CORRELATOR TECHNOLOGY

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Summary

The paper reviews a number of designs of cross-correlation receivers for the detection of active under-water transmissions.

Particular attention is given to the various structures of phase insensitive receivers, and to problems concerned with clipping of the input signal and the reference function.

0 INTRODUCTION

The purpose of this paper is to review a number of designs of crosscorrelation receivers for the detection of active underwater acoustic transmissions, and to show their mutual similarities and differences.

We shall only occupy ourselves with matched receivers, i.e. receivers using a reference signal indentical to the transmitted one. In doing so, we neglect the fact that a matched system is optimum only in a white noise background, but not in a mixed reverberation-noise situation where the background is effectively a coloured noise. From a theoretical viewpoint, a properly mismatched receiver should be used in the latter case. However, this refinement is seldom made because of many inherent difficulties. Besides, the majority of subjects treated in this paper would apply equally well to a mismatched system.

When looking through the available literature, a virtually endless number of variations on the same theme seems to exist. These differences are caused by the solution which is adopted for the problems summed up below. Each of them can be solved in more than one way. Hence we have at our disposal a multitude of elementary building blocks, of which a certain number has to be put together to obtain a complete receiver.

We shall not occupy ourselves with electronic details, but rather limit ourselves to the relevant operations, performed on the signals, and to the order in which they are applied. Hence, the results will mainly be presented as block diagrams.

1. The time problem. - In order to obtain the output $C(t_o+T)$ of a correlater receiver, the input signal must be correlated with a reference signal over a time interval from $t = t_o$ to $t = t_o+T$, where T = length of reference signal. Because we are interested in C - values at closely spaced time points, the procedure has to be repeated from $t=t_o+\Delta t$ to $t=t_o+\Delta t+T$, etc. Since Δt will generally be small with respect to T, the successive correlation intervals will overlap to a high degree. Thus, special measures must be taken to cope with the threat of time shortage for performing the required operations.

2. <u>The phase problem</u>. - The receiver must be capable of processing signals which have an arbitrary carrierphase with respect to their modulating waveform. In other words, the receiver must be phase insensitive.

3. <u>Sampled or continuous processing</u>. - The receiver output can either be obtained as a continuous function of time, or as a series of samples. Further, the input signal and the reference function can be processed either continuously or in sampled form.

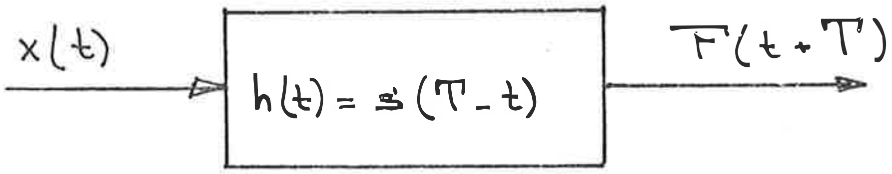
4. <u>Doppler channels</u>. - Dependent on the type of transmission and on the purpose of the equipment, the receiver has to be equipped with a number of doppler channels, or not.

5. <u>Coherent or incoherent integration</u>. - Sometimes it is desirable to give the receiver the capability of integrating the signal partly incoherently, rather than integrating fully coherently over the entire signal duration T.

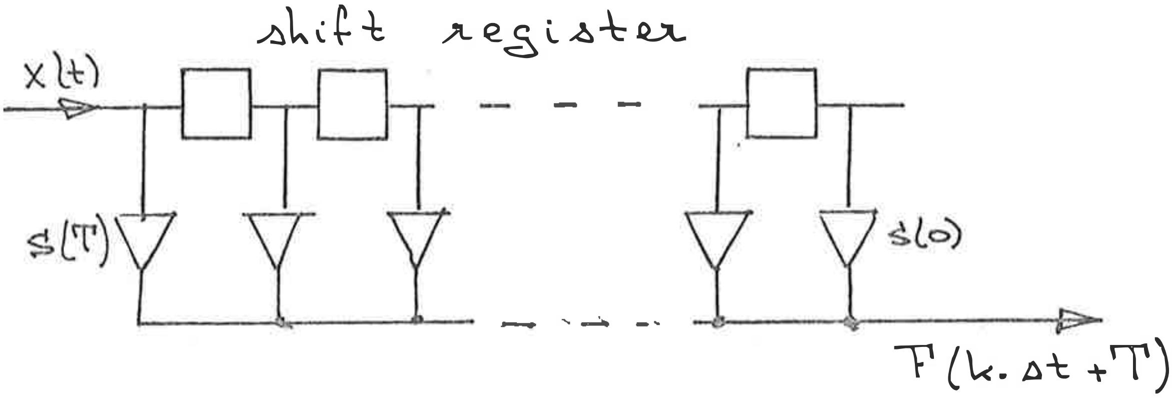
6. <u>Linear or clipped processing</u>. - Do we desire to process the input signal and/or reference function linearly, or are we satisfied with clipped processing (hard limiting)?

The most important items of this list, as for the differences between various systems, are items 1, 2 and 3.

A number of features which are believed to be of less importance from a fundamental viewpoint, has been omitted from the list above. Among these are the question whether the processing is on a digital or on an analog basis, and the question whether the received signal is heterodyned to another frequency before processing.



1 – Matched filter.



2- Matched filter with tapped shift register.

In optical processing systems, a solution is to write the signal in some form on a photographic film, which is continuously moved across an aperture where a light beam is modulated in amplitude or intensity by the film. This is equivalent to a continuously tapped delay line, and the correlator output can be obtained as a continuous time function F(t+T).

Time compression correlators. - A second solution to the time problem is the use of time compression. The principle is sketched in fig.3.

The signal x(t) is applied to a memory which stores the most recent (T-Δt) seconds. Read-out is performed by scanning the memory at an accelerated speed in order to obtain a time compression of the signal. Read-out starts at time $t=t_0+T-\Delta t$ and stops at time $t=t_0+T$. In this interval, a part of the original signal lying

1 THE TIME PROBLEM

The problem is to crosscorrelate a received signal $x(t)$ with a reference $s(t)$, which has a duration equal to the length of the transmitted signal, T sec. The integral to be computed is of the form

$$F(t+T) = \int_{t}^{t+T} x(\tau)\ s(\tau-t)\,d\tau, \qquad (1)$$

and the procedure must be performed in real time. There are two different approaches to this problem.

Matched filter. - The first is to try and build a matched filter (M.F.), with an impulse response equal to a time reversed copy of the reference signal:

$$\text{impulse response} = h(t) = s(T-t); \quad 0 < t < T \qquad (2)$$

The filter output, $u(t)$, will be the convolution of $x(t)$ and $h(t)$:

$$u(t) = \int_{t-T}^{t} x(\tau)\ h(t-\tau)\,d\tau = \int_{t-T}^{t} x(\tau)\ s(T-t+\tau)\,d\tau$$

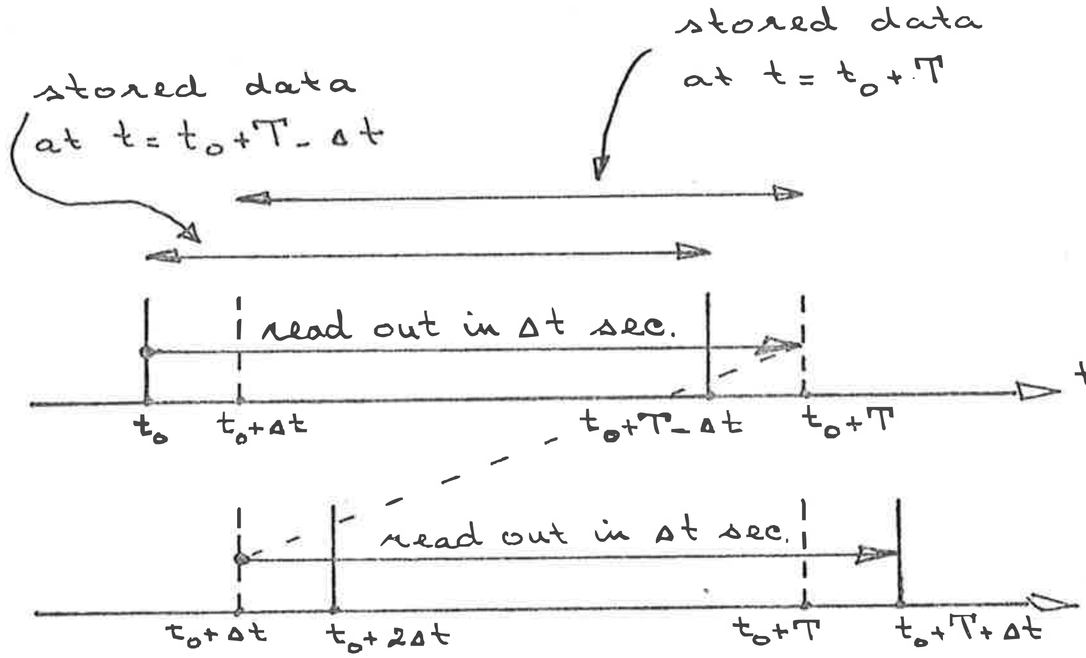
Thus, the output at time $(t+T)$ equals the required output (eq.1):

$$u(t+T) = F(t+T) \qquad (3)$$

Theoretically, the M.F. is the ideal solution to the time problem. In practice, however, the design can be very difficult since the impulse response $h(t)$ can be required to have a duration up to a few seconds and/or bandwidth up to a 1000 Hz. A matched filter for frequency modulated pulses, based on a dispersive delay line, is described by Tourneis, lit.1. A device of this type yields its output as a continuous function of time. (fig.1).

Another aproach is the use of a tapped delay line with weighting coefficients corresponding to $h(t)$. Usually, the input signal is sampled and a shift register is used, rather than a true delay line (fig.2). In this case, the output is obtained as a series of samples, $F(k.\Delta t+T)$.

between $t=t_0$ and $t=t_0+T$ is reproduced. Consequently, an interval of T seconds has been compressed to Δt seconds. As soon as the memory has been scanned, the procedure is repeated and the original signal interval from $t_0+\Delta t$ to $t_0+T+\Delta t$ is reproduced between $t=t_0+T$ and $t=t_0+T+\Delta t$. Etc.

