The large angle course of the disability glare function and its attribution to components of ocular scatter

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Keywords: Light scattering, Disability glare, Age, Pigmentation

Summary

Recent investigations have strengthened doubts on the validity of the Stiles-Holladay glare formula for large glare angles. Calculations partly based on an earlier theoretical analysis turned out to be helpful to understand the deviations from Stiles-Holladay and to extrapolate the glare function, in its dependence on age and pigmentation, into the large angle domain. A trend towards a $1/\theta$ rather than a $1/\theta^2$ fall off predominates.

1. Introduction

In the 1920s and 1930s Holladay[1)2)] and Stiles[3)4)5)] did basic studies on disability glare. Their results were more or less canonized by CIE[6)] in the so-called Stiles-Holladay glare formula

$$\frac{L_{eq}}{E_{gl}} = \frac{10}{\theta^2} \quad \text{with } \theta \text{ in degrees} \tag{1}$$

expressing that, for young adults, the masking effect of a glare source producing E_{gl} lux at the pupillar plane[7)] could be well described as that of an equivalent light veil with luminance L_{eq} cd/m² that decreases with $1/\theta^2$, θ being the glare angle in degrees. The factor 10 is a rough average, which actually may vary considerably between individuals. This formula, originally only established between about 4° and 30°, was later confirmed up to some 90° by Stiles and Crawford[5)]. Since then it has become increasingly clear that L_{eq} is more than only equivalent to, but rather a very real intraocular straylight veil indeed[8)].

Of course, Eq.(1) cannot be valid for small angles, and Vos et al.[9)], compiling data from various other literature sources, produced an extended glare function that was steeper between 8° and 1°, but then flattened to some bell-shaped spread function in the minutes of arc domain around the origin. But next to that, IJspeert et al.[10] have more recently raised doubts about the classic $1/\theta^2$ course for large glare angles. With their technique they could measure L_{eq} by direct flicker photometry at four pre-selected glare angles, viz. 3.5°, 7.0°, 13.6° and 25.4°. This technique allowed them, as firsts, to do reliable and quick field studies, and so to obtain data for an extensive population, in particular on the increase with age of glare susceptibility. Their main results on the angular dependence, averaged over many subjects, are summarized in Fig. 1, which is an adaptation of Figs 1 and 2 in Van den Berg et al.[11].

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insert Fig. 1 about here

One notices a flattening trend, a gradual change from $1/\theta^2$ to an approximate $1/\theta$ behavior, more pronounced in light than in dark eyes, and in red than in white light. This led the authors to investigate a possible attribution to light scatter through the ocular wall, either through iris or sclera. This trans-wall scatter they expected to be independent of glare angle and to be slightly reddish because of the absorption characteristics of melanin[12]. They could measure this component indeed, by the same photometric technique, and in two subjects they checked that there was no notable change indeed between $\theta = 9°$ and 28°. For the blue-eyed subject of Fig. 1b, its share in the total entoptic straylight (22% for white light, 36% for red light) turned out to be too small, however, to explain the uplift above $1/\theta^2$ (Fig. 1b). Table I gives Van den Berg et al.'s results for all their 6 subjects.

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insert Table I about here

The data show that, only in the combination of red light in light-blue eyes, the trans-wall component dominates large angle entoptic straylight luminance. In the other cases another explanation must be sought.

The authors speculated, then, whether the uplift might be attributed to scattering at the fundus, but did not elaborate on this point. Tacitly assuming fundus to fundus scattering to be uniform as well due to the integrating sphere effect, they postulated a glare formula which is virtually constant for large angles[13]. This extrapolation, however, requires better foundation. The present study deals with that. It was triggered by CIE's request to the first author to set up a committee to establish a better founded CIE glare formula.

However, before defining such an improved extended glare function, a further analysis is required to answer the following preliminary questions:

1 Are there sufficient indications indeed that the $1/\theta^2$ course is not valid for large glare angles?

2 What angular dependency should be expected for the various straylight components?

2. Earlier data on the large angle course

As a matter of fact most glare studies show a trend towards a slope less than -2 beyond $\theta \approx 10°$ (ref. [10] Fig. 4; ref. [14] Fig. 9). However, the main study underlying the CIE glare formula, that of Stiles and Crawford[5], the only one in fact extending—and even far—beyond $\theta = 30°$, is hold not to reveal such a tendency. But does not it, really? It seemed worthwhile to go back to the source and to replot their most relevant[15] Fig. 13 (Fig. 2a).

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insert Fig. 2 about here

We notice a few things. In the first place the evidence for a straight $1/\theta^2$ course all over the angular range is rather meager. In fact, there is a trend in both curves to a less steep course beyond $\theta = 10°$, only reverting at the very large angle end. Moreover, the apparent difference between the two subjects puts question marks at the reliability—or at least the representivity—of these extreme right data points. That Stiles and Crawford nevertheless concluded to a -2 slope is because of their obvious wish to describe the relation over its full angular range by one simple straight line.

