AN ANALYTICAL SOLUTION FOR SIGNAL, BACKGROUND AND SIGNAL TO BACKGROUND RATIO FOR A LOW FREQUENCY ACTIVE SONAR IN A PEKERIS WAVEGUIDE SATISFYING LAMBERT'S RULE

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Abstract: *Previously published equations for the time dependence of the echo and reverberation in a Pekeris waveguide are combined with an expression derived for surface-generated noise. These closed form solutions are applied to the calculation of signal to reverberation ratio and signal to total background ratio for three CW pulses with centre frequencies between 250 Hz and 3.5 kHz. The scenario considered is Problem A2.I from the 'Validation of Sonar Performance Assessment Tools' meeting of the Institute of Acoustics, held in Memory of David E. Weston, in April 2010. This scenario involves the detection of a spherical target in a Pekeris waveguide, against a background of rain noise and Lambert-rule reverberation from the seafloor.*

Keywords: *low frequency active sonar, sonar performance modelling, analytical solution, Pekeris waveguide, reverberation, rain noise, target echo*

1. INTRODUCTION

The target echo from an active sonar pulse is masked by a background of unwanted echoes (reverberation) and ambient noise. In order to understand or predict the performance of such a sensor it is therefore necessary to predict the levels of the echo, accompanying reverberation and ambient noise.

The need for reliable reverberation predictions led to the organisation of two reverberation modelling workshops sponsored by the US Office of Naval Research (ONR) and held at the University of Texas at Austin in November 2006 [1] and May 2008 [2]. The echo from a simple target was included in one of the scenarios specified for the second of these workshops. A third meeting, sponsored by the UK Institute of Acoustics (IOA), considered the signal (target echo), reverberation and ambient noise to estimate the overall sonar performance in the form of a signal to background ratio [3], using scenarios closely based on those developed for the ONR workshops. The present focus is on Problem A2.I of the IOA meeting [4], which combines ambient noise due to rainfall with reverberation from Problem XI from the first ONR workshop and an echo from the target used in Problem T (from the second ONR workshop). The signal processing required to form the signal to reverberation and signal to total background ratios for Problem A2.I is specified by Ref. [4]. Only the CW pulses are considered here.

The details of the problem specification relevant to this paper are described in Sec. 2. In Sec. 3 we consider the signal (target echo) and reverberation before processing, followed by the effects of the beamformer and matched filter on signal to reverberation ratio (Sec. 4). Ambient noise is introduced in Sec. 5, before and after processing.

2. PROBLEM DESCRIPTION

Aspects of Problem A2.I relevant to the present paper are summarised in this section. For full details see Ref. [4].

2.1. Sonar parameters

Three different CW pulses are considered, each with its own receiving array characteristics tuned to the properties of that pulse (Table 1). For all pulses the energy source factor is $S_E = 10^{20}$ μPa² m² s.

The CW pulse envelope $S_0(t)$ varies with time t according to

$$S_0(t) = \frac{S_E^{1/2}}{T_{\mathrm{eq}}^{1/2}} \exp\left(-\frac{\pi t^2}{2 T_{\mathrm{eq}}^2} \right), \tag{1}$$

where T_{eq} is the duration of an equal energy rectangular pulse. Equation (1) is normalised such that

$$\int_{-\infty}^{+\infty} S_0(t)^2 \, \mathrm{d}t = S_E \, . \tag{2}$$

transmitted pulse				receiving array	
pulse type	centre frequency f_m/ Hz	equivalent duration T_eq / ms	fwhm bandwidth B_fwhm / Hz	spacing Δx / m	n_hp Δx / m
lf CW	250	37.577	12.5	3.0	195
mf CW	1000	9.3944	50.0	0.75	48.75
hf CW	3500	2.6841	175.0	0.20	13

Table 1: Pulse parameters specified by Ref. [4]. The number of hydrophones n_hp is 65.