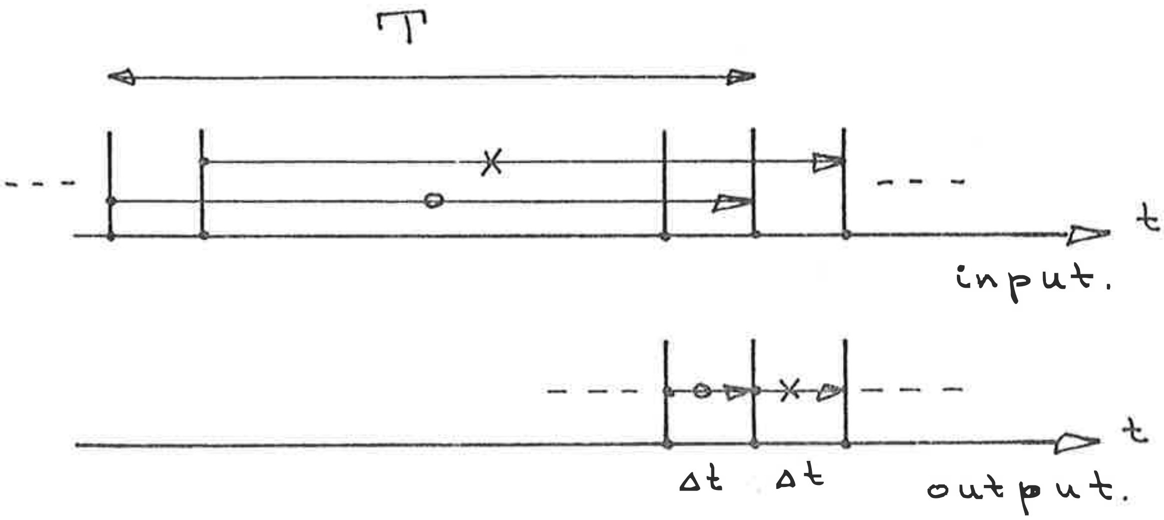


3- Time compression.

Fig.4 illustrates the way in which the input time axis is imaged on the output time axis. The result of time compression is that a large number of correlation intervals, originally overlapping almost completely, are ordered in time one after the other. Hence, the correlation procedure can be performed by correlating the compressor output with a similarly compressed reference function. The reference function is started every Δt seconds, in synchronism with the start of a new read-out period. See fig.4.

It will be clear that a time compression correlator (T.C.C.) yields a sampled output, $F(K.\Delta t+T)$, with a sample distance equal to the length of the compressed time interval.

Usually, time compression is applied to a sequence of signal samples, rather than to the continuous signal $x(t)$. The resulting systems are extensively described by Allen and Westerfield (lit.2).



4- Mapping of input time axis on output time axis.

Time compression of continuous signals x(t) can be obtained, a.o., with magnetic recording and by using rapidly moving pick up heads. A system of this type is described by Tournois (lit.1 , fig.10-a)

Sampled output and continuous output correlators. - It will be helpful to make a difference between correlators or matched filters yielding a continuous output function ("continuous output correlators) and those yielding only a sequence of output samples ("sampled output correlators").

This distinction is not directly related to the question whether input signal and reference are processed in continuous or sampled form. A sampled output correlator does not necessarily imply sampling of the input signals. On the other hand, a system with sampled inputs will necessarily yield a sampled output.

The majority of correlators for practical applications is of the sampled output type.

2 SIGNAL SAMPLING RATES

Since most correlation receivers use some form of sampling, either at the output or at the input, or both, we shall summarize the relevant results of sampling theory.

Suppose that a time function $x(t)$ with double sided amplitude spectrum (Fourier transform) $X(f)$ is sampled at a rate of R samples/sec, and let the sequence of samples be

$$x'(t) = \frac{1}{R} \sum_k x(\tfrac{k}{R})\delta(t-\tfrac{k}{R}) \; ; \quad k = \ldots, -1, 0, +1, \ldots \tag{4}$$

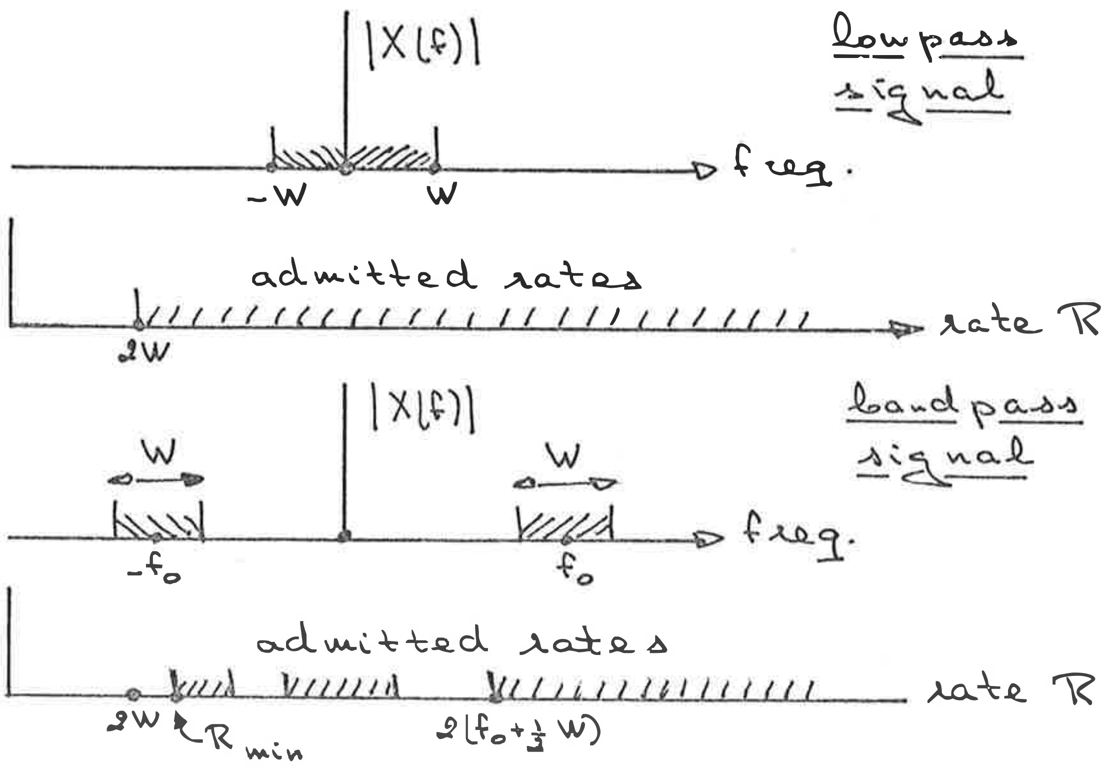
The spectrum $X'(f)$ of $x'(t)$ is a periodic repetition of the original spectrum $X(f)$, with repetition period R:

$$X'(f) = \sum_m X(f-mF) \; ; \qquad m = \ldots, -1, 0, +1, \ldots \tag{5}$$

When the sampling rate R is chosen in such a way that the repeated bands ($m \neq 0$) do not overlap the original band ($m = 0$), it is possible to restore the original signal from its samples by means of a properly designed restoration filter. The filter must pass the original frequency interval (uniform attenuation and linear phase over the original band), and must suppress all repeated bands. If this condition can be met, the only difference between original and restored signal is a delay corresponding to the filter group delay.

For a low pass signal $x(t)$, existing from DC up to a frequency W ($X(f) = 0$ for $|f| > W$), the required sampling rate is $R = 2W$, corresponding to Shannon's theorem. See fig. 5.

In underwater acoustic signal processing, the signal $x(t)$ will usually be a bandpass signal, occupying a spectral interval of width W around a relatively high center frequency f_o ($X(f) = 0$ for $f_o - \frac{1}{2}W < |f| < f_o + \frac{1}{2}W$). It turns out that a number of admitted intervals exists for the sampling rate R, in order to satisfy the requirement of no overlap. The situation is depicted in fig. 5. The exact position of the admitted intervals depends on both W and f_o. Usually, the lowest admitted rate, R_{min}, is somewhat higher than $2W$, although in practice the difference seldom exceeds a few percent. Literature on bandpass sampling can be found in lit.4.



5- Admitted sampling rates for lowpass and bandpass signals.

The capability of restoring the original signal from its samples is not the only condition to be met by the rate R. Usually, the purpose of sampling is to replace a correlation integral of the type

$$\int x(t)\ y(t)\ dt \tag{6}$$

by a sum of the type

$$\frac{1}{R}\Sigma\, x\left(\frac{k}{R}\right)\ y\left(\frac{k}{R}\right)\ . \tag{7}$$

Fortunately, it can be shown that the sum equals the integral when the rate R is chosen on one of the admitted intervals. Hence we can safely replace eq. 6 by eq. 7. (Actually, this statement can even be refined when a(t) and b(t) occupy frequency intervals of unequal width.)

When a matched filter or correlater operates on sampled input signals, the output usually takes the form

$$F\left(\frac{k}{R}+T\right) = \frac{1}{R} \sum_{i=k}^{i=k+RT} x\left(\frac{i}{R}\right) s\left(\frac{i}{R} - \frac{k}{R}\right) \tag{8}$$

fig.8 replaces the original eq.1:

$$F(t+T) = \int_{t}^{t+T} x(\tau)s(\tau-t)dt \tag{9}$$

Since eq.9 is essentially a linear filter operation, we conclude that the spectral interval occupied by the output F is the same as the one occupied by the input signal x(t). Further, the rate R at which the inputs x(t) and s(t) are sampled, is equal to the output sampling rate (eq.8). Consequently, a sampling rate R is an admitted rate for the output F(t+T) when it is an admitted rate for the input signals x(t) and s(t).

3 PHASE INSENSITIVE RECEIVERS

Before the various phase insensitive receiver structures can be derived, we must write down a few expressions for the required output.

Let the transmitted signal be a an amplitude and phase modulated pulse of duration T :

$$s(t) = a(t) \cos\left[\omega_o t + \alpha(t)\right] \; ; \quad 0 < t < T \tag{10}$$

Usually, s(t) will occupy a relatively narrow bandwidth B around its center frequency $f_o = \frac{\omega_o}{2\pi}$.

During propagation and reflection the carrier wave gets a phase shift φ with respect to the modulating wave forms. Putting delay and doppler equal to zero, the echo becomes:

$$x(t) = a(t) \cos\left[\omega_o t + a(t) + \varphi\right] ; \quad 0 < t < T \tag{11}$$

Since φ is an unknown random number, the receiver's signal response is required to be independent of φ. We shall define a phase insensitive receiver as one whose output is given by:

$$C(t+T) = \tfrac{1}{2}\left| \int_t^{t+T} \chi(\tau)\sigma^*(\tau-t)d\tau \right| , \tag{13}$$

where

$$\left.\begin{array}{ll} \chi(t) = x(t) + j\,\hat{x}(t) = & \text{analytic signal corresponding to } x(t) \\ \sigma(t) = s(t) + j\,\hat{s}(t) = & \text{" \quad " \quad " \quad "} \; s(t) \end{array}\right. \tag{14}$$

$\hat{x}(t)$ and $\hat{s}(t)$ are the quadrature signals of $x(t)$ and $s(t)$, respectively. Assuming that the signal bandwidth B is small with respect to its center frequency $f_o = \frac{\omega_o}{2\pi}$, they are given by:

$$\hat{x}(t) = a(t)\sin\left[\omega_o t + x(t) + \varphi\right] \text{ and } \hat{s}(t) = a(t)\sin\left[\omega_o t + \alpha(t)\right] \tag{15}$$

(Actually, the condition for validity of eqs 15 is only $f_o > \frac{1}{2}B$).

