PROCEDURE FOR TRACKING MANOEUVRING TARGETS WITH A MULTI-PURPOSE PHASED-ARRAY RADAR SYSTEM

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INTRODUCTION

This paper describes a tracking procedure for the FUCAS-experimental multi-purpose phased-array system. A phased-array system is always very complex. Upto this moment there are just a few systems, the results of experiments with those systems are not yet available. FUCAS is developed to acquire experience and to get insight in the problems, the possibilities and the limitations (Van der Spek (1)).

A phased-array system has many advantages in comparison with conventional radar systems. These advantages are the result of two properties of the antenna:

1. The possibility to direct the main lobe of the beam without inertia into any direction within the observable cone.
2. The possibility to change the shape of the antenna pattern when necessary (e.g. multi-beam, broad main lobe).

One of the advantages is the possibility to combine radar tasks which usually have to be carried out by several radars because the requirements for those tasks differ too much (compare search radar and tracking radar). Another feature is the possibility to maximize the quantity of useful radar information. During search the beam direction and the dwell time per beam can reflect the threat pattern. In particular there is no obligation to spend time and energy into directions in which there is no interest. While designing a data handling system for a conventional radar system we have to deal with a constant transmission rate and use that information to get results which are as good as possible. In a phased-array system the transmission rate for a task is controllable. When a task will be carried out as good as possible it may need all the observations possible and another task can not be carried out at all. It is obvious that this is not the operation mode in which the special facilities of a phased-array system are used optimally. Therefore the design philosophy is different; it can be summarized in the following statement:
"Do not carry out the radar tasks as good as possible, but as good as necessary with as least effort as possible".

REQUIREMENTS AND PERFORMANCE MEASURES

The consequencies of the statement about the task: "Track and Target" are: Track the target as accurate as necessary but minimize the number of target-observations and the load on the data handling system. These requirements are in conflict with each other. Tracking a manoeuvring target with the smallest number of observations can only be realized using a sophisticated tracking algorithm and such an algorithm requires a considerable load on the data processing part of the system. In what way these two aspects are balanced depends on the use of the system and the capacity of the radar and data handling system.

How difficult it is to track a certain target depends on two time-dependent processes.

1. The signal to noise ratio of the target returns which determines the measurement accuracy.
2. The manoeuvrability of the target.

The time dependency of the degree of difficulty is expressed by the change of the number of observations and the computer load.

Choice of performance measure

The performance measure for the position estimation algorithm is: "Minimize the maximal predicted position error". This choice is based on the following considerations: When the algorithm must obtain some performance with minimal effort it has to operate optimal in some sense. To find a performance measure the purpose of the algorithm has to be known. In a Track-While-Scan system for instance a new observation of the target is obtained independently of the tracking algorithm but in this phased-array system the result of the tracking algorithm is used to steer the pencil beam towards the expected target position at the next observation moment so the performance measure has to be related to the predicted position accuracy. After the target motion-state has been estimated, a prediction-box is calculated which is centered around the predicted position. The prediction-box is a volume inside which a possible return is correlated with the track, this means that the return is expected to be coming from the target in track. The size of this volume is chosen as small as possible to reduce the influence of false alarms and to obtain a high separation of neighbouring targets. On the other hand the prediction-box should not be too small for miss-probability has to be very small. A miss means that no correlated return is obtained, when it occurs the probability of loosing a track will increase significantly. There are three possible causes for a miss. The return can be too weak and no detection is possible. The difference between the predicted and the actual position is too large. In this case it is possible that the target is not detected because the beam direction differs too much from the target direction or it is detected but the target lies outside the prediction-box. The third possible cause is the observation error. When the errors are larger than the prediction-box the return will not be correlated to the track. When a return is correlated it is not very important how large the distance is between the search direction (predicted direction) and the actual one, for it is possible to obtain an estimate of the real position within the whole volume using the monopulse facility. So the maximal prediction error has to be kept within the prediction-box and this box has to be as small as possible.