And then, Stiles and Crawford expressed their data in terms of L_{eq}/E_N rather than of L_{eq}/E_{gl} since at large glare angles E_N is well defined, whereas E_{gl} is not, due to the somewhat complex perspective narrowing of the pupil through the intervening corneal optics.

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insert Fig. 3 about here

Eleven years later, however, Spring and Stiles[16] actually measured this perspective correction factor. They also made an effort to describe their data with a theoretical formula but did not publish. this, probably because the fit was not too satisfactory. We have made a new and more successful attempt (Fig. 3) and obtained a quite reasonable fit with[17]

$$\frac{A'_{pup}}{A_{pup}} \approx \frac{\cos[0.65(\theta-5)] \times \cos[0.84(\theta-5)]}{\cos[0.48(\theta-5)]} \approx \cos[0.92(\theta-5)] \qquad (2)$$

which can be considered as an adapted ordinary cosine correction factor[18] to account for the distorting effect of the corneal optics, with the subtracted 5° the angle between visual and optical axis[19]. Since $E_{gl}/E_N = A'_{pup}/A_{pup}$ we could replot, in Fig. 2b, the Stiles and Crawford data on the basis of L_{eq}/E_{gl} and this replot confirms the more upward trend more clearly indeed. That means that virtually all evidence, including Stiles and Crawford's, in fact points to a most probable large angle course between $1/\theta^2$ for the darkest and $1/\theta$ for the lighter colored eyes. The origin of the $1/\theta$ 'addition' will be the central issue in the further parts of this paper.

3. Components of entoptic scatter

a. Light scattered in the anterior eye media

Slitlamp pictures suggest that the main sources of straylight in the eye media are the cornea and the crystalline lens, due to their cellular structures. The anterior and posterior eye chambers are optically virtually empty, and the vitreous too, apart from minor smears. Light scattering by the cornea has been measured in the 20°–50° domain in excised steer eyes by DeMott and Boynton[20] and psychophysically by Vos and Boogaard[21]. They both found a $1/\theta^2$-course, roughly. Scattering in human donor eye lenses was recently measured by Van den Berg and IJspeert[22] between 2° and 50° and their data too follow a $1/\theta^2$ course, or even a bit steeper[23][24]. For the cornea, in vitro data from Freund et al.[25] fit with a $1/\theta^2$ course below 30°, but markedly deviate from that beyond about 50°: the scatter function tends towards a minimum at 90°, and increasing backscatter beyond.

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insert Fig. 4 about here

In Fig. 4 we have plotted together the DeMott and Boynton, the Vos and Boogaard, and the Freund et al. cornea data. The latter have been empirically fitted (by us) with $5\times10^5/\theta^2 + 0.01\times\theta^2$. Since Vos[26] found the glare contributions of the cornea and lens both to be approximately equal to one third of the total entoptic straylight veil in the 10° region, we can then describe the combined corneal and lens large angle glare contributions by

$$\left(\frac{L_{eq}}{E_{gl}}\right)_{C,L} = \frac{7}{\theta^2} + 7\times10^{-8}\times\theta^2 \qquad (3)$$

As indicated in the introduction, this course cannot continue down to very small angles, where we gradually enter the domain of the optical point spread function proper. Ray-trace studies[27] have shown that this central part can be attributed to diffraction and to focusing errors like spherical and chromatic aberration, located in the cornea and lens. As a consequence we can attribute the whole optical Point Spread Function up to some 20° to the anterior eye media. Linking IJspeert et al.'s[13] description[28][29] for the very central part to the just derived relation (3) and then recalibrating to

$$\int \left(\frac{L_{eq}}{E_{gl}}\right)_{C,L} d\omega = 1,$$

this cornea/lens part can be best described by

$$\left(\frac{L_{eq}}{E_{gl}}\right)_{C,L} = \frac{8.83\times10^{6}}{[1+(\theta/0.0046)^{2}]^{1.5}} + \frac{1.43\times10^{5}}{[1+(\theta/0.045)^{2}]^{1.5}} + \frac{7\times10^{2}}{1+(\theta/0.1)^{2}} + 7\times10^{-8}\,\theta^{2} \quad [\mathrm{sr}^{-1}] \qquad (4)$$

with θ in degrees

Eq.(4) clearly consists of two parts, which it makes sense to denote with special names: the 'core' part consisting of the first two terms, and the 'skirt' part consisting of the two last terms. This distinction will prove to be important when we want to introduce the age dependence. With increasing age the skirt part rises, of course at the cost of the core part. Both components, $(L_{eq}/E_{gl})_{core}$ and $(L_{eq}/E_{gl})_{skirt}$, the latter with a slight modification about which further on, are represented in Fig. 5. Note that $(L_{eq}/E_{gl})_{skirt}$ shows an upward trend with respect to $1/\theta^2$ above about 50°. It be further noted that A_{pup} does not appear in this formula, as both L_{eq} and E_{gl} have the pupil as their port of entry. This will be different for light entering the eye via the eye wall.