As defined in eq.13, the phase insensitive output is the absolute value of the crosscorrelation between the analytic signal $\chi(t)$ and the analytic reference $\sigma(t)$, where the reference equals the transmitted signal. Substitution of eqs 14, 15 into eq.13 shows immediately that $C(t+T)$ is indeed independent of φ.

After substitution of eqs 14 into eq.13, we obtain

$$C(t+T) = \tfrac{1}{2}\left|\left[\int x(\tau)s(\tau-t)d\tau + \int \hat{x}(\tau)\hat{s}(\tau-t)d\tau\right] + j\left[\int \hat{x}(\tau)s(\tau-t)d\tau - \int x(\tau)\,\hat{s}(\tau-t)d\tau\right]\right| \tag{16}$$

This expression can be simplified by observing that the integrals are pairwise equal, and that the second pair is the quadrature version of the first pair:

$$\left.\begin{aligned}\hat{F}(t+T) &= \int_t^{t+T} x(\tau)s(\tau-t)d\tau = \int_t^{t+T} \hat{x}(\tau)\hat{s}(\tau-t)d\tau \\ \hat{F}(t+T) &= -\int_t^{t+T} \hat{x}(\tau)s(\tau-t)d\tau = \int_t^{t+T} x(\tau)\hat{s}(\tau-t)d\tau\end{aligned}\right\} \quad (17)$$

Consequently, eq. 16 reduces to

$$C(t+T) = \left[F^2(t+T) + \hat{F}^2(t+T)\right]^{\frac{1}{2}} \quad (18)$$

Eqs 17, 18 express the required output in operations on the input signal. F is the correlation between signal x(t) and reference s(t), and $\hat{F}$ is the correlation between x(t) and the quadrature reference, $\hat{s}(t)$.

Since F(t+T) is obtained from x(t) via a linear filter operation (eqs 17), we expect F(t+T) to have the same structure as x(t), i.e. a carrierwave of frequency ω_o with a slowly varying envelope and phase. Actually, substitution of eqs 10, 11, 15 into eqs 17 leads to

$$\left.\begin{aligned}F(t+T) &= P(t+T)\cos(\omega_o t - \varphi) - Q(t+T)\sin(\omega_o t - \varphi) \\ \hat{F}(t+T) &= Q(t+T)\cos(\omega_o t - \varphi) + P(t+T)\sin(\omega_o t - \varphi)\end{aligned}\right\} \quad (19)$$

where

$$\left.\begin{aligned}P(t+T) &= \tfrac{1}{2}\int_t^{t+T} a(\tau)a(\tau-t)\cos\left[\alpha(\tau)-\alpha(\tau-t)\right]d\tau \\ Q(t+T) &= \tfrac{1}{2}\int_t^{t+T} a(\tau)a(\tau-t)\sin\left[\alpha(\tau)-\alpha(\tau-t)\right]d\tau\end{aligned}\right\} \quad (20)$$

Combination of eqs 18 and 19 leads to a second expression for the receiver output:

$$C(t+T) = \left[P^2(t+T)+Q^2(t+T)\right]^{\frac{1}{2}} = \text{envelope of } F(t+T) \text{ or } \hat{F}(t+T). \quad (21)$$

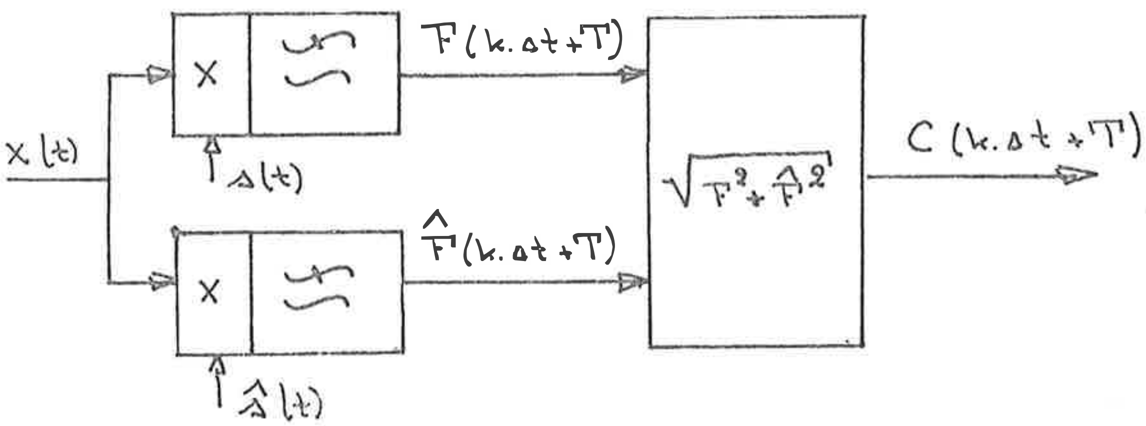
The expressions above suggest two structures for the phase insensitive receiver: 1. Correlation of x(t) with s(t) and ŝ(t) and subsequent combination in a square law combiner (eqs 17, 18), and 2. Correlation of x(t) with s(t), followed by envelope rectification of the resulting F(t+T).

Both systems are "D.C. or low-pass correlators" since the signal x(t), at frequency ω_0, is multiplied with a reference at the same frequency, and the output is obtained by integration or low pass filtering.

Another type of phase insensitive receiver, not immediately suggested by the formulae above, is based on "A.C. or band-pass correlators", in which the integrating element is a bandpassfilter tuned to a frequency υ, rather than a lowpass filter.

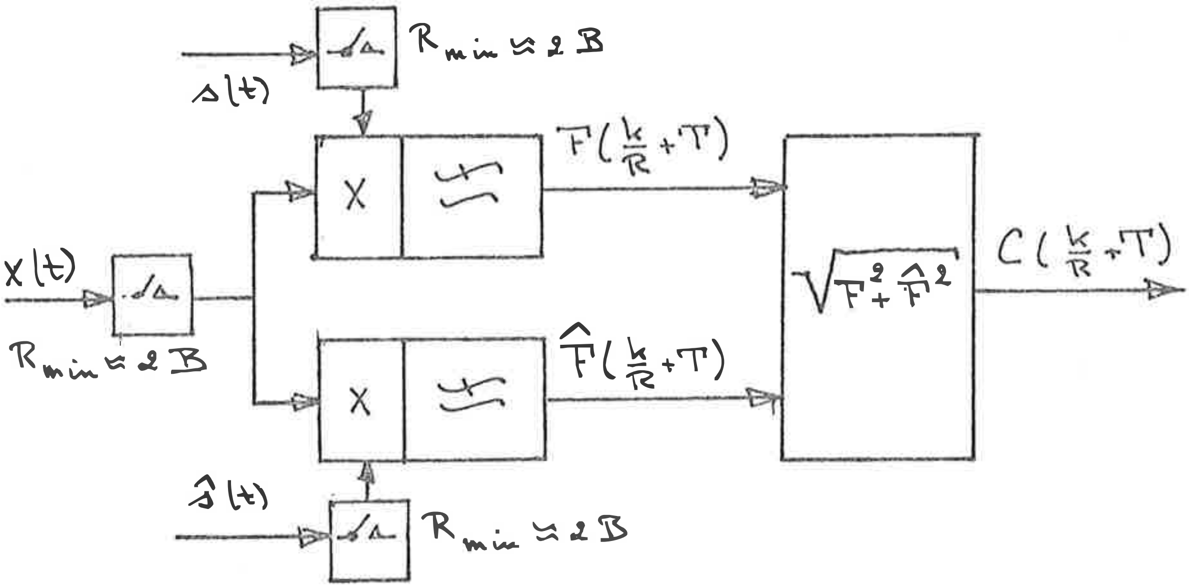
Phase - quadrature receiver with D.C.correlators

A block diagram is shown in fig.6. The system is based on eqs 17,18. The two D.C. correlators are indicated as multipliers, followed by lowpass filter. In a phase-quadrature receiver, the correlators will usually be of the sampled output type (if a continuous matched filter, the system of fig. 9 would be preferable). Hence, F and $\hat{F}$ and, consequently, also the output C, are obtained as a sequence of samples.



6- Phase-quadrature receiver with D.C. correlators.

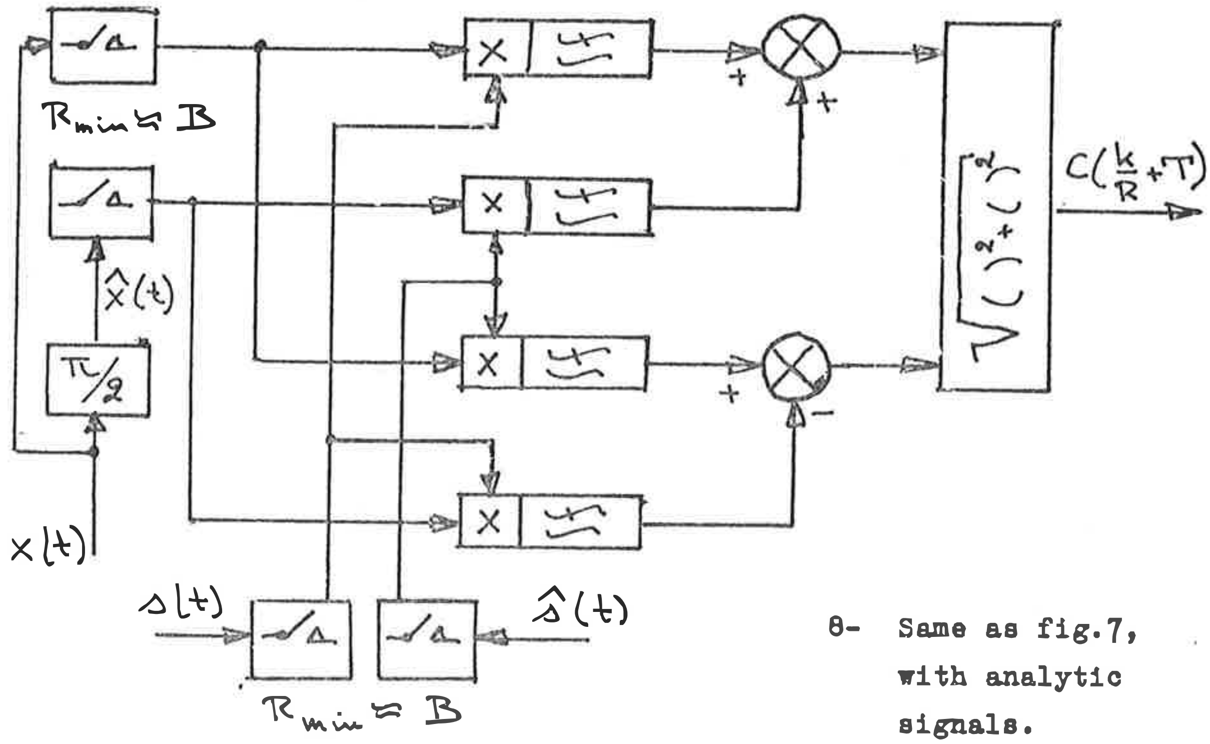
When the correlators are operating on sampled input signals, we get the block diagram of fig.7. The sampling rates are chosen according to the narrow band sampling theorem, i.e. the minimum rate is $R_{min} \approx 2B$ samples/sec.