Choice of Prediction Error Size

The size of the prediction-box depends on the sum of the observation errors and the prediction errors. Therefore the choice of the allowable size of the prediction error influences the prediction-box size. Tracking a specific target in a specific environment the prediction error size is connected directly with the number of used observation samples. Both the error and the number of samples have to be minimized. This is one of the many situations where no optimal choice is possible and a trade off has to be made between accuracy and system load. In FUCAS the maximal prediction error is chosen to be a constant times the maximal observation error so the prediction-box size is a constant times the minimal possible size. That minimal size is defined by the observation errors. When the observation errors decrease than the prediction error have to decrease too, but underneath some values the variance of the prediction errors is kept constant: the prediction is then accurate enough.

Balancing the Radar and Computer Load

As mentioned in the introduction there should also be a trade off between radar load, the number of observations, and the computer load or the complexity of the estimation procedure. In this concept an attempt has been made to simplify the algorithms as much as possible while the increase of the radar load needed is still small.

TARGET MODEL

One of the problems in tracking is the diversity in target trajectory characteristics. Therefore it is difficult to find a target-motion model which has all the important features and is not too complex. In literature several models have been introduced. In most models the target-motion is described by a random process for each dimensions where the three dimensions are supposed to be independent, for instance Singer (2). One type is a two stage chain driven by correlated noise, another type is a three stage chain driven by uncorrelated noise. In fact in both types the acceleration of the target is supposed to be correlated in time. In the mean these types fit well but one of the disadvantages is that the time-dependency of the acceleration process is neglected. Of course the acceleration is highly correlated apart from periods which are sharply edged at the beginning or end of a manoeuvre. Those acceleration discontinuities are sources of tracking problems. To obtain a model which describes that situation well, a two stage chain is chosen driven by a stepwise changing input. During a certain period of time the acceleration is constant. The model is described in formula (1).

$$\begin{bmatrix} X \\ \dot{X} \end{bmatrix}_{n+1} = \begin{bmatrix} 1 & T_{n+1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ \dot{X} \end{bmatrix}_{n} + \begin{bmatrix} 0 \\ T_{n+1} \end{bmatrix} g_n \quad \dots\dots\dots(1)$$

$$g_n = \dots, g_1, g_1, g_1, 0, 0, 0, g_2, g_2, g_2, g_2, \dots$$

T_{n+1}: time between observation n and n+1

TRACKING PROCEDURE

The estimation of the motion-state will be described for one dimension.
The estimator contains the target trajectory model. Using this model an estimate is made of the target position at the next observation moment (see Fig. 1). The difference between the predicted and the observed position (the residual) is the input of a control-box which consists of a part which controls the model input (the gain matrix) and a part which controls the observation interval. In the design the gain matrix only changes during the track-initiation phase. This is the period from a positive search-return which is the germ of a track, until the transient effects of the beginning of the track have faded away. The tracking filter was simplified this radical to save computing time. That the constant gain matrix does not affect the optimal operation of the filter too much is based on the fact that a heavy smoothing (small bandwidth) is not necessary, while the relative reduction of the prediction-box is consequence of the reduced influence of the observation errors will be small and the possible influence on occurring dynamic errors will grow fast. On the other hand a large bandwidth, necessary to obtain a high manoeuvre capability, can be obtained by increasing the observation frequency.

Position Estimation

The position estimation is described in the following formulas:

$$Y_n = H.\underline{X}_n + V_n \dots\dots\dots\dots\dots\dots\dots\dots(2)$$

$$\underline{X}_{n,n} = \underline{X}_{n,n-1} + K(Y_n - H.\underline{X}_{n,n-1}) \dots\dots\dots\dots(3)$$

$$\underline{X}_{n+1,n} = \emptyset.\underline{X}_{n,n} \quad \dots\dots\dots\dots\dots\dots\dots\dots(4)$$

$$H = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad \emptyset = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \quad K = \begin{bmatrix} \alpha \\ \beta/T \end{bmatrix}$$

$$\underline{X}_n = \begin{bmatrix} X \\ \dot{X} \end{bmatrix}_n \quad V: \begin{matrix} \{E\, V_n\} = 0 \text{ for any } n \\ \{E\, V_n . V_m\} = \delta_{n,m}\sigma^2 \text{ for any } n,m \end{matrix}$$