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insert Fig. 5 about here

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b. Light entering via the sclera and iris

The amount of light passing through the eye wall is $E_N \times A_{exp} \times \tau_{wall}$ [lm] in which A_{exp} is the area of those parts of the eye wall that are exposed to the glare light, and τ_{wall} is the average diffuse transmission through the eye wall (we will not differentiate here between iris and sclera). Its visual impact at the fovea (where L_{eq} is determined) is subject to the Stiles-Crawford effect beyond the pupil border by an unknown attenuation factor $\eta_{SC,lim}$: $E_N \times A_{exp} \times \tau_{wall} \times \eta_{SC,lim}$ [lm_{eff}]. This light is then redistributed more or less uniformly as L_{wall} over the entire fundus. It adds up to $\pi \times (L_{eq})_{wall} \times \eta_{SC,av} \times A_{pup}$ [lm_{eff}] in which $\eta_{SC,av}$ is the average luminous efficiency over the pupil[30] and π the effective solid angle over which L_{wall} is observed. Consequently

$$\pi \times (L_{eq})_{wall} \times \eta_{SC,av} \times A_{pup} = E_N \times A_{exp} \times \tau_{wall} \times \eta_{SC,lim}$$

or

$$(\frac{L_{eq}}{E_N})_{wall} = \frac{[\tau_{wall} \times \eta_{SC,lim}]}{[\pi \times \eta_{SC,av}]} \times \frac{A_{exp}}{A_{pup}} \quad [sr^{-1}]$$

which we can simplify to

$$(\frac{L_{eq}}{E_N})_{wall} = c_1 \times \frac{A_{exp}}{A_{pup}} \quad [sr^{-1}] \tag{5}$$

Evidently the ratio of the exposed area of iris and sclera to that of the pupil, A_{exp}/A_{pup}, is a rather critical element in determining the absolute height of $(L_{eq})_{wall}/E_N$, as already pointed out by Van den Berg et al.[11)]. It is determined both by the pupil size and by the degree of coverage by the eye lids which can only be determined by experiment. We have done this experiment by taking eye photographs and measuring both A_{exp} and the vertical diameter, D, of the pupil. From the latter $A_{pup} = \pi D^2/4$ can be calculated. Great accuracy cannot be expected in view of differences in squeezing behavior, in the embedding of the eye in its socket and variation in pupil size. Results are shown in Fig. 6 for three subjects.

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insert Fig. 6 about here

Fig. 6 shows that A_{exp}/A_{pup} can be approximated with $A_{exp}/A_{pup} = c_2 \times (105-\theta)$ with c_2 depending mainly on individual pupil size. Substituting this A_{exp}/A_{pup} relation in Eq.(5) we obtain: $(L_{eq}/E_N)_{wall} = c_3 \times (105-\theta)$ $[sr^{-1}]$. The constant c_3 is mainly determined by the degree of pigmentation (the darker, the smaller). Substituting, in addition, $E_{gl}/E_N = A'_{pup}/A_{pup}$ according to Eq.(2), we obtain for the trans-wall component:

$$(\frac{L_{eq}}{E_{gl}})_{wall} = c_3 \times \frac{(105-\theta) \times \cos[0.48(\theta-5)]}{\cos[0.65(\theta-5)] \times \cos[0.84(\theta-5)]} \quad [sr^{-1}] \tag{6}$$

This course is also represented in Fig. 5 on the basis of an assumed value for $c_3 = 3.7 \times 10^{-5}$, a value chosen in anticipation of what follows.

c. Light scattered at the ocular fundus

In his dissertation, Vos[26)] amply discussed the theoretically expectable angular dependence of glare due to fundus scattering. Within the framework of the present discussion a simplified version will suffice.

The contribution of fundus scattering to glare was rather early dismissed by Borschke[31)] when he argued that angular independent 'integrating sphere scattering' could never explain such a strongly angle dependent phenomenon as glare. Because Vos[32)] had experimentally found a sizable fundus component in the 1° to 6° glare angle range it became interesting to take a new look at Borschke's arguments. They turned out to be wrong. The so-called integrating sphere effect (a rather misleading term, since light integration has nothing to do with it) occurs when a small light spot (area dA, illuminance E) is projected on an opaque diffusely reflecting (ρ) sphere (Fig. 7).