7- Same as fig.6, with sampled input.

Another phase-quadrature system, with sampled signal processing, is shown in fig.8. It is based on eq.16. Use is made of both $x(t)$ and $\hat{x}(t)$, and of $s(t)$ and $\hat{s}(t)$. In other words, we use the analytic signals $\chi(t)$ and $\sigma(t)$ of eq.13. Since the analytic signals contain positive frequencies only, they occupy effectively a bandwidth of B, rather than two times B. Thus, the minimum sampling rate will now be $R_{min} = B$ samples/sec.

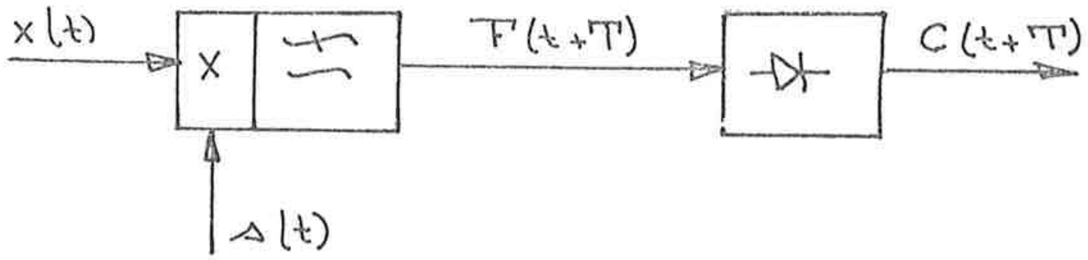
A disadvantage of fig.8 with respect to fig.7 is that $\hat{x}(t)$ has to be derived from $x(t)$, for instance by means of a $\pi/2$ phase shifter, and that 4 correlators are needed rather than 2. On the other hand, the total number of samples to be processed per second, is the same.



8- Same as fig.7, with analytic signals.

D.C.correlator with envelope rectifier

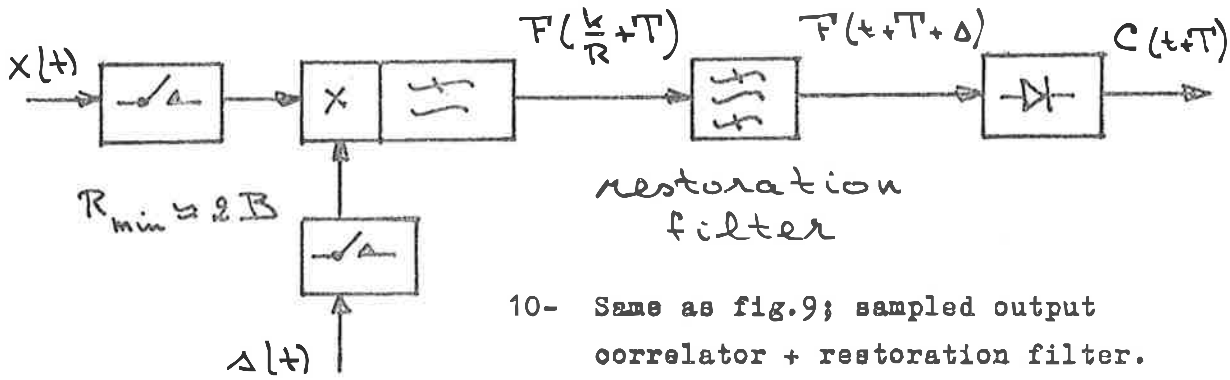
The system, based on eq.21, is shown in fig.9. The signal $x(t)$ is correlated with the reference $\Delta(t)$, followed by envelope rectification of the output $F(t+T)$.



9- D.C. correlator with envelope rectifier.

In fig.9 a continuous output correlator is required in order to feed the rectifier.

When using a sampled output correlator, the sequence $F(\frac{k}{R} + T)$ can be transformed into the required continuous time function by means of a restoration filter, which passes the spectral band corresponding to $F(t+T)$ and suppresses all repeated bands which have arisen in the process of sampling. (section 2). In other words, the filter should be a bandpass filter centered at $f_o = \frac{\omega_o}{2\pi}$ with a bandwidth B. The output will be $F(t+T+\Delta)$, where Δ is the filter group delay. The system is shown in fig.10 for the case of a sampled input and sampled output correlator. Minimum sampling rate is in the order of $R_{min} \approx 2B$ samples/sec.



10- Same as fig.9; sampled output correlator + restoration filter.

Actually, it is not necessary to center the restoration filter around the original centerfrequency $f_o = \frac{\omega_o}{2\pi}$. Any one of the repeated spectral bands, present in $F(\frac{k}{R}+T)$, may serve as well. These bands are centered at the frequencies

$$f = (f_o \pm n.R) \text{ and } f = (-f_o \pm n.R) \tag{22}$$

In order to facilitate the design of the restoration filter, it may prove helpful to select the lowest centerfrequency which still guarantees a dependable performance for the envelope rectifier.

A.C.correlator with envelope rectifier

The system is shown in fig.11 for the case of a time compression correlator. The correlator is an A.C. or bandpass-correlator, in which the integrating element is a bandpassfilter tuned to a frequency υ. Further, the reference is at an off-set frequency with respect to the transmitted signal:

$$s_\upsilon(\theta) = a(\theta) \cos\left[(\omega_o - \upsilon)\theta + \alpha(\theta)\right]; \tag{23}$$

In eq.23, θ represents compressed time, and the duration of $s_\upsilon(\theta)$ equals the length Δt of the compressed correlation interval. Let us see what happens inside one of these intervals.

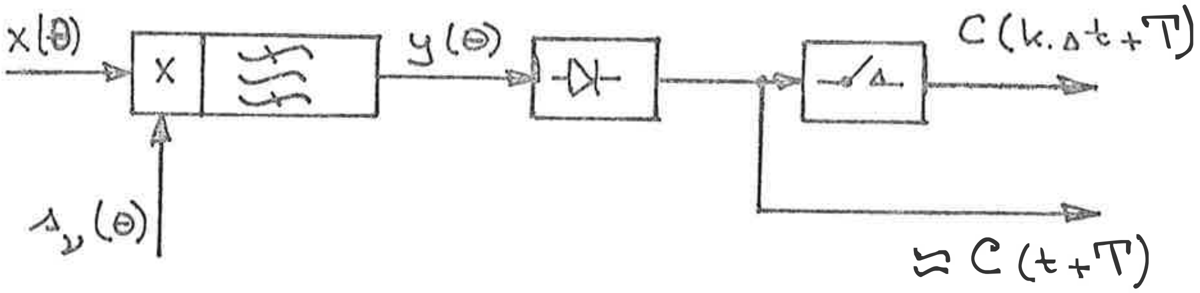
The received signal $x(\theta-t)$, similarly compressed, in multiplied with the reference, and the product signal is

$$y(\theta) = x(\theta-t)s_\upsilon(\theta) = a(\theta-t)\cos\left[\omega_o(\theta-t) + \alpha(\theta-t) + \varphi\right] \cdot$$
$$\cdot\ a(\theta)\cos\left[(\omega_o-\upsilon)\theta + \alpha(\theta)\right] \qquad (24)$$

$y(\theta)$ contains one component at frequency υ, and one at $2\omega_o-\upsilon$. The υ-component is

$$y_\upsilon(\theta) = \tfrac{1}{2}a(\theta-t)a(\theta)\cos\left[\upsilon\theta - \omega_o t + \alpha(\theta-t) - \alpha(\theta) + \varphi\right] \qquad (25)$$

The bandpassfilter, tuned to υ, starts ringing in response to $y_\upsilon(\theta)$, and its response will be an increasing vibration of frequency υ. The envelope of this wave will be independent of the phase φ of the input $y_\upsilon(\theta)$. Consequently, the required phase independent output can be obtained by feeding the filter response into an envelope rectifier, and by sampling the rectifier output at the end of each compressed time interval Δt, as indicated in fig.11. Calculation of the filter response to $y_\upsilon(\theta)$ shows that the result is indeed identical to eq. 21.



11- Time compressed A.C. correlator + envelope rectifier.

Since the procedure is repeated in each compressed interval, the filter should theoretically be emptied of its energy contents before starting the next correlation interval. However, the filterbandwith is in the order of $\frac{1}{\Delta t}$. Therefore, the filter response to the previous interval will approximately have died out at the end of the next one. Consequently, the "emptying" can be omitted.

It will be clear that the receiver of fig.11 yields its output only as a sequence of samples: $C(k.\Delta t+T)$. In practice, the operation of sampling the filter response's envelope at the end of Δt can be omitted. The filter output is then fed directly into the rectifier and its output will be a more or less continuous curve.

It should be observed that the diagrams of fig.11 and fig.10 show an essential difference. In fig.10, the restoration filter does not form a part of the correlator. It is tuned to the signal centerfrequency f_0 , and has a band width B. Both the filter and the rectifier operate on the normal time scale. Contrary to this, the filter in fig.11 is the integrating element in the correlator. It is tuned to a frequency $f_c = c.\frac{\upsilon}{2\pi}$ Hz, where c = the time compression factor, and its bandwidth equals $\frac{c}{T} = \frac{1}{\Delta t}$ Hz. Both the filter and the rectifier operate in compressed time.

