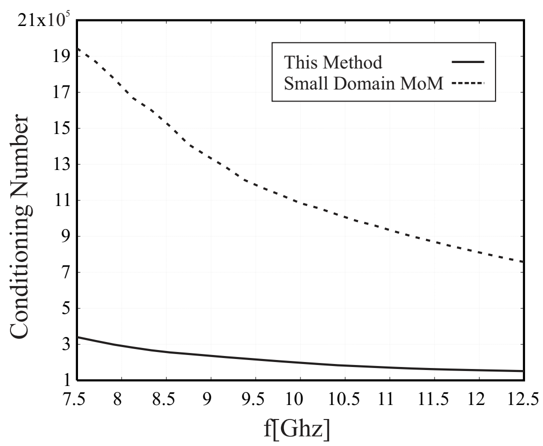

Fig. 4. Conditioning numbers for the structure in the inset of Fig. 3

importance of the method is in fact when dealing with array problem.

B. Non Resonant Microstrip-Coupled Leaky-Wave Slot

In this section we investigate a long slot coupled to a bent microstrip, this latter fed by a δ-gap (see Fig. 3). The structure realizes a leaky-wave slot antenna, whose basic physical properties have been investigated in [13], [14]. The slot is printed between a homogeneous half-space of relative permittivity 11.7, and a dielectric slab of relative permittivity 2.55 and thickness 0.275 mm. The amplitude of the reflection coefficient S_{11} at the input port obtained by our method is compared with that obtained by Ansoft Designer. (Note that the use of the travelling basis functions allows for a natural definition of the reflection coefficient).

Fig. 4 shows the conditioning numbers as a function of the frequency that are obtained for the structure investigated in Fig. 3, when using small domain basis functions and when using our entire domain ones. A drastic improvement is obtained, as expected. It should be noted that for long antennas (as this one) the gain in terms of reduction of unknown number is really significant. Only 6 entire domain basis functions are included in the slot against 150 small-domain functions of the conventional approach.

C. Slot Arrays

The use of travelling wave and fringe wave basis function is particulary beneficial in array problems.

Consider the array shown in Fig. 5, composed by microstrip-fed slots printed on a ground plane that separates a homogeneous silicon half space (ϵ_r=11.9) from a dielectric slab (ϵ_r=2.55, h=0.275mm) where the microstrips are printed. The slot's length and width are 6.6 mm and 0.3 mm respectively. The microstrip is bent for reason of reduced space, since the periodicity of the array in the E-plane, designed for imaging applications, should be small. The periodicities are 9 mm in the x-direction and 6 mm in the y-direction. The stub which excitates the slot is of length 7.25 mm and the microstrip's width is 0.5 mm. Lets first consider a subarray of 3x3 elements framed by dashed line in Fig. 5. Fig. 6 shows the active reflection coefficients on different elements of the 3x3 array. The elements of the upper and lower rows have almost the same reflection coefficient, which is around -12dB. In the central row, where the coupling in the y-direction is stronger, the minimum increases to -8dB. The amplitude of the active reflection coefficient for the corner slot ($S_{36,36}$) and for the central slot ($S_{16,16}$) (which is that with the zoom in Fig. 5) is presented.

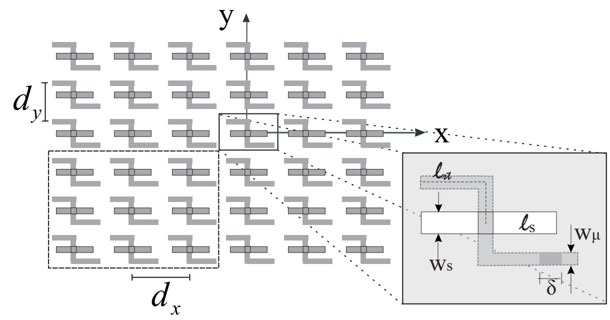

Fig. 5. 6x6 array of microstrip fed slots ($l_s = 6.6\ mm$, $w_s = 0.3\ mm$, $l_\mu = 7.25\ mm$, $w_\mu = 0.5\ mm$, $\delta = 1.5\ mm$, $d_x = 9\ mm$, $d_y = 6\ mm$, $h = 0.275\ mm$, $\epsilon_{r1} = 1$, $\epsilon_{r2} = 2.55$, $\epsilon_{r3} = 11.7$)

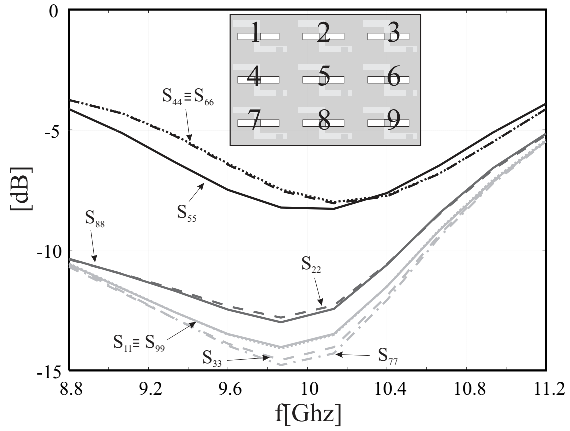

Fig. 6. Active reflection coefficients on the array elements.

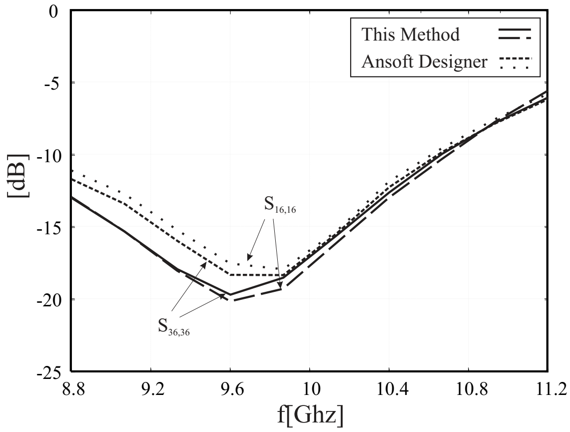

Fig. 7. Reflection coefficient of the 6x6 array (indicated in Fig. **??**) on the central element and on the down right corner element.

V. Conclusion

A method has been presents for the analysis of arrays whose beam forming network (BFN) and radiating elements are composed of pieces of transmission lines. The main advantageous feature of the basis functions used is their splitting into fringe wave and travelling wave components, which are associated to different properties of the coupling phenomena. A typical example is that of slot elements fed by microstrips. For this case, the method has been validated against element by element codes showing excellent accuracies and significant improvements for both calculation times and memory occupations requirements. The method is also well suited for analyzing integrated antennas in which the BFN cannot naturally be separated from the radiating elements, as occurs for coplanar-waveguide based feeding network [12]. The applications in which this happens more often are relevant to integrated receivers at millimeter and submillimeter wave frequencies where small dimension of δ-gap well represents realistic devices (mixer or direct detector).

The method is also particularly useful for the analysis of printed slot reflect-arrays, where each element is loaded by microstrip.

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