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insert Fig. 7 about here

The illumination at the fovea F via the retinal image of the glare source G, then, becomes

$$E_F = \frac{\rho \times E \times dA}{\pi \times s^2} \times \cos(\phi) \times \cos(\phi) = \frac{\rho\, E\, dA}{\pi\, s^2} \times \left[\frac{s}{2R}\right]^2 = \frac{\rho\, E\, dA}{4\, \theta\, R^2}$$

which means that the sphere is homogeneously reilluminated. However, this only holds for an opaque spherical surface. The fundus is not opaque, however, neither as a reflective nor as a receptive layer. The latter is immediately clear. With skimming light incidence the receptive layer can be taken as a homogeneous layer of diluted pigmentation, and so typically constitutes a volume receiver, rather than a surface receiver: its receptive efficacy as a transparent layer is independent of the angle of incidence as the apparent shortening of the area of reception is counteracted by an equally large lengthening of the absorptive pathway through the receptive layer. Consequently, the second $\cos(\phi)$ factor drops out. But similar arguments may be used for the fundus as a reflector, be it that the situation is more complicated. The term fundus is a collective noun for a complex of various superimposed layers: the sclera, the choroid, the pigment epithelium and the retina.

The retina is the most easy part to deal with, since it is typically a volume scatterer. That means that also the first $\cos(\phi)$ factor drops out, so that

$$E_{F,eff,ret} = \frac{\rho\, E\, dA}{\pi\, s^2}$$

corresponding to

$$\left(\frac{L_{eq}}{E_{gl}}\right)_{ret} \;(:)\; \frac{1}{\theta^2}$$

By this course it does not differentiate from that for the anterior eye media and we may include it in Eq.(4) by raising the 7×10^2 coefficient in the third term to 10^3, taking into account that its contribution to the entoptic straylight veil is estimated to be some 30%[32)]. By introducing at the same time the already discussed distinction between the core and the skirt part of the Point Spread Function, we thus obtain

$$\left(\frac{L_{eq}}{E_{gl}}\right)_{core} = c_4 \times \left\{ \frac{8.8\times10^6}{[1+(\theta/0.0046)^2]^{1.5}} + \frac{1.4\times10^5}{[1+(\theta/0.045)^2]^{1.5}} \right\} \; [sr^{-1}]$$

and

$$\left(\frac{L_{eq}}{E_{gl}}\right)_{skirt} = c_5 \times \left\{ \frac{10^3}{1+(\theta/0.1)^2} + 7\times10^{-8}\,\theta^2 \right\} \; [sr^{-1}]$$

both with θ in degrees.

The values c_4 and c_5 have been added to adjust these functions in height to the experimental data. Since $(L_{eq}/E_{gl})_{core}$ and $(L_{eq}/E_{gl})_{skirt}$ together constitute the PSF, c_4 and c_5 are linked by the condition

$$\int PSF\, d\omega = \int \left(\frac{L_{eq}}{E_{gl}}\right)_{core} d\omega + \int \left(\frac{L_{eq}}{E_{gl}}\right)_{skirt} d\omega = 1 \tag{7}$$

In view of the normalization data in Appendix II this translates to

$$\frac{(8.8+0.14)\times10^6}{9.3\times10^6}\times0.950\times c_4 + \frac{10^3}{390}\times0.050\times c_5 = 1$$

Anticipating the results of the fitting procedure we will take here $c_5 = 0.39$, and thus $c_4 = 1.04$, so that

$$\left(\frac{L_{eq}}{E_{gl}}\right)_{core} = \left\{ \frac{9.2\times10^6}{[1+(\theta/0.0046)^2]^{1.5}} + \frac{1.5\times10^5}{[1+(\theta/0.045)^2]^{1.5}} \right\} \; [sr^{-1}] \tag{8}$$

and

$$\left(\frac{L_{eq}}{E_{gl}}\right)_{skirt} = \left\{ \frac{3.9\times10^2}{1+(\theta/0.1)^2} + 2.7\times10^{-8}\,\theta^2 \right\} \; [sr^{-1}] \tag{9}$$

both with θ in degrees

As already announced, both curves are drawn in the collective graph of Fig. 5.

The contribution of the sclera, filtered by double passage through the choroid, may most probably be neglected for all glare angles. For small angles the site of the sclera is too far away from the receptors to adequately compete with the other straylight sources, and for large glare angles the reflected light is strongly absorbed as it has to pass in a striking direction through the choroid and the Pigment Epithelium[26)]. In fact, there is not any experimental evidence for a blood color tainted component in L_{eq}, as should be expected from a sizable scleral component[33)].