For a large time compression factor c, the filter tuning frequency $f_c = c.\frac{\upsilon}{2\pi}$ Hz often gets a prohibitively high value. When the correlator input signals are processed in sampled form, as is normally the case in a time compression system, the interaction between the repeated spectral bands of $x(t)$ and $s_\upsilon(t)$ will result in a set of repetitions of the difference frequency υ, a number of which will be at a lower frequency than f_c. These can also be used as centerfrequencies for the correlator's integrating filter.

4 DOPPLER SENSITIVE RECEIVERS

In the case of a doppler sensitive transmission, eg. a random modulation signal, the receiver is equipped with a number of doppler channels. The output correlation peak is supposed to appear in the channel corresponding to the signal doppler.

We shall only consider a doppler shift of the carrier frequency. Expansion or compression of the time scale of the modulating waveform, which occurs at higher target speeds (depending on the transmission's BT product) must be taken into account by a corresponding change of the reference function's time scale.

For a pure carrier frequency shift, the required receiver output is defined as the bidemensional crosscorrelation of signal and reference:

$$C(t+T,\delta) = \left| \int_t^{t+T} \chi(\tau)\sigma^*(\tau-t) \exp j\delta\tau \, d\tau \right| \tag{26}$$

As in the previous section, the absolute value serves to obtain a phase insensitive system. Eq.26 suggests two receiver structurers.

<u>Parallel receivers</u>

One interpretation of eq.26 is as the crosscorrelation function of a signal x(t) with a reference $\sigma_\delta(t) = \sigma(t)\exp{-j\delta t}$:

$$C(t+T,\delta) = \left| \int_t^{t+T} \chi(\tau) \, \sigma_\delta^*(\tau-t)d\tau \right| \tag{27}$$

$\sigma_\delta(t)$ is a frequency shifted version of the original reference $\sigma(t)$ which in itself equals the transmitted pulse. Hence, the system will be sensitive to a received signal with a corresponding doppler shift, $-\delta$.

Since the receiver's doppler region D must cover the entire range of expected doppler shifts, eq.27 leads to a bank of parallel receivers, each matched to an individual doppler δ_i . See fig.12.

Because the doppler response of each receiver is of the type

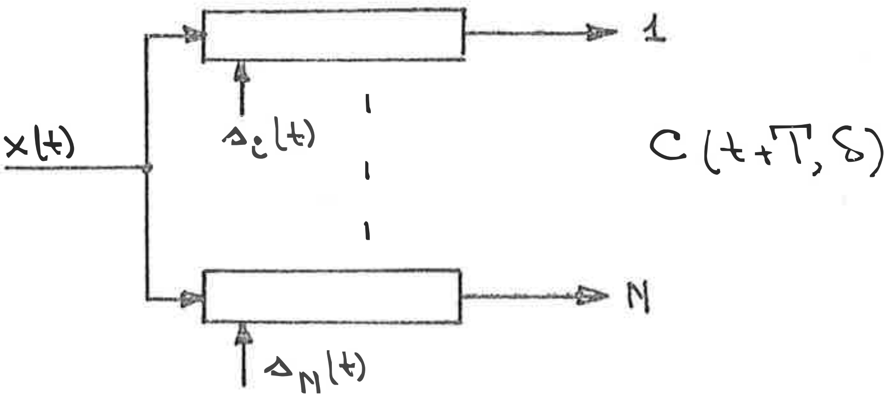
$$\frac{\sin \pi\left[\eta-(\omega_o-\delta_i)\right]T}{\pi\left[\eta-(\omega_o-\delta_i)\right]T},$$ where η = signal frequency and $(\omega_o-\delta_i)$ = tuning frequency of i^{th} receiver, the various receiver frequencies can be $\frac{1}{T}$ Hz apart (for fully coherent integration). Hence the number of receivers for covering a region D equals N = DT.

The receivers can be of one of the types described in section 3. When a compression system is used for time compression of the received signal, this system can be shared between the various receivers. The same holds for the delay line in a tapped delay line matched filter. Apart from this, however, the receivers are essentially individual systems.

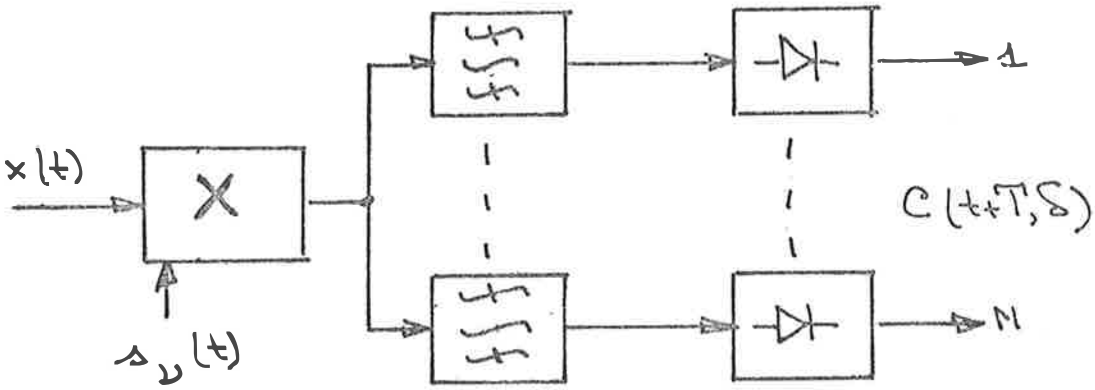
(A second, though similar, interpretation of eq.26 is to write

$$C(t+T,\delta) = \left|\int \left[\chi(\tau)\exp j\delta\tau\right] . \delta^*(\tau-t)d\tau\right|$$

This comes down to displacing the input signal over frequencies δ, and feeding the frequency shifted signals into a bank of identical receivers, each tuned to the original frequency ω_0).



12- Doppler sensitive system with individual receivers.



13- A.C. correlator + spectrum analysis.

<u>A.C.correlator + spectrum analysis</u>

A more attractive receiver structure is obtained by interpreting eq.26 as the Fourier transform of the product of χ and δ^*;

$$C(t+T,\delta) = \left|\int_t^{t+T}\left[\chi(\tau)\sigma^*(\tau-t)\right].\exp j\delta\tau \, d\tau\right| \qquad (28)$$

The input signal is multiplied by one reference only, the zero doppler reference, followed by a spectrum analysis of the product signal. The advantage is that the various doppler channels share the same multiplier unit. Usually, the system takes the form of an A.C.correlator, equipped with a bank of filters (fig.13).

As in fig. 11, the reference is generated at an offset frequency $(\omega_0-\upsilon)$. Rather than one filter tuned to υ, a number of filters is used, tuned to frequencies $(\upsilon+\delta_1)$. A signal x(t) at frequency $(\omega_0+\delta_s)$, δ_s = signal doppler, gives rise to a component $(\upsilon+\delta_s)$ in the product signal. Thus, a response is obtained from the filter with the corresponding tuning frequency.

5 MIXED COHERENT - INCOHERENT INTEGRATION

On some occasions one wishes to deliberately widen the pass-band of the integrating element in the correlators. One example is the case of rapid time variation of the communication channel within the received pulse duration.

Fully coherent integration of a pulse with length T is obtained with a filter bandwidth $\frac{1}{T}$ Hz. Widening the filter with a factor p reduces the length of the impulse response with the same factor. Hence, the integration interval is effectively divided in p coherently integrated parts, each of duration $\frac{T}{p}$ sec.

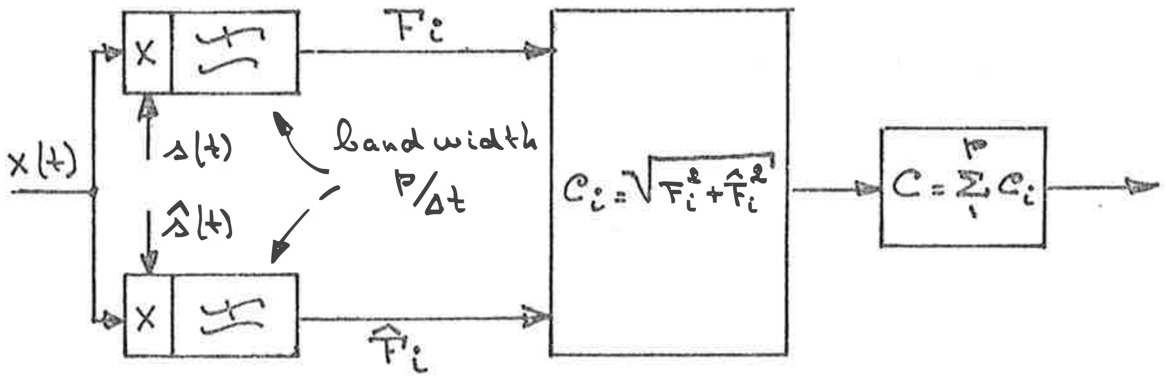
After having removed the phase information from the correlation results of each of the subintervals, they can be combined incoherently. In this way we get a mixed coherent-incoherent integrator, where the degree of incoherence is defined by p.

As usual in problems of this type, the question arises whether the subcorrelation results should be combined via a linear law, a square law, or still something else. Assuming Rayleigh-Rice statistics for the subcorrelation results, the combination should be on a square law basis for small SNR and on a linear law basis for large SNR.

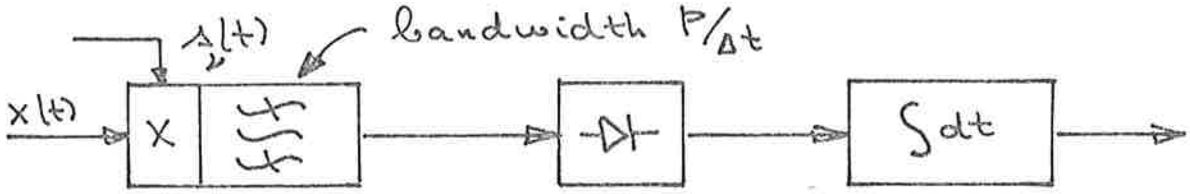
A few examples of mixed integrator block diagrams are shown in fig.14,15.

Fig.14 relates to a phase-quadrature receiver with time compressed DC correlators, equipped with binary integrators. The time compressed interval Δt is divided in p sub-intervals. At the end of the first sub-interval, the stored values in the integrating registers, F_1 and $\hat{F}_1$, are combined to give $C_1 = \left[F_1^2 + \hat{F}_1^2 \right]^{\frac{1}{2}}$.
The integrating registers are emptied and C_1 is stored in the C

register. This is repeated for the second sub-interval and C_2 is added to C_1. And so on, until p sub-intervals have been treated.



