Time analysis and processing of FM-CW SAR signals

A. Meta, P. Hoogeboom
International Research Centre for Telecommunications-transmission and Radar
Delft University of Technology, department of Electrical Engineering
Mekelweg 4, P.O. Box 5031, 2600 GA Delft, The Netherlands
Phone: +31152786257, Fax: +31152784046, E-mail: a.meta@irctr.tudelft.nl

Abstract *– Combining frequency modulated continuous wave (FM-CW) technology with synthetic aperture radar (SAR) methods leads to a cost-effective, high resolution imaging radar for small-scale applications. There is a growing interest in miniaturized versions of such systems. The radar delivers its output in the frequency domain rather then in the time domain so special processing algorithm, which accounts for the typical characteristics of FM-CW radar has to be used. A time domain analysis of a FM-CW SAR signal is derived and a first processing algorithm is presented.*

Introduction

For airborne earth observation applications, there is a special interest in cost effective, high resolution imaging radars with fast image generation capability and small enough to be mounted on a small, possibly unmanned, airborne platform. Examples of such applications are in monitoring of land use, water heights, but also military, e.g. observation of enemy lines with Unmanned Aerial Vehicles (UAV's). Imaging sensors based on pulsed radar technology are generally too heavy or too expensive for daily use. Frequency modulated continuous wave (FM-CW) radars, however are usually more compact and less expensive. An FM-CW Synthetic Aperture Radar (SAR) combines the advantage of FM-CW technology and SAR methods, leading to small, cost-effective imaging radar of high resolution. A demonstrator system is currently realized at IRCTR based on off-the-shell technology, [1-3]. An important issue in SAR is the signal processing. Processing algorithms for compressing the raw data in range as in azimuth direction have to be used. Existing SAR algorithms will not be readily applicable, mainly because pulse radars deliver signals in the time domain, whereas FM-CW radars deliver signal in the frequency domain. A description of the FM-CW signal for a SAR configuration is derived.

1. Basic FM-CW principle

Frequency modulated continuous wave radar is a technique for obtaining range information from a radar by frequency modulating a continuous signal, and thereby applying a timing mark to the carrier. In a linear FM-CW signal the modulated phase is:

$$\phi = 2\pi\left(f_0 t + \frac{1}{2}\alpha t^2\right) \tag{1}$$

where f_0 is the carrier frequency and α is the chirp rate. The echo from a single target at a range R will arrive at the receiver with a time delay of $\tau = 2R/c$, where c is the speed of light. When the received signal and the transmitted one are mixed, a beat frequency f_b is produced, which is proportional to the range, [4]:

$$f_b = \frac{2R}{c} \cdot PRF \cdot df \tag{2}$$

where PRF is the pulse repetition frequency and df is the RF frequency excursion. The product of PRF and df is equal to the chirp rate α. If the target (or the radar) is moving an additional Doppler frequency $f_d = 2v_{rt} / \lambda$, where v_{rt} is the radial velocity and λ the carrier wavelength, shifts the beat frequency.

2. SAR geometry

The generation of SAR images requires the coherent combination of the echoes recorded. The movement of the SAR platform with a certain constant speed introduces a Doppler effect, which is implicitly exploited in enhancing the image resolution in the direction of the movement. The geometry of the stripmap SAR system considered is shown in fig. 1: the earth surface is represented by the *xy*-plane and the radar sensor is moving with speed v along the straight flight trajectory $\{(x,y,z) \mid y=0, z=h\}$, where h is the altitude of the aircraft above the earth surface. The platform's *x*-coordinate is related to the time variable t according the relation $x = vt$; the time variable interval is chosen to be $-T/2 \le t < T/2$, where T is the SAR integration time, that is the time the target is in the antenna beamwidth. The distance between the antenna position and the point target is a function of the time and denoted with $R(t)$, while the nominal range R_0 is defined as the closest distance and in this case (no squint angle) $R_0 = R(0)$. The distance $R(t)$ can be expressed as, [5]:

$$R(t) = \sqrt{R_0^2 + x^2} = R_0 + \frac{x^2}{2R_0} - \frac{x^4}{8R_0^3} + \ldots \approx R_0 + \frac{v^2 t^2}{2R_0} \tag{3}$$

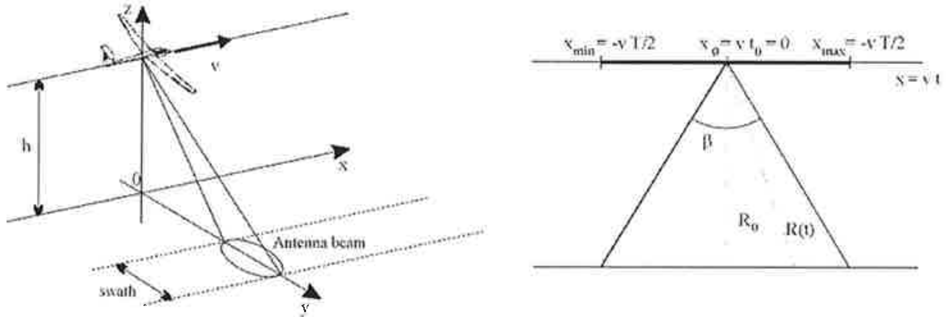


Figure 1. Geometry of the SAR system. The point target is in the antenna beam as long as x-coordinate of the antenna position is within the interval (x_{min},x_{max}). β is azimuth beamwidth of the physical antenna.