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insert Fig. 8 about here

The most important fundus component within the context of this paper, probably, is the reflection against the Pigment Epithelium itself. For large scatter angles (from A to the Fovea, in Fig. 8) the situation is rather simple. The Pigment Epithelium can be considered as an opaque scatterer co-spherical with the foveal retina as a volume receiver. So only one $\cos(\phi)$ factor remains in the scatter formula, so that

$$\frac{L_{PE}}{E_{gl}} \;(:)\; \frac{1}{\theta} \tag{10}$$

For smaller glare angles the Fovea and the place of scattering at the Pigment Epithelium cannot be considered any longer as co-spherical, the receptor outer-segments lying definitely inside the Pigment Epithelium spherical surface. As a result the perspective shortening of the glare source is markedly reduced so that the angular dependence of L_{PE}/E_{gl} becomes more pronounced. The impact of the light scattered at the Pigment Epithelium at G and caught in the outer segments at F, the Fovea, then, is proportional to this perspective shortening, i.e. to $\cos\chi = \sin\delta \approx \delta$, and inversely proportional to the square of distance, i.e. $1/\theta^2$. In the triangle AFC the sine rule reads

$$\frac{\sin\chi}{R-h} = \frac{\sin(\chi+\theta)}{R}$$

or

$$\tan\delta = \frac{R\,/\,(R-h)-\cos\theta}{\sin\theta}$$

For $\delta \ll 1$, $\theta \ll 1$, $H \ll R$, this reads

$$\delta = \frac{1+h/R-1+\theta^2/2}{\theta} = \frac{h}{R\theta} + \frac{\theta}{2}$$

so that

$$\delta_{average} = \frac{1}{H}\int_0^H [\frac{H}{(R\theta)}+\frac{\theta}{2}]\, dh = \frac{H}{(2R\theta)} + \frac{\theta}{2}$$

Consequently, the impact at F, in our terms L_{eq}/E_{gl}, becomes

$$(\frac{L_{eq}}{E_{gl}})_{PE}\ (\because)\ [\frac{H}{(R\theta)}+\theta]\,\frac{1}{\theta^2} = \frac{H}{(R\theta^3)} + \frac{1}{\theta}$$

At the fovea H $\approx$ 60 μm and R $\approx$ 12500 μm, so

$$(\frac{L_{eq}}{E_{gl}})_{PE}\ (\because)\ \frac{0.005}{\theta^3} + \frac{1}{\theta}$$

Conversion from radians to degrees then gives

$$(\frac{L_{eq}}{E_{gl}})_{PE}\ (\because)\ \frac{16}{\theta^3} + \frac{1}{\theta} \qquad (11)$$

with θ in degrees

Finally, of course, the increase of L_{PE}/E_{gl} with decreasing glare angle stops at retinal distances comparable with the thickness of the outer segment layer, i.e. at about 30 μm $\approx$ 0.1°. This is most easily attained by introducing this value as a constant angle to Eq.(11), and in such a way that the equation becomes analytical at θ=0°:

$$(\frac{L_{eq}}{E_{gl}})_{PE} = c_6 \times \left[\frac{16}{[\theta^2+0.1^2]^{1.5}} + \frac{1}{[\theta^2+0.1^2]^{0.5}}\right] \qquad (12)$$

with θ in degrees

The course of this Pigment Epithelium fundus component is also indicated in Fig. 5 on the basis of an, again anticipating, value c_6 = 0.14.

4. A synthesis

The final product of our analysis is Fig. 5 in which the course of L_{eq}/E_{gl} for the four components is sketched, described by Eqs (6), (8), (9) and (12), respectively. The line for the anterior eye media (including the retinal scatter) is drawn at a height corresponding to mentioned experimental studies. The other two curves have been given a height which is in the expected range, but of course the degree of pigmentation and the wavelength of the glare light will have a sizeable influence. In darkly pigmented eyes and with short wavelength light these components may be smaller and vice versa. How well do these calculated curves together account for the experimentally observed data? To answer this question we can use the recently published further analysis of IJspeert et al.'s[10)] population study in which both age and complexion (pigmentation of eye and skin) were variables. In this analysis Van den Berg[34)] argues that their data (which only cover the range between 3.5° and 25.4°) can be described in terms of three empirical 'basic' curves[35)]:

$$\frac{L_{eq}}{E_{gl}}(\theta) = (\frac{L_{eq}}{E_{gl}})_{base} + (\frac{A}{70})^4 \times (\frac{L_{eq}}{E_{gl}})_{age} + p \times (\frac{L_{eq}}{E_{gl}})_{pig} \qquad (13)$$

in which p = pigmentation factor, which ranges from 0 for heavily pigmented non-caucasians, via 0.5 for brown eyed and 1.0 for blue-green eyed caucasians to 1.2 for blue-eyed caucasians. The symbols in Fig. 9 mark the derived angular courses for $(L_{eq}/E_{gl})_{base,age,pig}$. The drawn curves are descriptions in terms of our calculated components, viz.

