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ON THE VALIDITY AND APPLICATION OF THE RECIPROCITY PRINCIPLE IN ACOUSTICS

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PREFACE

The development and improvement of underwater weapons and detection systems such as acoustic torpedo's, mines and sonar causes an increasing interest in the reduction of underwater sound radiated by warships. Moreover there are the ever rising standards of comfort on board of these ships.

The Royal Netherlands Navy has the assistance of the Technisch Physische Dienst TNO-TH to advise her on the problems concerned with noise reduction and noise control.

To obtain a better understanding of the transmission of sound the TPD has performed many experiments. It appeared to be advantageous to do some of these experiments in the reciprocal way. An example in this respect is the measurement of the acoustical transfer functions which describe the transfer of sound from resiliently mounted machines to the water.

The authors have investigated both the theoretical problems around the reciprocal measurement and the difficulties they met when applying this principle. The interesting results of the investigations give rise to more publicity, which purpose this report may fulfil.

IR. J. F. MEIJER
Director of Naval Construction RNL.N

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by

H. F. STEENHOEK * and T. TEN WOLDE *

Summary

The reciprocity principle is treated for acoustical-acoustical, mechanical-acoustical and mechanical-mechanical systems. Experiments showing the validity of the principle are presented. The possible advantages of reciprocal measurements are dsicussed.

1 Introduction

To solve problems in acoustics it is often necessary to obtain information on the transfer of sound from one position to another. Generally the first position will be that of a sound source (position 1), the second will be the position at which one is interested in the sound pressure (the "observation position", position 2).

In this report two types of sources will be distinguished: "acoustical" ones and "mechanical" ones. Acoustical sources directly generate a sound pressure in the surrounding medium (for instance air or water). Mechanical sources directly excite a mechanical construction. In practice many sound sources have a mixed character: they produce as well a sound pressure in the surrounding medium as vibrations in the supporting structure. Both excitations may result in a sound pressure at the observation position. Thus, generally, both must be taken into consideration.

In many cases experiments on the transfer of sound are very much simplified if a reciprocal method of measurement is employed. This means that the positions of sound source and observation point are interchanged. Reciprocal experiments are of course only useful if the relationship with direct experiments is known. Chapter 2 of this report deals with the derivation of this relationship and with a discussion of the conditions that must be satisfied. In chapter 3 the available theoretical evidence for the validity of reciprocity is briefly considered. Chapter 4 presents a number of experimental checks. Chapter 5 deals with the possible advantages of the reciprocal method of measurement.

2 Reciprocity relations

Acoustical-acoustical system

If the sound source can be considered to be a point source an acoustical-acoustical system can be treated as a four-pole (fig. 1).

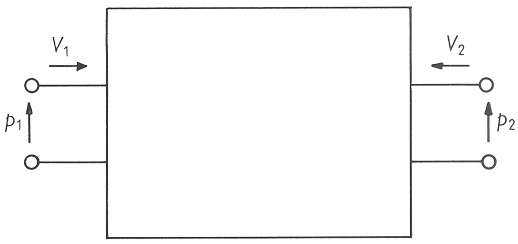

Fig. 1. Four-pole representation of an acoustical-acoustical system.

The sound pressures p_1 and p_2 are called positive if the total pressure is higher than the static pressure. The volume velocities V_1 and V_2 are called positive if the particles move away from the positions 1 and 2, respectively. All signals are supposed to be harmonic. The arrows in fig. 1 define the positive directions at the four-pole connections.

The system is supposed to contain only linear, passive elements. Under these conditions we may write:

$$a_{11}V_1 + a_{12}V_2 = p_1 \tag{1}$$

$$a_{21}V_1 + a_{22}V_2 = p_2 \tag{2}$$

Equations (1) and (2) are called "impedance equations". The coefficients a_{11}, a_{12}, a_{21} and a_{22} are the "impedance parameters".

The system is supposed to be "reciprocal", what means that $a_{12} = a_{21}$. Whether or not this is true for a particular system will be discussed in chapters 3 and 4, but at the moment we will assume reciprocity. Thus it follows:

$$a_{11}V_1 + a_{12}V_2 = p_1 \tag{1a}$$

$$a_{12}V_1 + a_{22}V_2 = p_2 \tag{2a}$$

If during a "direct" experiment V_2 is kept zero** it follows from (2a):

$$\frac{V_1'}{(p_2')_{V_2'=0}} = \frac{1}{a_{12}} \tag{3}$$

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** for practical situations: $V_2' \ll p_2'/a_{22}$

If during a "reciprocal" experiment V_1 is kept zero* it follows from (1a):

$$\frac{V_2''}{(p_1'')_{V_1''=0}} = \frac{1}{a_{12}} \tag{4}$$

Thus:

$$\boxed{\frac{V_1'}{(p_2')_{V_2'=0}} = \frac{V_2''}{(p_1'')_{V_1''=0}}} \tag{5}$$

If the sound pressures are measured with small hydrophones or microphones, the volume velocities at the particular positions can supposed to be zero and the measured sound pressures are equal to the "undisturbed" ones. So in practice the conditions $V_2' = 0$ and $V_1'' = 0$ can be satisfied.

In a similar way it follows from equations (1a) and (2a):

$$\boxed{\frac{p_1'}{(p_2')_{V_2'=0}} = -\frac{V_2''}{(V_1'')_{p_1''=0}}} \tag{6}$$

This equation is generally not very useful because of the condition $p_1'' = 0$ which is usually not satisfied.