3. Time domain FM-CW SAR signal

In this paragraph the characteristic of a frequency modulated continuous wave signal is analyzed taking into account the particular synthetic aperture radar geometry. Let the transmitted signal written in complex form be:

$$S_T(t) = \exp\left[j2\pi \left(f_0 t + \frac{1}{2}\alpha t^2 \right) \right] \tag{4}$$

In the derivation of this paragraph the constant amplitude factors of the signal are discarded because they have no effect on the phase calculation. The received signal is a delayed version of the transmitted one, that is $S_R(t)=S_T(t-\tau)$; after it is mixed with a portion of the transmitted signal, the IF signal is produced:

$$S_{IF}(t)=\exp\left[j2\pi\left(f_0\tau+\alpha t\tau-\frac{1}{2}\alpha\tau^2\right)\right] \tag{5}$$

This description of the intermediate frequency signal is valid for one sweep time duration, that is, until there is no discontinuity in the modulated signal phase. To describe the signal characteristic for more then one sweep time duration the periodicity of the modulation has to be accounted for. Let's break down the time variable into:

$$t=t'+k\cdot PRI=t'+t_k \tag{6}$$

where $-PRI/2\leq t'<PRI/2$ and $-N/2\leq k<N/2$. N is the number of sweeps processed during the azimuth integration time and k is an integer; t_k is used for notation simplicity. From (6):

$$t'(t)=\mathrm{mod}(t,PRI) \tag{7}$$

The travel time of the signal from the transmitter to the target and back is also a function of time:

$$\tau(t)=\frac{2R(t)}{c}\approx\frac{2R_0}{c}+\frac{x(t)^2}{2R_0}=\frac{2R_0}{c}+\frac{v^2(t'+t_k)^2}{2R_0} \tag{8}$$

Using (8) and $t'(t)$ instead of t in (5), after some manipulation and neglecting terms with a factor c^2 in the denominator, the IF signal can be written in the following way:

$$S_{IF}(t'+t_k)=\exp\left\{j2\pi\left[\begin{array}{l}\frac{2R_0}{c}\left(f_0-\frac{\alpha R_0}{c}\right)+\frac{t_k^2v^2}{c}\left(\frac{f_0}{R_0}-\frac{2\alpha}{c}\right)\\+\left(\frac{2\alpha R_0}{c}+\frac{2t_kv^2}{c}\left(\frac{f_0}{R_0}-\frac{2\alpha}{c}\right)+\frac{\alpha t_k^2v^2}{cR_0}\right)\cdot t'\\+\left(\frac{v^2}{c}\left(\frac{f_0}{R_0}-\frac{2\alpha}{c}\right)+\frac{2\alpha t_kv^2}{cR_0}\right)\cdot t'^2+\frac{\alpha v^2}{cR_0}\cdot t'^3\end{array}\right]\right\} \tag{9}$$

From the linear term in t in (9) it can be seen that the first term $2\alpha R_0/c$ is proportional to the target nominal range; the second term $2t_kv^2(f_0/R_0-2\alpha/c)/c$, which is linear in t_k, is due to the Doppler frequency shift, while the third term $\alpha t_k^2v^2/cR_0$, quadratic in t_k, describes the variation of the radar-target distance around its nominal value. The term $(f_0/R_0-2\alpha/c)t_k^2v^2/c$ is the azimuth phase shift history.

4. Signal Processing

The main purpose of the SAR processing algorithms is to compress raw data in range as well in azimuth direction. A demonstrator system is currently realized at IRCTR based on off-the-shell technology and it will be attached under the wing of a motorglider. Demonstrator characteristics and parameters are summarized in table 1; more details can be found in [2], [3]. The IF raw data coming from the radar sensor are recorded in a matrix form in such a way that every column consists of data of a single sweep; hence the columns contain the range information whereas the rows contain the azimuth information. The dimension of the raw data matrix for a single target is $(f_s / PRF) \times N$, where f_s is the sampling frequency. For $f_s = 5MHz, PRF = 1KHz$ and $N = 1024$ it would be a 5000×1024 matrix. The raw data are range compressed performing a Fourier transform over the columns to get the beat frequency, related to the range according (2). After this FFT processing the matrix dimension is drastically reduced covering only those ranges that are of interest, that is in the range direction there would be only $2(R_{far} - R_{near}) df / c$ cells. From the values of table 1 this means that the range-compressed data for a single target would be a 1400×1024 matrix. Before azimuth compression the azimuth phase shift history has to be compensated multiplying the range compressed data by the azimuth reference function:

$$\exp\left(-j2\pi \frac{t_k^2 v^2}{c} \left(\frac{f_0}{R_0} - \frac{2\alpha}{c} \right) \right) \tag{10}$$