$\underline{X}_n$: target motion state at moment -n-

Y_n : observation at moment -n-

$\underline{X}_{n,m}$: estimate of $\underline{X}_n$, using data up to and including Y_m

V_n : observation error sample at moment -n-

T : observation interval

The alpha and beta values after initiation are calculated in such a manner that the maximal prediction error will be minimized while supposing a quasi stationary behaviour of the observation interval. The filter is linear, so the error response on an observation sequence will be the sum of the responses on the observation noise and the dynamic errors. Using the described model the maximum error due to the trajectory will occur in consequence of an acceleration step. The error response on a unit acceleration step is:

$$E(t) = \frac{2}{\beta}\left[1-(1-\alpha)^{t/2T}(\cos\omega t + K\sin\omega t)\right] \dots\dots\dots(5)$$

$$\omega = 1/T \arccos\left(\frac{1-\frac{\alpha+\beta}{2}}{\sqrt{1-\alpha}}\right) \dots\dots\dots\dots\dots\dots(6)$$

$$K = \frac{\alpha}{\sqrt{4\beta-(\alpha+\beta)^2}} \quad \dots\dots\dots\dots\dots\dots\dots\dots(7)$$

The errors due to the observation noise represent a gaussian noise process with variance:

$$\sigma_o^2 = \sigma^2 . \ \mathrm{VRR}(\alpha, \beta) \dots\dots\dots\dots\dots\dots\dots(8)$$

The function VRR is the Variance Reduction Ratio, it is defined as the variance of the output noise when the input is gaussian white noise with unity variance. The maximum noise error is stated to be -a- time the standard deviation. When a is larger then 2.5 the probability of crossing that value is very small (less than .0124). The maximum prediction error is the sum of the two components and can be written as:

$$F_{max} = a.\sigma\sqrt{\mathrm{VRR}(\alpha, \beta)} + f(\alpha,\beta)\,\frac{gT^2}{2} \dots (9)$$

The first term on the right hand side is the observation noise component and the second term the component due to a manoeuvre with acceleration -g-. $f(\alpha,\beta)$ is the solution of formula 5 when t is the time at which the maximum occurs.
The alpha and beta values are calculated so that $F_{max}/a\sigma$ is minimized. The results are shown in Fig.2. The alpha and beta values are still a function of a parameter P.

$$P = \frac{\frac{1}{2}\, gT^2}{a.\sigma} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

This parameter has the same function as the quotient of the standard deviations of the dynamic and observation errors in a kalman filter (Singer (2)). The maximal value of the prediction error versus P is shown in Fig. 3 and versus α in Fig. 4.The minima of the curves for constant P are the optimal values.

The adjustment of P depends on the expected observation accuracy, the manoeuvring capability of the occurring aircraft and the maximal observation rate. Suppose that a standard deviation of the prediction error of half the standard deviation of the observation errors is allowable then P has to be 10^{-3} (see Fig. 3). When the value $\frac{1}{2}\, gT^2/a\sigma$ equals 10^{-3} the filter operates optimal. The filter is not very sensitive for a mismatch between that value and P. It can change a factor 30 while the error is still within 25% of the best reachable. Fig. 4 shows the flat optima of the filter.

Adaptation Algorithm for the Observation Interval

The observation interval T influences the prediction incertainty due to the dynamic errors (errors caused by the target acceleration).
The change of the uncertainty due to the observation error is a second order effect; only the change of T during the effective filter length influences that uncertainty. The dynamic errors are proportional to gT^2 where g equals the target acceleration. The adaptation algorithm controls T in such a manner that the dynamic errors are kept constant. The dynamic errors results in a bias of the residual because of the correlated acceleration. That bias is estimated and compared with a norm. The difference between bias and norm is used to obtain the value with which T has to be multiplied to get the new interval.