There are two other reciprocity relations which most easily can be derived from the admittance equations instead of the impedance equations:

$$b_{11}p_1 + b_{12}p_2 = V_1 \tag{7}$$

$$b_{21}p_1 + b_{22}p_2 = V_2 \tag{8}$$

The admittance parameters can be related to the impedance parameters and it can be shown that if $a_{12} = a_{21}$ necessarily $b_{12} = b_{21}$. If so it follows:

$$\boxed{\frac{p_1'}{(V_2')_{p_2'=0}} = \frac{p_2''}{(V_1'')_{p_1''=0}}} \tag{9}$$

and

$$\boxed{\frac{V_1'}{(V_2')_{p_2'=0}} = -\frac{p_2''}{(p_1'')_{V_1''=0}}} \tag{10}$$

The last equation is in fact identical with equation (6) and generally not of interest. Also equation (9) will seldom be used.

Mechanical-acoustical system

We consider a mechanical-acoustical system which is excited by a harmonic mechanical point source. In general a mechanical point source will excite the structure with as well a force as a couple which both may cause as well translational as rotational movements. Each force, couple, translational or rotational movement can be resolved into three components. From this statement it follows that a mechanical-acoustical system can be considered to be a fourteen-pole (fig. 2).

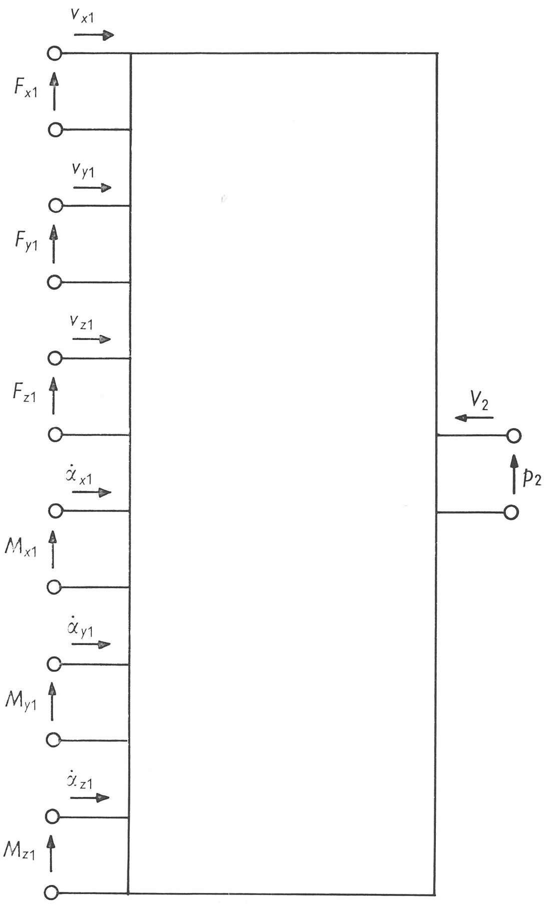

Fig. 2. Fourteen-pole representation of a mechanical-acoustical system.

Under the condition that this fourteen-pole contains only linear, passive elements the following impedance equations are valid (for nomenclature see page 13):

$$a_{11}v_{x1} + a_{12}v_{y1} + a_{13}v_{z1} + a_{14}\dot\alpha_{x1} + a_{15}\dot\alpha_{y1} + a_{16}\dot\alpha_{z1} + a_{17}V_2 = F_{x1} \tag{11}$$

$$a_{21}v_{x1} + a_{22}v_{y1} + a_{23}v_{z1} + a_{24}\dot\alpha_{x1} + a_{25}\dot\alpha_{y1} + a_{26}\dot\alpha_{z1} + a_{27}V_2 = F_{y1} \tag{12}$$

$$a_{31}v_{x1} + a_{32}v_{y1} + a_{33}v_{z1} + a_{34}\dot\alpha_{x1} + a_{35}\dot\alpha_{y1} + a_{36}\dot\alpha_{z1} + a_{37}V_2 = F_{z1} \tag{13}$$

$$a_{41}v_{x1} + a_{42}v_{y1} + a_{43}v_{z1} + a_{44}\dot\alpha_{x1} + a_{45}\dot\alpha_{y1} + a_{46}\dot\alpha_{z1} + a_{47}V_2 = M_{x1} \tag{14}$$

$$a_{51}v_{x1} + a_{52}v_{y1} + a_{53}v_{z1} + a_{54}\dot\alpha_{x1} + a_{55}\dot\alpha_{y1} + a_{56}\dot\alpha_{z1} + a_{57}V_2 = M_{y1} \tag{15}$$

* for practical situations: $V_1'' \ll p_1''/a_{11}$

$$a_{61}v_{x1}+a_{62}v_{y1}+a_{63}v_{z1}+ \\ +a_{64}\dot{\alpha}_{x1}+a_{65}\dot{\alpha}_{y1}+a_{66}\dot{\alpha}_{z1}+a_{67}V_2 = M_{z1} \tag{16}$$

$$a_{71}v_{x1}+a_{72}v_{y1}+a_{73}v_{z1}+ \\ +a_{74}\dot{\alpha}_{x1}+a_{75}\dot{\alpha}_{y1}+a_{76}\dot{\alpha}_{z1}+a_{77}V_2 = p_2 \tag{17}$$

If during a direct experiment v_{y1}, v_{z1}, $\dot{\alpha}_{x1}$, $\dot{\alpha}_{y1}$, $\dot{\alpha}_{z1}$ and V_2 are kept zero it follows from equation (17):

$$\left(\frac{v'_{x1}}{p'_2}\right)_{\substack{\text{all velocities}\\ \text{at position 1}\\ \text{zero except } v_{x1}';\\ V_2'=0}} = \frac{1}{a_{71}} \tag{18}$$