Compensation focuses the data and determines the relative azimuth position inside the illuminated area during the integration time. Fig. 2 illustrates this principle: a target A is displaced of $\Delta x = v\Delta t$ with respect to the origin where the azimuth reference function is centered. After the phase compensation the azimuth line will contain a frequency tone:

$$\Delta f = \Delta t \tan \gamma = \frac{\Delta x}{v} \frac{B_d}{T} \tag{11}$$

where B_d is the Doppler bandwidth and $\tan \gamma$ is the slope of the frequency curve. This frequency

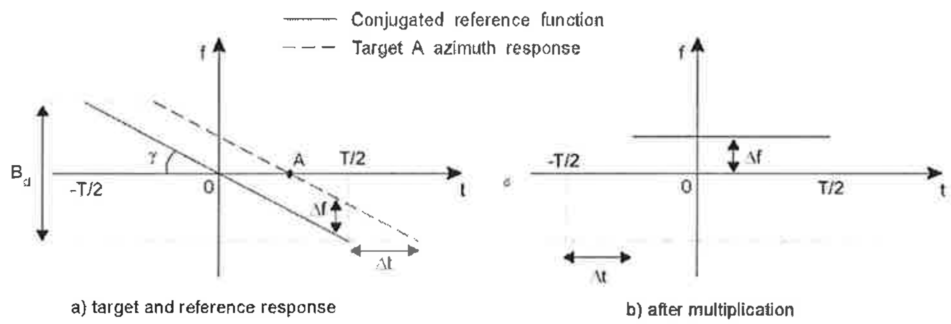


Figure 2. a) The target A is displaced of $\Delta x = v\Delta t$ with respect to the origin where the azimuth reference function is centered and it is illustrates how Δt is proportional to Δf; b) after multiplication the frequency is constant and equal to Δf in the shadowed area, that is the time the target A is in the antenna beamwidth centered in zero.

tone can be detected by processing the compensated data with a Fourier transform in the azimuth direction. After inverting (11) the azimuth displacement Δx can be obtained. If the flight path deviates from the ideal one errors are introduced, leading to a resolution loss. A Kalman filter is used to integrate GPS and Inertial Navigation System (INS) data in order to obtain position and velocity information and reconstruct the actual flight path, [8]. The basic logical flow diagram is described in fig.3.

Fourier transform methods obtain frequency resolution that is constant as the inverse of the observation time, [7]. In range direction the observation time is $1/PRF$ leading from (2) to a range resolution:

$$\rho_R = \frac{c}{2df} \tag{12}$$

For the azimuth direction (i.e. cross range) the resolution is, [6]:

$$\rho_A = \frac{\lambda}{2\beta} \tag{13}$$

where $\lambda = c/f_0$ is the transmission wavelength and β is the azimuth beamwidth. Using the demonstrator parameters listed in table 1, the theoretical range resolution is 0.3 m. Practical range and azimuth resolution will be larger due to windowing and other smearing effects such as sweep non-linearities, platform motion and even due to the fact that only part of the real antenna bandwidth will be used. Range as well azimuth resolution of 1 m should be within reach for the first test flight campaign.

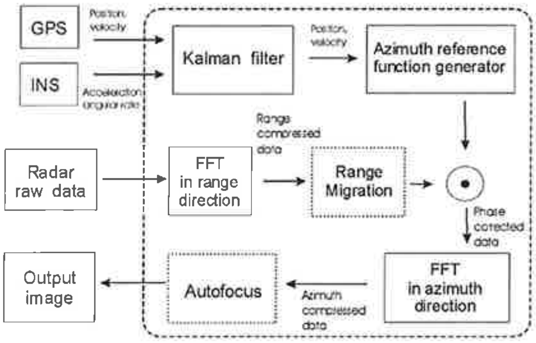


Figure 3. Basic logical flow diagram of the SAR processor to be applied. GPS and INS input data are used to reconstruct the actual flight path. Raw data are processed with FFT's in range and in azimuth direction to get the output image. Dotted blocks will be the object of near future work.