The position estimation will operate near the optimal value of P when the norm is chosen properly. In fact the numerator of the quotient $\frac{1}{2}\, gT^2/a\sigma$ is kept constant. P changes only due to the occuring standard deviation of the observation noise σ and the dynamic range of that value is small.

Manoeuvre Detection

The adaption of T after the start of an high -g- turn is not fast enough, so it is possible that the prediction error will become too large. A manoeuvre detection has to guard against that situation. It consists of two steps. First a threshold crossing of the residual occurs. When this happens the next observation interval is executed as fast as possible. When that observation has a residual less than the threshold the old value of T is used again, when it is larger than the threshold T is forced to the minimal value and from that point T converges to a new point of equilibrium.

No Correlated Returns

When no correlated return has been obtained (miss) the next observation interval will be executed with minimal value for T. A new position prediction will be calculated and the prediction-box is enlarged to keep the "miss-probability" constant. This will be repeated, if necessary, an adjustable number of times. When still no correlated returns is obtained the track is killed.

RESULTS

Fig. 5 shows the results of a test program. The value of the estimated input, the residual, the prediction errors and the observation frequency are plotted versus time. At the beginning the observation frequency decreases after the initiation. The spikes in the observation frequency plot are due to single excursions of the turn-detection threshold.
The trajectory starts with a straight line parallel to the X-axis followed by a 180 degree turn with normal acceleration of 10 m/s^2. At t = 180 sec. another turn starts. It is a 3 g-turn (30 m/s^2).

Results of the real system are not yet available for it will not be completed before January 1978.

REFERENCES

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3. Singer, R.A. and Belinke, K.W., 1971, Transactions on Aerospace and Electronic Systems, Vol. AES-7, 100-110.

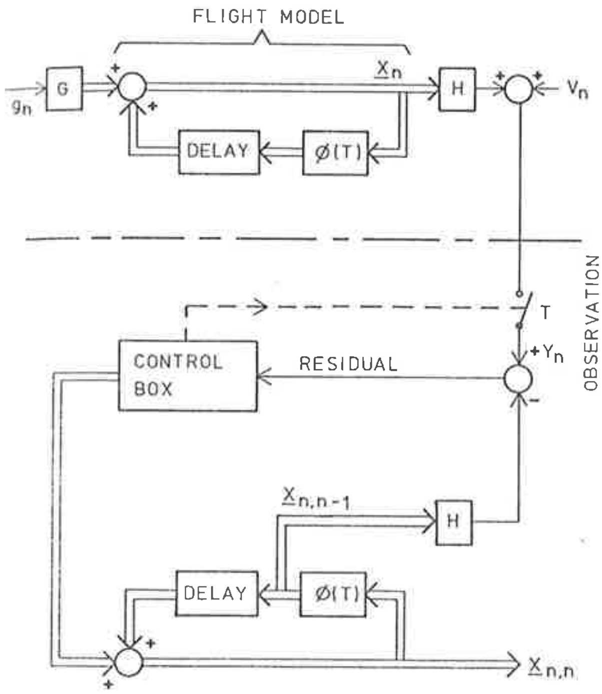

Figure 1 Block scheme of estimator.