For the corresponding reciprocal experiment we require that v_{x1}, v_{y1}, v_{z1}, $\dot{\alpha}_{x1}$, $\dot{\alpha}_{y1}$ and $\dot{\alpha}_{z1}$ are zero. Then it follows from equation (11):

$$\frac{V_2''}{(F_{x1}'')_{\substack{\text{all velocities}\\ \text{at position 1}\\ \text{zero}}}} = \frac{1}{a_{17}} \tag{19}$$

If $a_{71} = a_{17}$ it follows:

$$\boxed{\left(\frac{v'_{x1}}{p'_2}\right)_{\substack{\text{all velocities}\\ \text{at position 1}\\ \text{zero except } v_{x1}';\\ V_2'=0}} = \frac{V_2''}{(F_{x1}'')_{\substack{\text{all velocities}\\ \text{at position 1}\\ \text{zero}}}}} \tag{20}$$

In a similar way it follows for instance that if $a_{72} = a_{27}$:

$$\left(\frac{v'_{y1}}{p'_2}\right)_{\substack{\text{all velocities}\\ \text{at position 1}\\ \text{zero except}\\ v_{y1}';\ V_2'=0}} = \frac{V_2''}{(F_{y1}'')_{\substack{\text{all velocities}\\ \text{at position 1}\\ \text{zero}}}} \tag{21}$$

and if $a_{76} = a_{67}$:

$$\left(\frac{\dot{\alpha}'_{z1}}{p'_2}\right)_{\substack{\text{all velocities}\\ \text{at position 1}\\ \text{zero except}\\ \dot{\alpha}_{z1}';\ V_2'=0}} = \frac{V_2''}{(M_{z1}'')_{\substack{\text{all velocities}\\ \text{at position 1}\\ \text{zero}}}} \tag{22}$$

Equations (20), (21) and (22) state that the reciprocal experiments ask for the measurement of "blocked" forces or "blocked" couples.

Besides reciprocity relations of the type of equation (20) – type 1 – three other types can be derived from the impedance or the admittance equations:

type 2

$$\boxed{\left(\frac{F'_{x1}}{p'_2}\right)_{\substack{\text{all forces and}\\ \text{couples at position 1}\\ \text{zero except}\\ F_{x1}';\ V_2'=0}} = -\frac{V_2''}{(v_{x1}'')_{\substack{\text{all forces and}\\ \text{couples at position 1}\\ \text{zero}}}}} \tag{23}$$

type 3

$$\boxed{\left(\frac{F'_{x1}}{V'_2}\right)_{\substack{\text{all forces and}\\ \text{couples at position 1}\\ \text{zero except}\\ F_{x1}';\ p_2'=0}} = \frac{p_2''}{(v_{x1}'')_{\substack{\text{all forces and}\\ \text{couples at position 1}\\ \text{zero}}}}} \tag{24}$$

type 4

$$\boxed{\left(\frac{v'_{x1}}{V'_2}\right)_{\substack{\text{all velocities at}\\ \text{position 1 zero}\\ \text{except } v_{x1}';\ p_2'=0}} = -\frac{p_2''}{(F_{x1}'')_{\substack{\text{all velocities at}\\ \text{position 1 zero}}}}} \tag{25}$$

The equations show that the reciprocal experiments require the measurement of free velocities (types 2 and 3) or the measurement of blocked forces or blocked couples (types 1 and 4).

We expect that the relations of types 3 and 4 will seldom be of interest. Types 1 and 2, however, can often be very useful as will be shown in chapter 5.

The conditions for the validity of the reciprocity relations are such that in fact the original fourteen-pole has been reduced to a four-pole. For the equations of types 1 and 4 this is the four-pole which remains if at five of the six mechanical inputs (outputs) the velocities are permanently kept zero. For the direct case this can be realized by non excitation, for the reciprocal case generally the only possibility is blocking by high impedances. For the equations of types 2 and 3 the fourteen-pole has been reduced to the four-pole which remains if at five of the six mechanical inputs (outputs) the forces and couples are permanently kept zero.

Mechanical-mechanical system

In general a mechanical-mechanical system should be considered to be a twentyfour-pole. If we assume that the system is linear and that the impedance parameters $a_{ij} = a_{ji}$, four types of reciprocity relations can be derived:

type 1

$$\boxed{\left(\frac{v'_{x1}}{F'_{y2}}\right)_{\substack{\text{all velocities}\\ \text{at positions 1 and 2}\\ \text{zero except } v_{x1}'}} = \left(\frac{v''_{y2}}{F''_{x1}}\right)_{\substack{\text{all velocities}\\ \text{at positions 1 and 2}\\ \text{zero except } v_{y2}''}}} \tag{26}$$

type 2

$$\boxed{\left(\frac{F'_{x1}}{F'_{y2}}\right)_{\substack{\text{all forces and}\\ \text{couples at position}\\ \text{1 zero except } F_{x1}';\\ \text{all velocities}\\ \text{at position 2 zero}}} = -\left(\frac{v''_{y2}}{v''_{x1}}\right)_{\substack{\text{all velocities}\\ \text{at position 2 zero}\\ \text{except } v_{y2}'';\ \text{all}\\ \text{forces and couples}\\ \text{at position 1 zero}}}} \tag{27}$$