$$\begin{aligned}
(\frac{L_{eq}}{E_{gl}})_{base} &= 1.00 \times (\frac{L_{eq}}{E_{gl}})_{skirt} + 0.60 \times (\frac{L_{eq}}{E_{gl}})_{PE} \\
(\frac{L_{eq}}{E_{gl}})_{age} &= 1.50 \times (\frac{L_{eq}}{E_{gl}})_{skirt} + 1.00 \times (\frac{L_{eq}}{E_{gl}})_{PE} \qquad (14) \\
(\frac{L_{eq}}{E_{gl}})_{pig} &= 1.25 \times (\frac{L_{eq}}{E_{gl}})_{PE} + 1.00 \times (\frac{L_{eq}}{E_{gl}})_{wall}
\end{aligned}$$

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insert Fig. 9 about here

These descriptions were derived by making best visual fits on the basis of the presumption that $(L_{eq}/E_{gl})_{base}$ and $(L_{eq}/E_{gl})_{age}$ should not have a trans-wall component and that $(L_{eq}/E_{gl})_{pig}$ should not have a skirt-component. The fit seems rather satisfactory in view of the experimental uncertainty. As a result we may transform Van den Berg's formula into

$$\frac{L_{eq}}{E_{gl}}(\theta) = [1.00+1.50(\frac{A}{70})^4] \times (\frac{L_{eq}}{E_{gl}})_{skirt} + [0.60+1.00(\frac{A}{70})^4] \times (\frac{L_{eq}}{E_{gl}})_{PE} + p \times [1.25 \times (\frac{L_{eq}}{E_{gl}})_{PE} + (\frac{L_{eq}}{E_{gl}})_{wall}] \qquad (15)$$

The significance of Eq.(15), of course is that it allows for a semi-theoretical extrapolation of IJspeert et al.'s experimental data, both in the small angle and in the large angle domain. For the large angle domain we can just use our Eqs (6), (8), (9) and (12). For the small angle domain we have to add the core component, but multiplied by an age reduction factor to compensate for the rise of the skirt component with age. The expression for this age reduction factor is then obtained by integrating over the forward hemisphere. It reads

$$\text{Age reduction factor} = 1.00 - 0.08 \times \left(\frac{A}{70}\right)^4$$

Consequently we obtain

$$\begin{aligned}
\frac{L_{eq}}{E_{gl}} = {} & \underbrace{[1.00-0.08(\tfrac{A}{70})^4] \times \left(\frac{9.2\times 10^6}{[1+(\theta/0.0046)^2]^{1.5}} + \frac{1.5\times 10^5}{[1+(\theta/0.045)^2]^{1.5}} \right)}_{\text{core component}} + \\
& + \underbrace{[1.00+1.50(\tfrac{A}{70})^4] \times \left(\frac{3.9\times 10^2}{1+(\theta/0.1)^2} + 2.7\times 10^{-8}\times\theta^2 \right)}_{\text{skirt component}} + \\
& + \underbrace{p\times[0.60+1.00(\tfrac{A}{70})^4] \times \left(\frac{2.2\times 10^3}{[1+(\theta/0.1)^2]^{1.5}} + \frac{1.4}{[1+(\theta/0.1)^2]^{0.5}} \right)}_{\text{PE component}} + \\
& + \underbrace{p \times \left(3.7\times 10^{-5} \times \frac{(105-\theta)\times\cos[0.48(\theta-5)]}{\cos[0.65(\theta-5)]\times\cos[0.84(\theta-5)]} \right)}_{\text{wall component}} \qquad \text{with } \theta \text{ in degrees}
\end{aligned} \tag{16}$$

In this expression the last term, though theoretically derived, is needlessly complicated since it is virtually constant. For this reason we are justified to finally reduce Eq.(16) to

$$\begin{aligned}
\frac{L_{eq}}{E_{gl}} = {} & \underbrace{[1.00-0.08(\tfrac{A}{70})^4] \times \left(\frac{9.2\times 10^6}{[1+(\theta/0.0046)^2]^{1.5}} + \frac{1.5\times 10^5}{[1+(\theta/0.045)^2]^{1.5}} \right)}_{\text{core component}} + \\
& + \underbrace{[1.00+1.50(\tfrac{A}{70})^4] \times \left(\frac{3.9\times 10^2}{1+(\theta/0.1)^2} + 2.7\times 10^{-8}\times\theta^2 \right)}_{\text{skirt component}} + \\
& + \underbrace{p\times[0.60+1.00(\tfrac{A}{70})^4] \times \left(\frac{2.2\times 10^3}{[1+(\theta/0.1)^2]^{1.5}} + \frac{1.4}{[1+(\theta/0.1)^2]^{0.5}} \right)}_{\text{PE component}} + \underbrace{2.5 \times 10^{-3}\, p}_{\text{wall component}} \\
& \qquad \text{with } \theta \text{ in degrees}
\end{aligned} \tag{17}$$

This extrapolation is elaborated in Fig. 10 for three 35 year old subjects of various complexion and one 80-year-old caucasian subject of average complexion. For comparison the mean of the curves of Fig. 1a and Fig. 2b are also given, shifted by one log unit. Numerical data underlying Fig. 10 are listed in Appendix II.