14- Time compressed phase-quadrature D.C. correlators; mixed integration.



15- Time compressed A.C. correlator + rectifier; mixed integration.

Fig.15 shows an analogous operation for an A.C.correlator + rectifier. The bandwidth of the integrating bandpassfilter is widened by a factor p. The filter response is fed continuously to the envelope rectifier and the required incoherent combination is simply obtained by inserting a smoothing filter with time constant Δt (= length of time compressed interval) after the rectifier.

In both examples, we have restricted ourselves to a linear law incoherent combination of the sub correlation results.

6 CLIPPING

By clipped processing we mean that the input signal $x(t)$ is passed through a clipper, or hard limiter, which transforms $x(t)$ into a square wave of amplitude $\pm$ 1 in which only the original zero crossings are retained. Usually, the reference functions are clipped as well. Although the electronic design of a clipper receiver may differ greatly from a linear one, the operations on the clipped signal remain the same from a functional point of view Hence, the clipper receiver can be regarded as an ordinary linear processor, preceded by one or more clippers.

The advantage of clipping are that the dynamic fluctuations of $x(t)$ are removed, which considerable simplifies the electronic requirements, and that at the same time the output $C(t+T)$, or $C(t+T,\delta)$, is automatically normalized to a standard amplitude scale, independent of the input amplitude.

Disadvantages are mainly caused by two sources:

1) Typical nonlinear interaction of the various signalcomponents in the clipper, 2) The self normalizing property, mentioned above. Though in itself a desirable feature, it can also be a cause of trouble when the receiver's processing gain is too small.

Below, we shall give a few examples of non linear effects, and finally we shall try to find the restrictions under which clipping is allowed from the viewpoint of self normalization.

Non linear effects

When a narrowband time function (bandwidth B Hz around relatively high carrier-frequency f_o Hz) is clipped, two gross effects can be distinguished: 1) a distortion, usually a widening, of the original spectrum around f_o, 2) the generation of new spectral bands around the uneven harmonics of the carrier, $3f_o$, $5f_o$, etc.

The first phenomenon is responsible for differences between the detailed output functions of a linear and a clipped processor. Since these effects are inherent to the clipper, they can not be circumvented by changing the receiver structure.

The second class of phenomena can be troublesome when clipping is combined with sampled processing. However, these effects can to a reasonable extent be overcome by proper design, in particular by a proper choice of sampling frequencies.

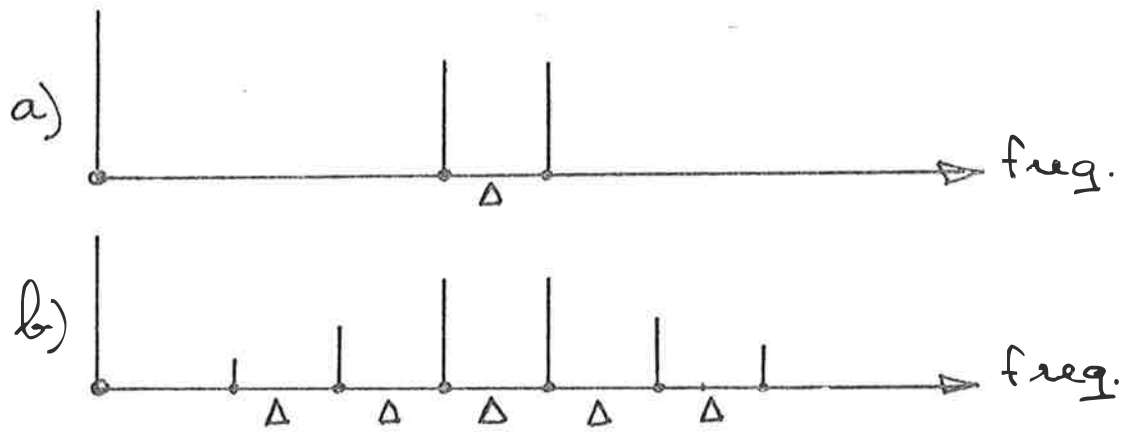
As a simple example of a phenomenon belonging to the first class, we consider a signal consisting of two equal amplitude sine waves with a frequency difference Δ:

$$x(t) = \cos 2\pi(f_o+\tfrac{1}{2}\Delta)t + \cos 2\pi(f_o-\tfrac{1}{2}\Delta)t \qquad (29)$$

Interference of the two components results in a carrier at f_o, modulated sinusoidally with a beatfrequency $\frac{1}{2}\Delta$:

$$x(t) = \tfrac{1}{2} \cos 2\pi.\tfrac{1}{2}\Delta t. \cos 2\pi\, f_o t \qquad (30)$$

Clipping of x(t) transforms the sinusoidal modulation into a rectangular one. Consequently, the spectrum of sgn x(t) is a suppressed carrier with sidebands consisting of the (uneven) harmonics of a rectangular wave. (Fig.16).



16- Spectrum of two sine waves before and after clipping.

Suppose, now, that instead of two sine waves, we have two random modulation pulses, without mutual time delay, but with a carriershift Δ. When x(t) is fed into a linear doppler sensitive receiver, such as an A.C.correlater + spectrum analysis (fig. 13), the product signal of x(t) and the reference will consist of two sine waves, at the moment of time matching. Output peaks will appear from the corresponding two doppler filters. When x(t) is clipped, however, the product signal will also contain the parasitic frequency components of fig.16-b, and ghost peaks appear from doppler channels in which no output should be present. The result is a distortion of the signal's doppler structure.

A similar effect occurs effect occurs with (linear) frequency modulated pulses. See fig.17. (Computer calculated results; sweep-width B=400 Hz; pulse duration T=1 sec; receiver of the phase-quadrature DC correlator type, fig. 6). The upper traces show various ~~scatterer~~ structures (independent random phases), the middle and lower traces show the outputs of a linear and a clipper receiver, respectively.

In the second picture from the left, two equal strength scatterers are present with a separation of $\tau \approx 24$ msec. Thus, $x(t)$ will consist of two overlapping FM pulses with a frequency difference $\Delta = \tau . \frac{B}{T}$. As in the previous example, clipping produces parasitic sweeps at multiples of Δ. These are the sweeps which are responsible for the ghost peaks on the time axis in the case of clipped processing.

For overlapping pulses of unequal strength, the situation is complicated in the sense that the spectral symmetry disappears. The ghost peaks will now get different amplitudes. In addition, the ratio between the main peaks will differ from the original ratio. Quantitative information about this phenomenon is given in lit.3

The net result is shown in fig.17, third picture from the left, where a larger number of spiky, isolated scatterers is present. There is a striking difference between the outputs of the linear and clipped systems. The amplitude ratios of the various scatterers have changed, and a significant response exists outside the time interval over which the scatterers extend.

Effects belonging to the second class of non linear phenomena are related to the generation of harmonics when a narrowband time function is applied to a clipper. When both the signal and the reference are clipped, it can be expected that mutual correlation will occur between the 3rd harmonics, between the 5th harmonics, etc. Naturally, the 3rd harmonics are the most important since they have the largest amplitude $(\frac{1}{3})$ with respect to the 1st harmonic.

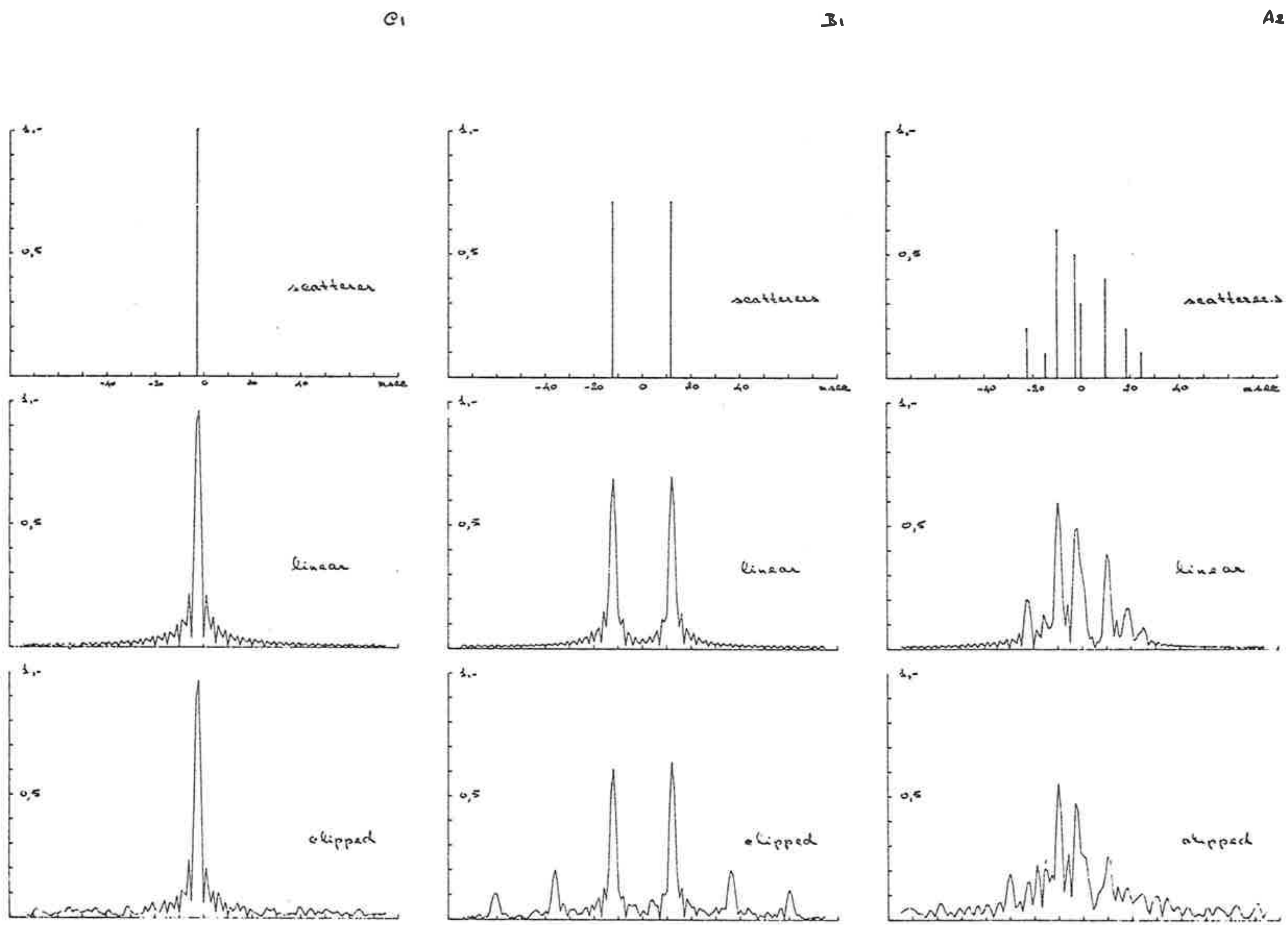
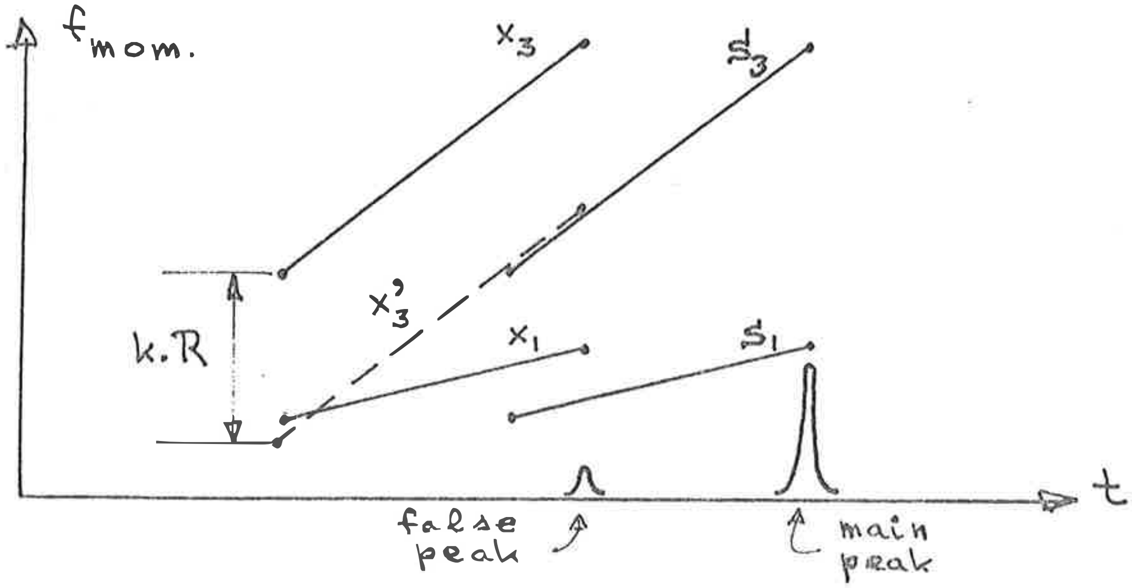