type 3

$$\left(\frac{F'_{x1}}{v'_{y2}}\right)_{\substack{\text{all forces and}\\ \text{couples at}\\ \text{positions 1 and 2}\\ \text{zero except } F_{x1}'}} = \left(\frac{F''_{y2}}{v''_{x1}}\right)_{\substack{\text{all forces and}\\ \text{couples at}\\ \text{positions 1 and 2}\\ \text{zero except } F_{y2}''}} \tag{28}$$

type 4

$$\left(\frac{v'_{x1}}{v'_{y2}}\right)_{\substack{\text{all velocities at}\\ \text{position 1 zero}\\ \text{except } v_{x1}'\text{; all}\\ \text{forces and couples}\\ \text{at position 2 zero}}} = -\left(\frac{F''_{y2}}{F''_{x1}}\right)_{\substack{\text{all forces and}\\ \text{couples at position}\\ \text{2 zero except } F_{y2}''\text{;}\\ \text{all velocities}\\ \text{at position 1 zero}}} \tag{29}$$

The conditions for the validity of equation (26) state that at five of the six inputs at position 1 and at five of the six inputs at position 2 the velocities are permanently kept zero. Thus in fact the twentyfour-pole has been reduced to a four-pole. In the case of equation (28) at five of the six inputs at position 1 and at five of the six inputs at position 2 the forces and couples are permanently kept zero and another four-pole remains. Equation (27) asks for zero forces and couples at five inputs at position 1 and for zero velocities at five inputs at position 2. Equation (29) requires zero velocities at five inputs at position 1 and zero forces and couples at five inputs at position 2. Thus the four types of reciprocal relations are valid for four different mechanical-mechanical four-poles! The types 1, 2 and 3 are essentially different, type 4 is in fact identical with type 2 (interchange position 1 and 2 and the direct and the reciprocal experiment).

3 Theoretical evidence for the validity of reciprocity

Acoustical-acoustical system

HEMHOLTZ [1] was the first to point out the existence of the reciprocity relation as given in equation (5) for certain types of acoustical-acoustical systems. Substituting the wave equation in GREEN'S formula he showed that equation (5) is valid for the system between two points in a space filled with a homogeneous medium if the boundaries of the space are rigid or infinitely far away. By the same method it can also be shown that the relation still holds if the boundaries of the space are locally reacting and if the space contains rigid bodies [2].

Lord RAYLEIGH [3] indicated that the principle is still valid if the medium is not homogeneous, if the system contains elastic bodies and if the boundaries are elastic. LYAMSHEV [4] supported these statements. He showed that the presence of different media is allowed and that elastic boundaries and bodies do not disturb the validity of equation (5).

Lord RAYLEIGH furthermore stated that linear dissipation in the system is probably also allowed [3, chapter V, p. 111].

In [5] JANSSEN has treated porous sound absorbing materials with respect to reciprocity. On the basis of a certain set of differential equations he showed that the acoustical reciprocity principle was violated in this case. However, it can be shown* that the set of equations employed by JANSSEN was not fully correct and that also porous sound absorbing materials behave reciprocally.

Due to the work of especially RAYLEIGH and LYAMSHEV there is great evidence that the reciprocity principle will hold for linear, passive, acoustical-acoustical systems which, as RAYLEIGH says, "vibrate about a configuration of stable equilibrium". A number of checks is presented in chapter 4.

Mechanical-acoustical and mechanical-mechanical systems

CHERTOCK [6] proved the validity of a reciprocity relation for the mechanical-acoustical system between a position on a vibrating surface and a point in the adjacent medium. Also the reciprocity has been demonstrated for the case of heaving, rolling, pitching, etc. motions of a floating body excited by a train of surface waves [7].

Lord RAYLEIGH [3] proved that linear, passive, dynamic systems are reciprocal if they have a finite number of degrees of freedom. The number of degrees of freedom is the number of co-ordinates necessary to define the displacements in the system.

We suppose that RAYLEIGH'S reasoning is correct but some doubt may remain concerning the possible influence of damping, flow and gyroscopic movements (see [3, chapter IV, p. 104]). Consequently it was thought to be useful to check also some mechanical-acoustical and mechanical-mechanical systems on reciprocity.

4 Experimental checks

Methods of measurement

For checks on the reciprocity of

acoustical-acoustical systems

two different methods were employed:

<u>method A1</u>

The positions of the electrodynamic sound source and the receiving element (hydrophone or microphone)

* This statement is based on private communications with D. W. VAN WULFFTEN PALTHE and J. H. JANSSEN.

were interchanged. In both set-ups the same electric current was supplied to the sound source. The criterium for reciprocity is that the detected sound pressures in both set-ups are the same. This method is only valid if the sound source and the hydrophone (or microphone) have the same directivity and orientation. Generally it will be required that both transducers are omni-directional. Furthermore the hydrophone (or micro-phone) should have a high acoustical input impedance and the sound source should supply the same volume velocity at both positions. The latter will be realised if the sound source has a high acoustical output impedance.

method B1

The reciprocity of the electrical-electrical system be-tween the electrical terminals of an electro-acoustical transducer at position 1 and an electro-acoustical transducer at position 2 was considered (see fig. 3).