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insert Fig. 10 about here

In previous publications both Vos[8)] and IJspeert et al.[10)] have concluded that the glare level increased with age by a factor

$$[1+(\frac{A}{70})^4]$$

Eq.(17) now replaces and refines that conclusion in the sense that the age dependence is more complicated, apparently. One can illustrate that by comparing the original prediction that the glare level doubles between 35 and 72 years of age with the calculated results on the basis of Eq.(17) as shown in Table II. Roughly, the increase lies between 2 and 2.5 times, rather than 2 times.

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insert Table II about here

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5. Discussion

The calculations in the previous sections may here and there suggest great precision, but of course this is not true, if only for the rather great inter-individual variations in age-effects which mainly shows up as a difference in level. No doubt, therefore, that standardization on behalf of CIE should involve considerable simplification of the semi-theoretical Eq.(17). But there is more. In section 2 we opted for Stiles and Crawford's Fig. 13, rather than their Fig. 15 because of 'anomalous' effects in the latter graph. We now reproduce their Fig. 15 in our Fig. 11, just like Fig. 2b in terms of L_{eq}/E_{gl}, i.e. after correction for the perspective shortening of the pupil.

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insert Fig. 11 about here

It will be clear at once that whatever synthesis of our four components cannot but fail to produce similar results, simply due to the internal inconsistency. However, our analysis may suggest an escape. The contribution of the trans-wall component was the only one that depends on the exposure of the whole eye to the glare light. For the intraocular straylight components the size of the pupil dropped out because both the glare light and the equivalent veil light entered the eye via the pupil. This does not hold for the trans-wall light, however, and variations in light exposure of the sclera by distending or squeezing the eyes may have consequences for this component. Of course we do not know anything of Stiles and Crawford's distending or squeezing behavior and this way out is purely speculative in the absence of accurately controlled experiments. At the same time it underlines our remark about the intrinsic lack of precision when it comes to daily practice. Fortunately, the very large angle part of the glare function is not of great importance for daily practice.

6. Conclusions

Our road was a bit complex, and the many details which needed to be discussed may have easily obscured our final goal. Therefore we may give a short review. We noted that the literature revealed a trend towards a gradual decrease in the slope of the glare function at greater glare angles, definitely deviating from the 'standard' Stiles-Holladay glare formula. Question was whether we could understand this upward trend and if this understanding might help to extrapolate the glare function into the less well investigated very large glare angle domain, in particular also in its dependence on age and pigmentation. This question could be answered in a positive way. A more detailed analysis of the nature of scattering at the pigment epithelium and of the entry of glare light through the ocular wall enabled us indeed to make more reliable predictions than before on the shape of the entire glare function for subjects of various age and pigmentation. Eq.(17) and Fig. 10 show the main results. They confirm our surmise that the main cause of the slope reduction towards large angles is the contribution to entoptic scatter of the Pigment Epithelium, and that the light passing through sclera and iris becomes only significant at very large glare angles, beyond about 50° and then, only, for the lighter pigmented eyes. It will be clear, though, that the experimental basis at large glare angles needs to be strengthened.

Appendix I The apparent reduction of the pupil size in side view

In Fig. 12, with a more detailed view at the right, the light incidence situation at the cornea is sketched.

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insert Fig. 12 about here

With $\delta/d_{cornea} = \cos(i)$, $D/d_{cornea} = \cos(r)$ and $D/d_{pup} = \cos(\Omega)$ we obtain

$$d_{pup} = \frac{\delta \times \cos(r)}{\cos(i) \times \cos(\Omega)}$$

Hence

$$\frac{A'_{pup}}{A_{pup}} = \frac{\delta}{d_{pup}} = \frac{\cos(i) \times \cos(\Omega)}{\cos(r)}$$

The angles i, Ω and r are related through

$$\Omega = r + \phi \qquad \underset{\text{Snell's rule}}{\sin i = \sin r \times n} \qquad \underset{\text{sine rule}}{\sin \Omega : 7.8 = \sin r : 4.5} \qquad ß = \phi + i$$

which enables exact calculation, with $n = 1.336$. For small angles—but one may easily verify by actually carrying out the calculations that the derived expression holds as well for large angles in good approximation—we may substitute

$$\Omega = 0.84 \times ß, \quad r = 0.48 \times ß \text{ and } i = 0.65 \times ß$$

so that

$$\frac{A'_{pup}}{A_{pup}} = \frac{\cos(0.65 \times ß) \times \cos(0.84 \times ß)}{\cos(0.48 \times ß)}$$

Since the line of sight on the average deviates about 5° nasally from the optical axis, we obtain

$$\frac{A'_{pup}}{A_{pup}} = \frac{\cos[0.65(\theta-5)] \times \cos[0.84(\theta-5)]}{\cos[0.48(\theta-5)]} \tag{18}$$

As Fig. 3 shows, this formula may within reasonable limits be replaced beyond 80°, by

$$\frac{A'_{pup}}{A_{pup}} = \cos[0.92 \times (\theta - 5)] \qquad \textbf{(19)}$$

a formulation which has the advantage that it makes clear that it is a slightly distorted and displaced ordinary cosine formula.