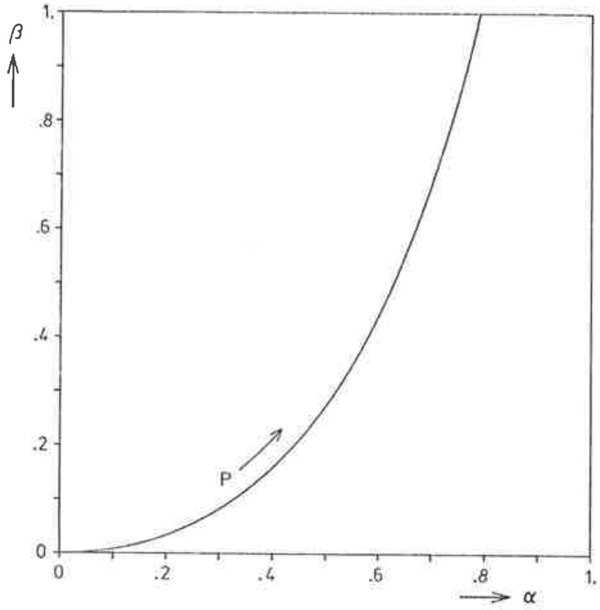

Figure 2 αβ-combinations with minimax transient-error response.

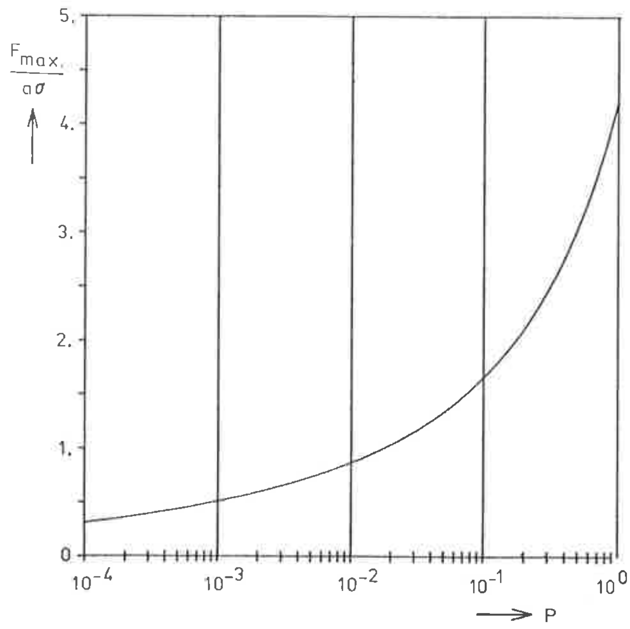

Figure 3 Relative F_{max} versus P.

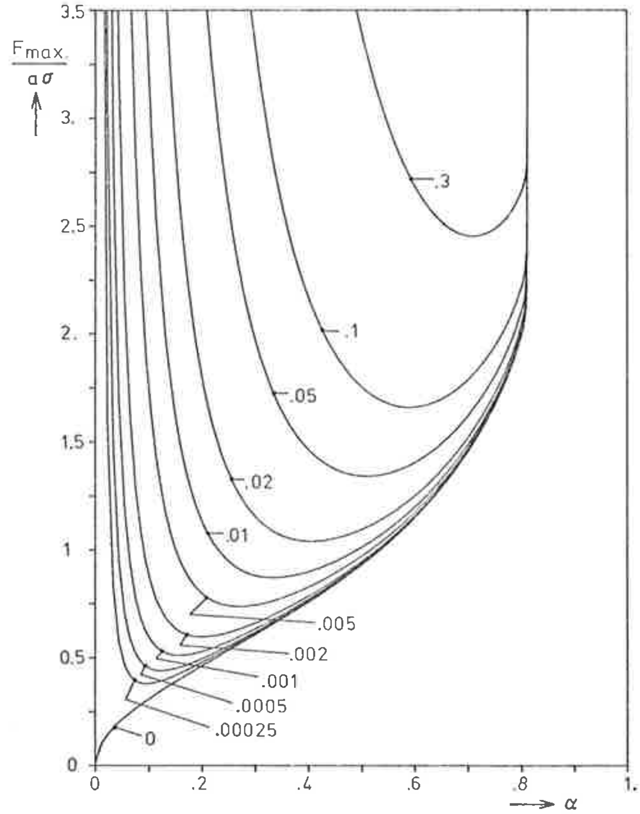

Figure 4 Relative F_{max} versus α for constant values of P.