The system is reciprocal if

$$\frac{i_1'}{(e_2')_{i_2'=0}} = \frac{i_2''}{(e_1'')_{i_1''=0}} \tag{30}$$

which can be checked easily. If equation (30) is valid it is not necessary that the acoustical-acoustical system is reciprocal. This is only true if both transducers are reciprocal or both "anti-reciprocal". Anti-reciprocal means that the impedance parameters show the relation $a_{ij} = -a_{ji}$. If the representation of fig. 3 is adopted – voltage analogous to acoustic pressure and electric current analogous to dynamic volume velocity – it can be shown that electrodynamical transducers are reciprocal and electrostatic transducers anti-reciprocal. It can furthermore be shown that:

a. a series connection of a reciprocal and an anti-reciprocal four-pole results in an anti-reciprocal four-pole;
b. a series connection of two anti-reciprocal four-poles yields a reciprocal four-pole;
c. a series connection of two reciprocal four-poles results in a reciprocal four-pole.

If two electrodynamical transducers are applied it follows from these theorems that with the reciprocity of the whole electrical-electrical four-pole also the reciprocity of the acoustical-acoustical four-pole in the middle has been shown.

For checks on the reciprocity of

mechanical-acoustical systems

the two following methods were employed:

method A2

The positions of source and receiver were interchanged. During the direct experiment an electrodynamical exciter was employed at position 1 and the force or the velocity at position 1 and the sound pressure at position 2 were measured. During the reciprocal experiment an electrodynamical sound source with known volume velocity was active at position 2 and the free velocity or the blocked force at position 1 was measured (in the same direction as the excitation during the direct experiment). In this way equations (20) and (23) could be checked. A condition for the validity of this method is that "point sources" and "point receivers" are used. If equation (20) is checked, the movements at position 1 must be sufficiently blocked when the reciprocal experiment is done. If equation (23) is checked the vibration pick-up used during the reciprocal experiment should not mechan-ically load the structure at position 1.

method B2

The reciprocity of the electrical-electrical system be-tween the electrodynamical transducers at positions 1 and 2 was considered. If this system is reciprocal the mechanical-acoustical system between positions 1 and 2 is also reciprocal provided that both transducers can be considered as being a point source in the active stage and a point receiver in the passive stage.

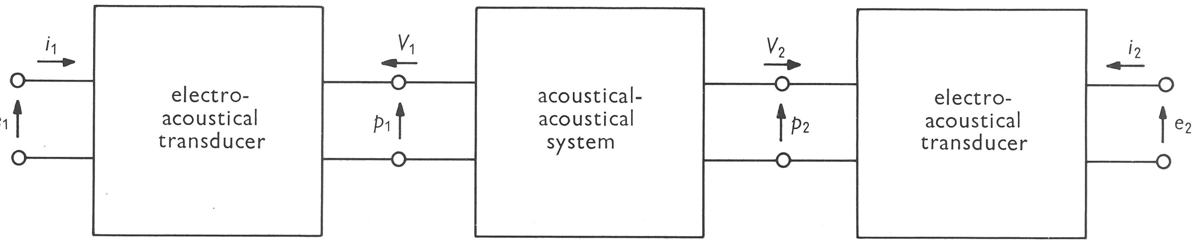

Fig. 3. Four-pole representation of the system between the electrical terminals of two electro-acoustical transducers at positions 1 and 2.

For the checks on the reciprocity of

mechanical-mechanical systems

the two following methods were used:

method A3

The positions of the exciter and the vibration pick-up (or force gauge) were interchanged. The necessary conditions are similar to those of methods A1 and A2.

method B3

The reciprocity of the electrical-electrical system between the electrical connections of two electrodynamical transducers at positions 1 and 2 was considered. The necessary conditions are similar to those of methods B1 and B2.

All experiments were limited to rms values. In most cases $^1/_3$ octave band filtered noise was used.

The application of method A2 asks for data on the volume velocity of the acoustical sound source (for method A1 only a constant acoustical source is needed and the source strength need not to be known). The value of the volume velocity of a harmonic acoustical point source can be obtained from a measurement of the radiated sound pressure in a free field. In textbooks (see for instance [8]) it has been shown that the sound pressure due to a point source in a free field is:

$$p = \frac{\varrho}{4\pi r}\frac{\mathrm{d}}{\mathrm{d}t}\left\{V\left(t - \frac{r}{c}\right)\right\} \tag{31}$$

For harmonic motions and rms values $\mathrm{d}\{V(t-r/c)\}/\mathrm{d}t$ can be replaced by $V2\pi f$ and it follows that:

$$\boxed{V = \frac{2pr}{\varrho f}} \tag{32}$$

Investigated systems

The following

acoustical-acoustical systems

were investigated (see also table 1, page 14):

a. the system between two points in a water tank (positions 1 and 2); the sound field in the water tank was reverberant but not perfectly diffuse
b. the system between a point in a ship model and a point in the air outside the ship (positions 3 and 4, respectively); the latter position was at a distance of about 3 m from the model in a reverberant but not perfectly diffuse sound field
c. the system between a point in the water tank and a point in the air outside the ship model (positions 2 and 4, respectively)
d. the system between a point in the water tank and a point in the air inside the ship model (positions 2 and 3, respectively).

All four systems appeared to be reciprocal in the frequency ranges in which reliable measurements could be performed. The lower limits of these frequency ranges were defined by the signal-to-noise ratio which was not sufficiently large at lower frequencies. Generally the upper limits of the frequency ranges were determined by the dimensions of the transducers which must be small compared with the wavelength.

In most frequency bands the direct and reciprocal results agreed within 2 dB, that is within the margin of experimental error. In frequency ranges in which the spectra showed a sharp minimum, a sharp maximum or a large slope, the deviations were sometimes as large as 5 dB. However, in these bands deviations up to 5 dB were also examined if a direct or a reciprocal experiment was repeated.