Appendix II Numerical values for the various components of the glare function

$\log \theta$	θ	core	skirt	PE	wall
θ in degrees					
$-\infty$	0	9.3×10^{6}	390	2.2×10^{3}	3.7×10^{-3}
−3.0	.001	8.7×10^{6}	390	2.2×10^{3}	3.7×10^{-3}
−2.5	.0032	5.3×10^{6}	390	2.2×10^{3}	3.7×10^{-3}
−2.0	.01	8.2×10^{5}	387	2.2×10^{3}	3.7×10^{-3}
−1.5	.032	1.1×10^{5}	355	2.2×10^{3}	3.7×10^{-3}
−1.0	.10	1.1×10^{4}	196	1.1×10^{3}	3.7×10^{-3}
−0.5	.32	448	36	68	3.7×10^{-3}
0	1.0	14	3.9	2.3	3.6×10^{-3}
+0.5	3.2	--[36]	.39	.11	3.6×10^{-3}
+1.0	10	--	.0396	.016	3.3×10^{-3}
+1.5	32	--	.0039	.0045	2.9×10^{-3}
+1.8	63	--	.0011	.0022	2.5×10^{-3}
+1.955	90	--	.00071	.0015	2.2×10^{-3}
Integral over forward hemisphere		0.950	0.05	0.04	0.02

Figure Legends

Fig. 1 L_{eq}/E_{gl} as a function of glare angle θ. **a.** for three pigmentation groups with white glare light; **b.** for one blue eyed subject with three colors of the glare light. The indicated horizontal levels are the measured base lines toward which the respective curves should flatten off if that light level were due only to trans-wall straylight. Replot of Van den Berg et al.[11], Figs 1 and 2.

Fig. 2 **a.** L_{eq}/E_N as a function of θ according to Stiles and Crawford[5], Fig. 13. **b.** The same data, converted to L_{eq}/E_{gl}.

Fig. 3 Perspective narrowing of the pupil as measured by Spring and Stiles[16], with calculated exact course, approximate course according Eq.(2) left, and simplified description according to Eq.(2) right. For details, see Appendix I.

Fig. 4 Corneal scatter functions obtained by DeMott and Boynton[20], by Vos and Boogaard[21] and by Freund et al.[25], brought to approximately fitting height.

Fig. 5 Postulated contributions of the core part ('core'), the skirt ('skirt') part, the Pigment Epithelium part ('PE') and the trans-wall part ('wall') to the entoptic straylight veil (according to data Appendix II).

Fig. 6 A_{exp}/A_{pup} as a function of θ, determined photographically for three subjects. Insert illustrates the method. The results per subject are scaled to the value 1 at 0°. Line drawn is $(A_{exp}/A_{pup})_{rel} = 0.0095\times(105-\theta)$.

Fig. 7 Glare light incidence and scattering at the fundus.

Fig. 8 Schematical view of light scattering at the Retinal Pigment Epithelium.

Fig. 9 Angular dependence of $(L_{eq}/E_{gl})_{base,age,pig}$ according to Van den Berg[34] (symbols), compared with best fits with Eq.(14).

Fig. 10 Resulting semi-theoretical description of L_{eq}/E_{gl} over the whole angular domain, for three 35-year-old subjects of various complexion and for one 80-year-old caucasian subject of average complexion. Two experimental courses are shown for comparison.

Fig. 11 Results of Stiles and Crawford's[5] Fig. 15, replotted in terms of L_{eq}/E_{gl}.

Fig. 12 Light incidence by a glare beam. Right a more detailed view at the location of corneal incidence.

Table I Share of trans-wall straylight in the total entoptic light scatter veil at 25.4°. Van den Berg et al.'s[11] data for white and red light for their six subjects.

subject	1	2	3	4	5	6
eye color	light-blue	blue	brown	brown	dark-brown	dark-brown
white light	29%	22%	6%	6%	1%	1%
red light	72%	36%	19%	28%	4%	3%

Table II Factor of increase in glare level between 35 and 72 years of age.

glare angle	3°	10°	30°
p=0 (non-caucasian, dark eyes)	2.46	2.46	2.46
p=0.5 (caucasian, brown eyes)	2.47	2.43	2.19
p=1.0 (caucasian, average eyes)	2.47	2.40	2.08

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36. Values below $10^{-6} \times$ peak value have been set at zero.