Fig 17, clipped and linear processing of FM pulses.

When the signal and reference are processed as continuous time functions, the effect of inter-harmonic correlation can normally be circumvented without too much difficulties. The situation becomes rather complicated, however, when the clipper correlator operates on sampled time functions, Frequency repeated copies of the harmonics will now be present over the entire spectrum, with repetition distances of R Hz (R=sampling rate), and the problem of possible interaction between them should be studied carefully.

As an example, we consider a DC correlator for the reception of FM pulses (fig.18). The reference and its 3rd harmonic are denoted by s_1 and s_3, respectively. An FM sweep x_1, with 3rd harmonic x_3, is present at the input. It is not yet in time opposition with the reference. However, a repeated copy of x_3, denoted by x_3', will correlate with s_3, which results in a false output peak on the range axis. Although the false peak will be below $\frac{1}{9}$ of the main peak, it can still have a considerable magnitude.



18- Interaction between 3rd harmonics.

A similar phenomenon can occur, for example, in a doppler sensitive receiver for RM pulses, for instance of the type of fig. 13 . After frequency repetition due to sampling, the interaction between x_3 and s_3 can result in a false doppler output from a channel where nothing ought to be present.

By a proper choice of the sampling rate R, and of the signal and reference frequencies, the unwanted responses can be suppressed to a reasonable degree. It should be remarked, however, that one is almost invariably forced to increase R significantly over the minimum rate $R_{min} \approx 2B$, which is required by the bandpass sampling theorem as applied to non-clipped signals.

Self-normalization

We shall now neglect the typical nonlinearity of a clipper and just regard its self-normalizing property. By this we mean that, whatever its input may be, the output power is always normalized to 1 $volt^2$.

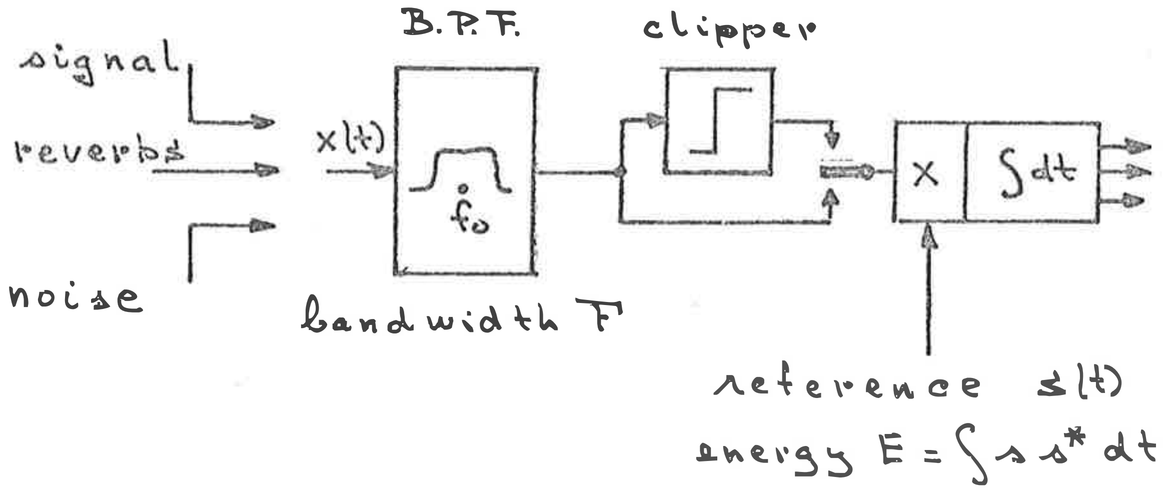
Thus, the peak output of a clipper correlator never exceeds a maximum power M, which is obtained when one signal, perfectly matched to the reference, is applied at the input.

As a simple example we consider N strongly overlapping, but resolvable, signals $x_i(t)$ of approximately equal power. The power at the clipper output will be $\frac{1}{N}$ $volt^2$ for each of them. Consequently, the N correlation peaks at the receiver output will have a power of $\frac{M}{N}$. The maximum output power is shared between the N correlation peaks.

Suppose now, that we have at the clipper input a white noise of bandwidth F. The number of "resolvable noise components" can be shown to be FT, where T=duration of reference function. Hence, the correlator's mean output power will now be $\frac{M}{FT}$. When the aggregate of N signals appears, the output power rises to $\frac{M}{N}$(assuming the input signal-to-noise ratio to be very high). Since dependable detection is only possible when the signal output exceeds the noise output by a given factor, we conclude that clipping is only allowed when $FT > N$. Otherwise we end up in a situation where the signals are perfectly visible at the input, but have disappeared at the output.

We shall consider the problem of self-normalization in a little more detail, and find out how the performance of a clipper correlator depends on the system's resolution, on the extent of the target in delay and doppler, and on the type of interference, noise or reverberation. It will appear that clipping is allowed when the correlator's processing gain, defined in a proper way, is sufficiently great.

Since we shall compare the performance of a clipper correlator and a linear one, we shall start by writing down the equations for a linear system (Lit.5). Analytic signal notation is used. Contrary to the previous sections, however, we shall abandon the greek symbols. The receiver's block diagram is shown in fig.19. Note that the receiver is preceded by a bandpass filter of width F. Further, the receiver is a doppler sensitive one, with a number of doppler output channels.



19- Linear or clipper correlator with input bandpass filter.

The following assumptions are made:

concerning the pulse:

- bandwidth B, time duration T.
- ambiguity function:

$$v(t,f) = |\varphi(t,f)|^2 , \quad \text{with} \tag{31}$$

$$\left.\begin{array}{l}\varphi(t,f) = \frac{1}{E}\int s(\tau)s^*(\tau-t)\exp 2\pi j f\tau \, d\tau \\ E = \text{pulse energy} = \varphi(0,0)\end{array}\right\} \tag{32}$$

concerning the target:

- delay spread L, around fixed delay t_1,
  doppler spread W around fixed doppler f_1.
- statistically independent scatterers of uniform strength over entire target area WL, closely spaced with respect to resolvable doppler and delay.
- power scattering coefficient $\sigma dt df$ for infinitesimal area $dt\, df$
- target length L<< pulse duration T.

concerning the reverberation:

- doppler spread Q around doppler zero
- independent, closely spaced scatterers of uniform strength $\rho dt df$ over Q and T.

concerning the noise:

- white noise, power density N_0
- bandwidth F (input bandpass filter)

Under these assumptions, the following equations are derived for the signal powers at input and output of the linear processor.

$$\left.\begin{aligned} \text{input - echo power} \quad & \overline{|e(t)|^2} = \frac{E\sigma WL}{T} \\ \text{reverb power} \quad & \overline{|r(t)|^2} = \frac{E\rho QT}{T} \\ \text{noise power} \quad & \overline{|n(t)|^2} = N_0 F \end{aligned}\right\} \qquad (33)$$

The averaging relates to the ensemble of scatterer structures, defined above.

$$\left.\begin{aligned} \text{output - echo power} \quad & \overline{|u_e(t,f)|^2} = \sigma E^2 \iint_{WL} v(t-\tau, f-\eta)\, d(\tau,\eta) \\ \text{reverb power} \quad & \overline{|u_r(t,f)|^2} = \rho E^2 \iint_{QT} v(t-\tau, f-\eta)\, d(\tau,\eta) \\ \text{noise power} \quad & \overline{|u_n(t,f)|^2} = N_0 E \end{aligned}\right\} \qquad (34)$$

u(t,f) is the output at time t in the doppler channel tuned to f. The integrals in eqs 34 are over the extent of target and reverberation, respectively. They represent the volume of the ambiguity function which is intercepted by the scattering object. In simplified notation:

$$\iint_{WL} v(\ )d\tau d\eta = I\,tar \;; \qquad \iint_{QT} v(\ )d\tau d\eta = I\,rev \tag{35}$$

Let us now restrict our attention to those resolution cells which are "on the target", i.e. to the cells within the region

$$f_1 - \frac{1}{2}W < f < f_1 + \frac{1}{2}W; \quad t_1 - \frac{1}{2}L < t < t_1 + \frac{1}{2}L \tag{36}$$

For these cells, Itar reaches a maximum value Itarmax, since the ambiguity function's peak is on top of the target:

$$\left.\begin{aligned} &\text{output - echo power} \quad \overline{|u_e|^2} = \sigma E^2 I\,tar\,max \\ &\qquad\text{reverb power} \quad \overline{|u_r|^2} = \rho E^2 I\,rev \\ &\qquad\text{noise power} \quad \overline{|u_n|^2} = E\,N_o \end{aligned}\right\} \tag{37}$$

For simplicity, we shall only consider the limiting cases of noise - only, or reverberation - only. The output signal -to-interference ratios will then be:

$$\left.\begin{aligned} d_r &= \frac{\text{echo power}}{\text{reverb power}} = \frac{I\,tar\,max}{I\,rev} \cdot \frac{\sigma}{\rho} \\ d_n &= \frac{\text{echo power}}{\text{noise power}} = \frac{\sigma E\;I\,tar\,max}{N_o} \end{aligned}\right\} \tag{38}$$

Actually, the receiver's performance can still be increased by post correlation integration over the target's extent. However, we shall not consider this since the improvement is the same for the linear and clipped receivers.