Buiten and Janssen [9] checked the reciprocity of several other acoustical-acoustical systems all being scale models of exhaust systems. The measurements were performed with discrete frequencies but again only rms values were considered. The results were very favourable.

The

mechanical-acoustical systems

which were checked on reciprocity are (see also table 2):

a. the system between a point on the hull of a 1:10 scale model of a part of a submarine and a point in the water of the lake in which the model was immersed
b. the system between a point of a steel box floating on water and a point in a reverberant water tank
c. the system between a point of a 1:10 scale model of a part of a minesweeper (constructing materials wood and aluminium) and a point in a reverberant water tank.

As indicated in table 2 all three systems appeared to be reciprocal within the margin of experimental error.

Some of the

mechanical-mechanical systems

which were investigated on reciprocity are indicated in table 3 and figures 4, 5 and 7. This set of systems approximately covers the range of structures that can be met in ships or model ships, especially with respect to resilient mounting systems.

All investigated systems appeared to be reciprocal within the limits of accuracy and repeatability.

The first results of measurements on many of the systems were not as good as indicated in tabels 1, 2 and 3, and several times doubt about the validity of reciprocity arose. Every time, however, it appeared that one or another measuring mistake was made. These mistakes could be:

a. a too high excitation signal giving rise to non-linearities in the system (in the mechanical, acoustical or electrical parts)
b. the boundary conditions were not sufficiently fulfilled; in the case of acceleration measurements on light structures using method B2 or B3, difficulties could often be overcome by considering the accelerometer to be a part of the structure; if method A2 or method A3 was applied the exciter and the accelerometer were sometimes not sufficiently mechanically equal and the investigated systems were in fact not the same during the direct and the reciprocal experiment; other mistakes originated in the dimensions of the transducers which should be small with respect to the wavelength
c. in some cases it appeared that the radiated airborne sound from a mechanical exciter caused higher levels at the observation position than the excited structureborne sound; this introduced deviations between the direct and reciprocal results for the mechanical-acoustical system which was supposed to be under investigation
d. electrical cross-talk.

Again and again we experienced that every refinement of the experimental technique improved the agreement between the direct and the reciprocal results. At present, we are fully convinced that the usual mechanical-mechanical, mechanical-acoustical and acoustical-acoustical systems as can be found in and around ships or buildings, are reciprocal.

5 Applications

Possible advantages of reciprocal experiments are:

a. simpler experiments
b. better signal-to-noise ratio.

Especially the first advantage is often very obvious if a mechanical-acoustical system is investigated. Generally at the mechanical side, which is supposed to be the source side in the direct case, six degrees of freedom must be considered. At the acoustical side only one degree of freedom is present. So direct experiments ask for the separate excitation of three translations and three rotations and the measurement of the six resulting sound pressures. The separate excitation of six vibration components is generally impossible as well on model scale as on full scale. This is partly due to space limitations at the mechanical side. But, also if sufficient space is available, independent excitation of all six components is generally extremely difficult. The corresponding reciprocal experiment is much easier: excitation of the system with one omni-directional acoustical sound source and measurement of three free translational velocities and three free rotational velocities, or the measurement of three blocked forces and three blocked couples. The independent measurement of six vibration components can be done with the aid of a set of accelerometers on a stiff base. This measuring device must be so small and light that it does not affect the structure. This can often be arranged with the aid of the modern light-weight accelerometers. Independent measurement of blocked forces and couples may be performed with a set of piezo-electric force gauges. Especially for measurements on top of springs the following method may be employed. A solid mass is shaped in such a way that its centre of gravity coincides with the position at which it is mounted on the top of the spring. On the mass a set of accelerometers is mounted and the six acceleration components (rotational and translational) can be measured. For frequencies higher than the resonant frequencies of the mass-spring system the blocked forces and couples can be derived from

$$F = m\dot{v} \tag{33}$$

and

$$M = I\ddot{\alpha} \tag{34}$$

m being the mass and I the relevant moment of inertia.

Often the situation is such that the complete investigation of a mechanical-acoustical system with the aid of direct measurements is in fact impossible while reciprocal experiments are rather easy to perform. This means that new fields of experimental investigation are opened. Especially the radiation due to rotational excitation can now be studied.

Even if direct mechanical excitation is possible it is sometimes still advantageous to use a reciprocal measuring method. This may for instance be the case if the mechanical exciter also radiates a considerable amount of airborne sound. This airborne sound instead of the structureborne sound may determine the sound pressure at the observation position and wrong conclusions may be drawn. The corresponding reciprocal experiment is safer because it is guaranteed that only one way of excitation occurs.

Another reason why reciprocal measurements may be easier is a better signal-to-noise ratio. This is dependent on the available equipment and the background noise at both positions. The latter is of course dependent on the position and strength of the sources which cause the background noise.

In general it is useful to consider the possibilities of as well the direct as the reciprocal measuring method. Which one will be applied depends on the available equipment, the available space at both positions and on the origin of the background noise. Often it will be sensible to apply both methods in order to obtain a check on the results by an independent measuring method.

We have experienced that the reciprocal measurement technique is extremely useful in many model and full scale experiments especially with respect to the investigation of resilient mounting systems.