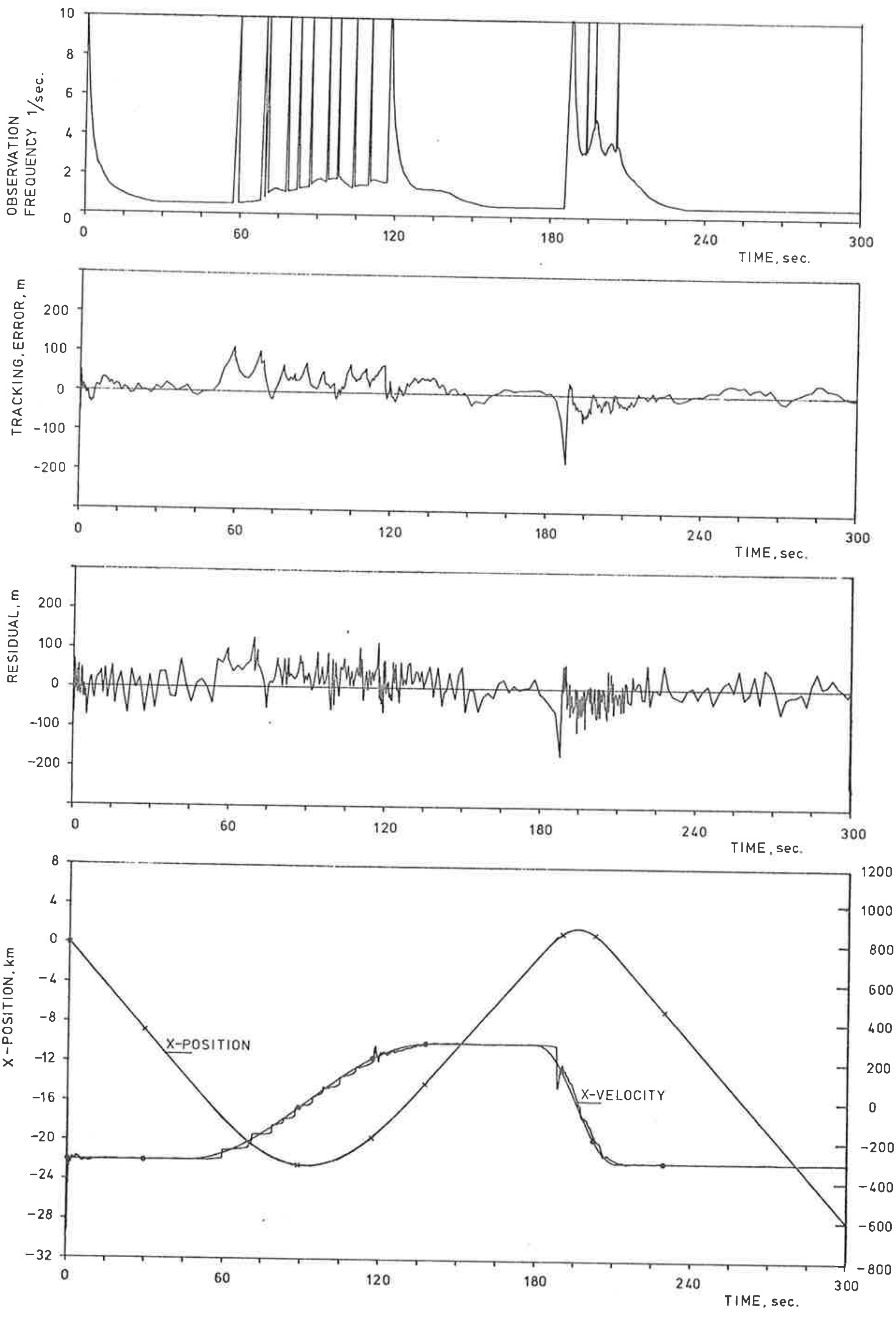

Figure 5 Output of Testprogram.