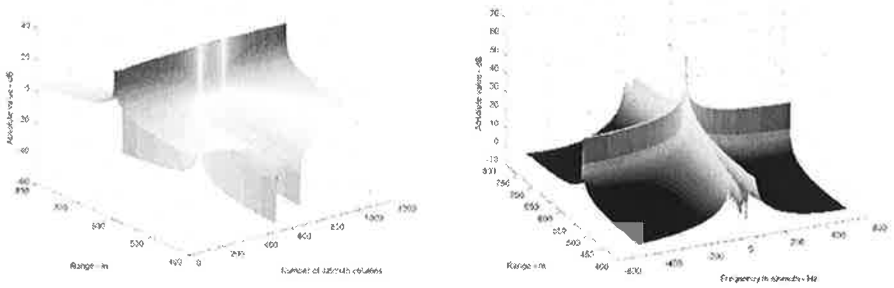
A computer simulation of an airborne FM-CW SAR system has been developed with the parameters of table 1. As the radar platform flies by, a target is illuminated for a certain period of time and during this integration time a number of sweeps is transmitted. The IF signal is simulated and it represents the raw data. After the raw data are generated, they are processed with the algorithm previously described and illustrated in fig. 3. The range migration correction and autofocus algorithms are not implemented yet and they will be the object of near future work. Simulation results of a single target at a nominal range of 600 m and processing 1024 sweeps are reported in fig. 4 showing graphics after range compression and azimuth compression. In fig. 4.a along x-axis range values are reported while in the y-axis the number of azimuth columns is shown. In fig. 4.b along y-axis there are frequency values in azimuth direction, which extend in an interval of one PRF, that is $1\ KHz$.

5. Conclusion

The use of FM-CW SAR can be useful for all that kind of applications that require small and not expensive sensors because it combines the advantage of compact FM-CW technology and high resolution SAR methods. A time domain analysis of FM-CW SAR signals has been derived and a first processing algorithm has been described. A computer simulation has been developed to perform a first validation of the algorithm and the range and azimuth resolutions agree with the theoretical resolutions. In the near future a flight test campaign with the demonstrator will be carried out and data will be used to further validate the developed algorithm. Future steps in this project are the development of autofocus algorithms, Moving Target Indicator (MTI) mode in the FM-CW SAR and the correction of range migration. The data processing for the demonstrator will be performed off-line; it is the final goal to enable real time production of all resulting image products.

TABLE 1. FM-CW SAR demonstrator specifications.

FM-CW front end		SAR configuration	
Carrier frequency	35 GHz	Platform velocity	25 m/s
Frequency sweep	500 MHz	Altitude	300 m
PRF	1000 HZ	Ground swath	500 m
Modulation	sawtooth	Near range	400 m
Beamwidth elevation	28°	Far range	820 m
Beamwidth azimuth	6°	Range resolution	0.3 m
		Azimuth resolution	0.3 m



a) range compressed data

b) azimuth compressed data

Figure 4. Simulation results of a single target at a nominal range of 600 m and processing 1024 sweeps are reported. In a) it is shown the range compressed data while b) illustrates the response after phase correction and azimuth compression.

Acknowledgment

This research is supported by the Technology Foundation STW, applied science division of NWO and technology programme of the Ministry of Economic Affair. The project is in close cooperation with the TNO Physics and Electronics Laboratory (TNO-FEL) in The Hague, The Netherlands. Eonic in Delft is involved too in cooperation on the real-time data processing

References

[1] A. Meta, P. Hoogeboom, "High resolution airborne FM-CW SAR: digital signal processing Aspects", in Proc. IGARSS 03, Toulouse, France, 21-25 July 2003.

[2] J.J.M. de Wit, P. Hoogeboom, "High resolution airborne FM-CW SAR performance analysis", in Proc. IGARSS 03 Toulouse, France, 21-25 July 2003.

[3] J.J.M. de Wit, P. Hoogeboom, "Performance analysis of a high resolution airborne FM-CW synthetic aperture radar", in Proc. IRS 03 Dresden, Germany, 30 September - 2 October 2003.

[4] M. Skolnik, "Introduction to radar systems", McGraw-Hill, Inc., London, 1980.

[5] D.R. Wehner, "High-resolution radar", Artech House, Boston, 1995.

[6] N. Levanon, "Radar principles", John Wiley, New York, 1988.

[7] A.G. Stove, "Linear FMCW radar techniques", Proc. IEE, vol. 139, No. 5, October 1992, pp 343-350.

[8] M.S. Grewal, L.R. Weill, A.P. Andrews, "Global positioning systems, inertial navigation and integration", John Wiley & Sons, New York, 2001.

International Radar Symposium

IRS 2003

30 September - 02 October 2003

Dresden, Germany

Proceedings

German Institute of Navigation

Deutsche Gesellschaft für Ortung und Navigation e.V. (DGON)