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NOMENCLATURE

a_{11}, a_{12}, a_{22}, etc.	impedance parameters
b_{11}, b_{12}, b_{22}, etc.	admittance parameters
c	velocity of sound
e_1, e_2	voltage at positions 1 and 2
f	frequency
F_{x1}, F_{y1}, F_{z1}	forces at position 1 in directions X, Y, Z
F_{x2}, F_{y2}, F_{z2}	forces at position 2 in directions X, Y, Z
i_1, i_2	current at positions 1 and 2
I	moment of inertia
m	mass
M_{x1}, M_{y1}, M_{z1}	moments of couples at position 1 about axis X, Y, Z
M_{x2}, M_{y2}, M_{z2}	moments of couples at position 2 about axis X, Y, Z
p	sound pressure
p_1, p_2	sound pressure at positions 1 and 2
r	distance to sound source
t	time
v_{x1}, v_{y1}, v_{z1}	translational velocities at position 1 in directions X, Y, Z
v_{x2}, v_{y2}, v_{z2}	translational velocities at position 2 in directions X, Y, Z
$\dot{v}$	translational acceleration
V_1, V_2	volume velocities at positions 1 and 2
$\dot{\alpha}_{x1}$, $\dot{\alpha}_{y1}$, $\dot{\alpha}_{z1}$	angular velocities at position 1 about axis X, Y, Z
$\ddot{\alpha}$	angular acceleration
ϱ	density
$'$	single dash: indicates a direct experiment
$''$	double dash: indicates a reciprocal experiment

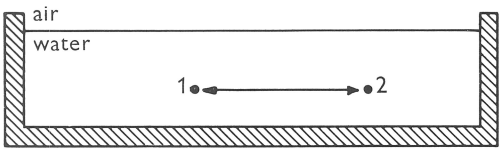
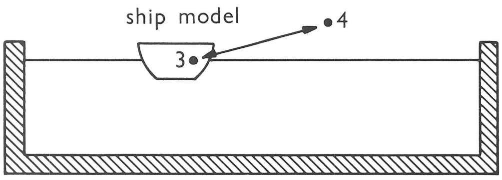
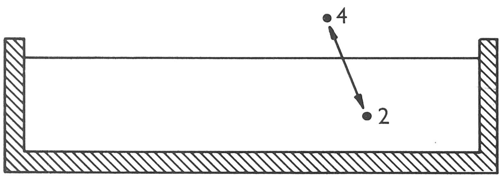
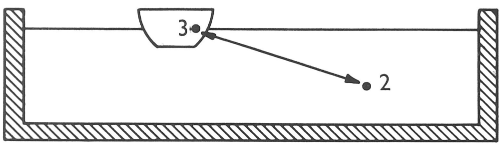
	system	method	frequency range Hz	agreement within
1		A1	25 . . . 4000	1 dB
2		B1	100 . . . 4000	2 dB
3		B1	100 . . . 1600	2 dB
4		B1	100 . . . 1600	2 dB

Table 1. Investigated acoustical-acoustical systems.

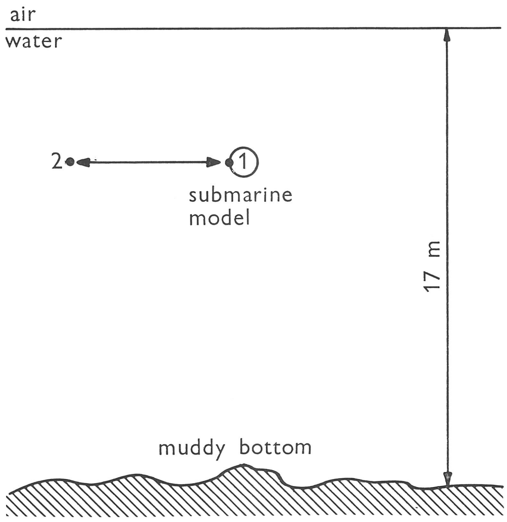
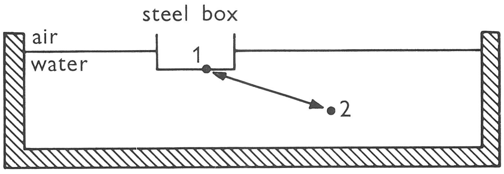
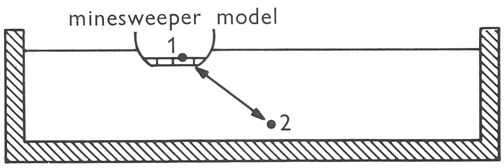
	system	method	frequency range Hz	agreement within
1		A2	200 . . . 3150	5 dB
2		B2	125 . . . 10 000	3 dB
3		A2	250 . . . 8000	5 dB

Table 2. Investigated mechanical-acoustical systems.

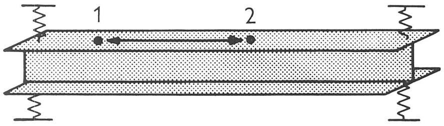
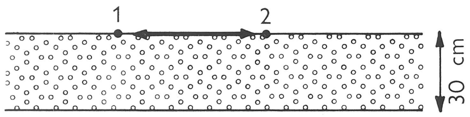
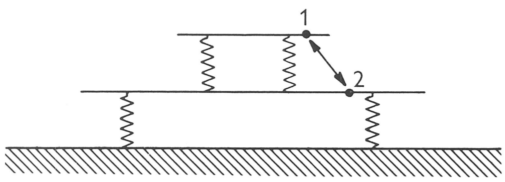
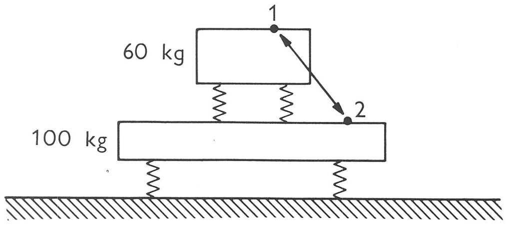
	system	method	frequency range Hz	agreement within
1	steel bar  supported by air springs	A3	25 . . . 40 000	3 dB
2	 concrete floor	A3	25 . . . 40 000	3 dB
3	see figure 4	B3	25 . . . 40 000	1 dB
4	see figures 5 and 6	A3	25 . . . 40 000	4 dB
5	 two, 2 mm thick steel plates; rubber springs	A3	1000 . . . 40 000	2 dB
6	 two steel blocks; rubber springs	B3	25 . . . 40 000	4 dB
7	see figure 7	B3	25 . . . 6 000	2 dB

Table 3. Investigated mechanical-mechanical systems.