Finally, we define a processing gain G as

$$G = \left(\frac{\text{echo power}}{\text{interference power}}\right) \text{output} \Big/ \left(\frac{\text{echopower}}{\text{interference power}}\right) \text{input} \qquad (39)$$

The result is

$$\left.\begin{aligned} G_r &= \frac{I\,tar\,max}{WL} \cdot \frac{QT}{I\,rev} \quad \text{(reverberation)} \\ G_n &= \frac{I\,tar\,max}{WL} \cdot FT \quad \text{(noise)} \end{aligned}\right\} \qquad (40)$$

Note that eq 39 is an unusual gain definition since it does not only depend on the transmitted pulse, but also on the properties of the target and the reverberation.

Let us now turn our attention to the clipper system. (See also lit.6). Its maximum possible output power, with a normalized input power of 1 volt^2, equals:

$$M = \left[\int \sqrt{\frac{T}{E}}\, s(t) \cdot s(t) \cdot dt\right]^2 = ET \qquad (41)$$

Again, we restrict ourselves to the pure reverberation limited and the pure noise limited case.

In both situations, the output power is found by dividing the powers of eqs.37 by the input powers of eqs.33. (This involves an approximation since, theoretically, we should divide by the power of the individual input wave, rather than by the ensemble averaged power). After some algebra, we find the results below:

<u>Reverberation limited case</u>

input "reverbs only"
output power: $\dfrac{M}{QT/I\,rev}$

input "signal + reverbs"
output signal power: $\dfrac{M}{QT/I\,rev} \cdot \dfrac{d_r}{1+\dfrac{d_r}{G_r}}$

output reverbs power: $\dfrac{M}{QT/I\,rev} \cdot \dfrac{1}{1+\dfrac{d_r}{G_r}}$

(42)

Noise limited case

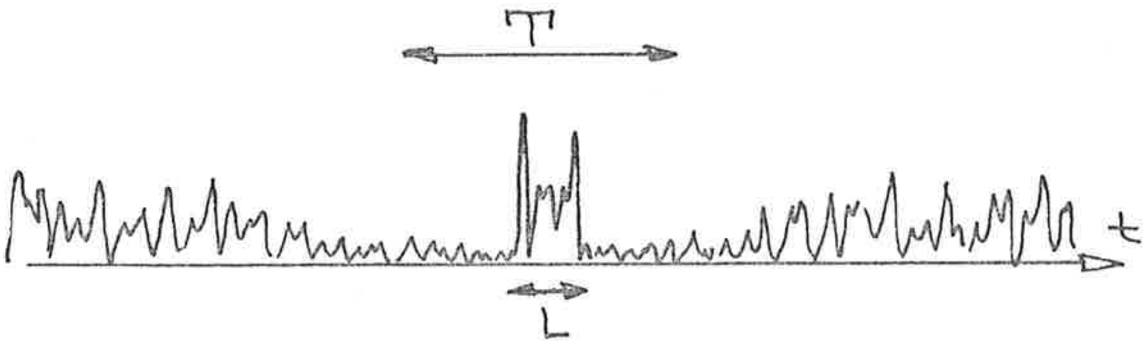
input "noise only"
output power: $\frac{M}{FT}$

input "signal+noise"
output signal power: $\frac{M}{FT} \cdot \frac{d_n}{1+\frac{d_n}{G_n}}$ (43)

output noise power: $\frac{M}{FT} \cdot \frac{1}{1+\frac{d_n}{G_n}}$

Eqs.42, 43 clearly indicate what happens in a clipper system.

Before the signal is present, the interference output power is a factor $\frac{QT}{I\ rev}$ or FT, respectively, under the correlator's maximum output M. When a signal occurs, the interference output is reduced, and the signal output will increase. The net effect is, that at the output the signal peaks appear to be situated in an interference valley. See fig.20.



20- Output time function from clipper system.

When detection is based on the signal exceeding a threshold value which has been set for the interference-only situation, we require the target peaks to exceed this threshold by approximately the same factor as in a linear system. Eqs.42, 43 indicate that this is obtained when

$$\frac{d_r}{G_r} \ll 1, \text{ or } \frac{d_n}{G_n} \ll 1, \text{ respectively or, roughly, when} \qquad (44)$$

the processing gain G is sufficiently great. Eqs.40 describe the way in which G depends on the properties of target, reverberation, noise, and on the transmission.

N.B. For random modulation pulses, the detection will sometimes be self clutter limited, rather than noise or reverberation limited This is due to the particular shape of the ambiguity function (thumb-tack type), which picks up self clutter from the target via its side lobes.

In this case, the input power is normalised with the same factor, either for the echo, or for the self clutter. Thus, the output signal -to- self clutter ratio is the same for a linear and a clipper system.

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Discussion lecture Van Schooneveld.

1. Dr. D. Nairn.

Q. - The requirement for sampling is that the repeated spectral bands do not overlap the original ones. What are the consequences of this for the minimum sampling rate? How wide must the holes between adjacent bands be for a realizable restoration filter?

A. - This depends on the filter's quality. Amplitude and phase transfer must be uniform and linear, respectively, over the pass band. Further, the skirts must be sufficiently steep to keep the holes between adjacent bands and, consequently, the minimum sampling rate, within reasonable limits. These requirements are contradictory. The final trade-off will depend on the amount of effort, spent on the filter's design.

Q. - It has been said that the minimum theoretical rate $R \simeq 2B$ should be replaced, in practice, by rates in the order of 5B to 7B.

A. - In reference 4 of the literature list ("a sampling theorem for narrow band signals", Tijdschrift NERG, vol 32, No. 1, 1967), the author describes a restoration experiment with a signal of 270 Hz, centered around 5125 Hz. Hence, 2B = 540 samples/sec. Perfect signal restoration (correlation between original and restored signal 0,99 or better) was obtained using a rate of 760 samples/sec., i.e. $R \simeq 2,8.B$, rather than 2B. Hence, a rate of 5B to 7B is unnecessarily high for many applications.

2. Dr. P.L. Stocklin.

Q. - Sometimes, one does not want to restore the original signal. In such a case, the sampling rate is increased to the region of 5B to 7B in order to be able to observe the detailed signal structure.

A. - The author agrees. This is the required rate when no restoration is done, at least for low-pass signals. In other cases, however, one is forced to restore. One example is the correlator of fig. 9, where an input signal for an envelope detector must be generated. The interpolation property of the restoration filter can then be exploited to reduce the sampling rate to values in the order of 3B samples/sec.

3. ....

Q. - A potential danger of sampling a narrow band signal is the "blind phase" problem, due to sampling synchronously with the carrier frequency.

A. - This is true. It turns out, however, that these dangerous rates are in the inadmissible intervals for the sampling rate (fig. 5). As soon as one chooses a sampling rate inside one of the admissible intervals, the danger of synchronous sampling disappears automatically.

4. Prof. R. Griffiths.

Q. - Has non-uniform sampling been applied?

A. - No, because the technology of uniform sampling is easier, and because the effective number of signal samples would still remain the same.

5. ....

Q. - What happens when the reference of a correlator is clipped, but the signal is not?

A. - Essentially, we now have the linear correlator system again. The signal will only correlate with the 1^{st} harmonic of the reference. The problem of avoiding interaction between the signal and the higher harmonics of the reference is simple, compared with the case where both time functions are clipped, since the number of repeated bands on the frequency axis of the multiplied time function (when clipping is combined with sampling) is much lower.

6. R. Unger.

Q. - When the correlator's output peaks are in an interference valley, due to the clipper's self normalization, it would be possible to adapt the detection threshold to the varying output level.

A. - Yes, this is certainly possible. In the present paper, however, the author has restricted himself to constant thresholds, set for the interferenceonly situation.

7. Dr. E.J. Risness.

Q. - The author has discussed the problem of incoherent integration in the frequency domain. There is also the problem of incoherent integration in the range dimension, and with certain signals there is an interaction between the two types of integration. Would the author comment on this?

A. - For F.M. signals, the two types of integration are related, due to the particular shape of the F.M. ambiguity function.

Q. - But with other signals, one can distinguish between incoherent integration in range and doppler. It's easy to see how the problem can be extended.

In retrospect, the author wishes to give the following comment:

A. - There are two, theoretically different, ways of obtaining incoherent integration in the frequency (doppler) domain:
1 - widening of the integrator's passband by decreasing the duration of coherent integration. This is the method described in the paragraph on mixed integrators. It is equivalent to the use of a new reference function, which is the sum of a number of the original reference functions, slightly shifted in frequency.
2 - leaving the original channel bandwidth at its minumum

value $(\frac{1}{T})$, but combining a number of adjacent doppler channels incoherently. after having removed the signal's phase information.

In the same way, there exist two corresponding methods for incoherent integration in the delay (range) domain:
1 - The use of a new reference function, which consists of the sum of a number of original references, slightly shifted in range, followed by phase removal after correlation.
2 - The use of the original reference function, and incoherent combination of the outputs from adjacent range bins after phase removal. This is the normal form of post-correlation-integration.

8. Dr. D. Nairn.

Q. - Is it desirable to add a few bits to a fully clipped system in order to obtain a modest degree of linearization?

A. - Yes. Actually, this is one of the few things one can do when not satisfied with the behaviour of a clipped system. In itself, 3 or 4 bits will already give a significant increase in performance. However, a serious side problem will be to keep the input signal voltages within the restricted aperture window which corresponds to a low-bit quantization. Sudden signal excursions will lead rapidly again to saturation (clipping) of the receiver.

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