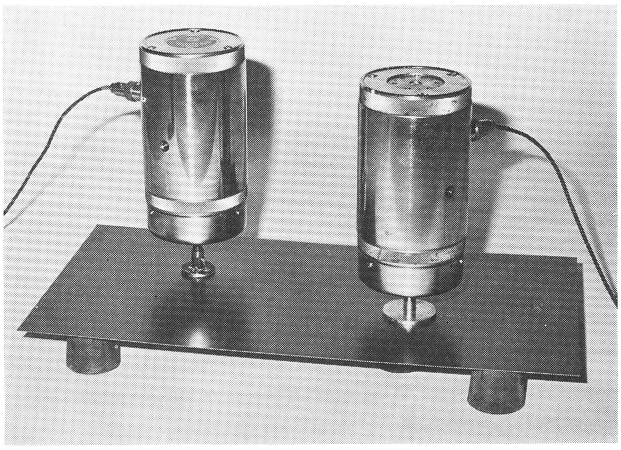


Fig. 4. Mechanical-mechanical system 3: thin steel plate on rubber springs. The metal feet on which the electrodynamical exciters were mounted were considered to be parts of the system under investigation.

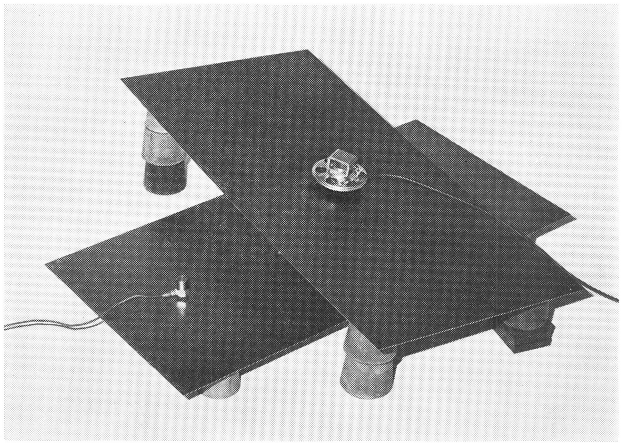


Fig. 5. Mechanical-mechanical system 4: two thin steel plates on rubber springs. The transmission of sound from one plate to the other mainly occurred via the air. When the photograph was taken an electrodynamical exciter was mounted on the top plate and an accelerometer on the lower plate. The exciter was in fact a small loudspeaker with a metal block glued to the voice coil. This block was equal in size and mass to the accelerometer (see also fig. 6). The metal block and the accelerometer were considered to be parts of the mechanical-mechanical system under investigation and this system did not change when the positions of source and receiver were interchanged.

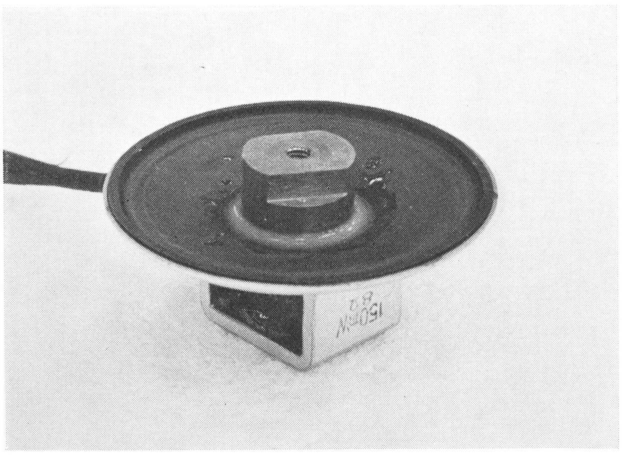


Fig. 6. The "loudspeaker-exciter".

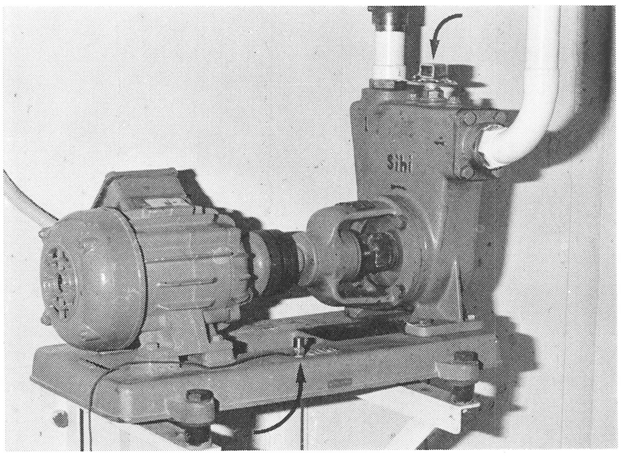


Fig. 7. Mechanical-mechanical system 7: a small water pump. Again the loudspeaker-exciter was used.

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