2.2. APL source model for rain noise

The noise calculation of Sec. 5 requires as input a value of the spectral density of the areic dipole source factor [5, p424], denoted K_f. The problem specification requires this parameter to be calculated according to the APL 1994 rain noise source model [6], which is valid between 1 kHz and 100 kHz. In the frequency range 1-10 kHz, and for the specified conditions (wind speed = 0, rainfall rate = 1 mm/h) this model varies with frequency f according to

$$K_f = \left(10^{7.16} \, \mu\mathrm{Pa}^2\right) / f \, , \tag{3}$$

leading to $K_f \approx 14500$ μPa²/Hz at 1 kHz and 4130 μPa²/Hz at 3.5 kHz. The 250 Hz scenario is specified with zero rain noise (detection against reverberation only).

2.3. Other parameters

The environment regarding propagation and scattering is identical to that of Problem XI [1; 7], with one additional parameter – the rate of rainfall – needed for the ambient noise level. The target is a vacuum sphere of radius 5 m, the target strength of which is approximately equal to its high frequency limit of 8.0 dB re m² at all three frequencies.[8]

3. SIGNAL AND REVERBERATION LEVELS BEFORE PROCESSING

In this section we consider signal and reverberation before processing. For the echo in a waveguide of depth h and sound speed c we take a result from Ref. [9], making the assumption that the transmitted pulse is shorter than the duration of the two-way impulse response, but longer than the separation between successive multipaths, to express the echo as the following function of range r

$$E_{\mathrm{HN}}(r) = S_E \frac{c\sigma_{\mathrm{back}}}{2h^2r^3} \exp(-4\alpha r), \tag{4}$$

where σ_{back} (≈ 78.5 m²) is the backscattering cross-section [5, pp 41, 607]. The peak RMS pressure [i.e., (4)], unlike the total echo energy [10], is independent of seabed properties. The attenuation coefficient α, and all other frequency dependent parameters are understood to be evaluated at the pulse centre frequency.

Reverberation is calculated as a function of the Lambert parameter μ (≈ 0.00200) and the reflection loss gradient η (≈ 0.274) using the Zhou-Harrison (ZH) formula [10; 11]

$$R_{\mathrm{ZH}}(r) = S_E \frac{\pi\mu c}{4\eta^2 r^3} [1 - \exp(-Qr)]^2 \exp(-4\alpha r), \tag{5}$$

where

$$Q = \theta_{\mathrm{c}}^{\ 2}\eta / h\,. \tag{6}$$

The symbols E and R represent the locally averaged squared pressure of echo and reverberation, respectively. They are converted to sound pressure level in decibels and plotted vs target range in Figure 1.

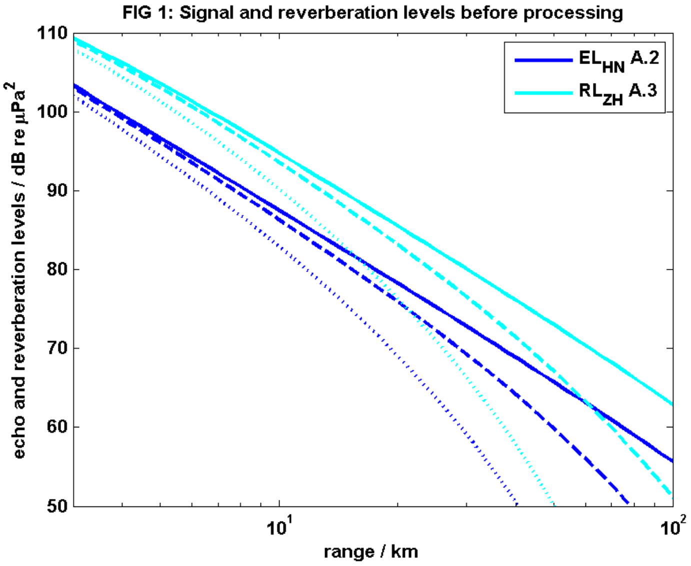

Fig.1: Echo and reverberation levels before processing. Solid: 250 Hz; dashed: 1 kHz; dotted: 3.5 kHz.

4. SIGNAL TO REVERBERATION RATIO BEFORE AND AFTER PROCESSING

Before processing, the signal to reverberation ratio (SRR) is the ratio of E to R. For a CW pulse, the gain of the matched filter against reverberation may be neglected. We therefore assume that the signal to reverberation ratio (SRR) after all processing, denoted ρ_{mf}, is related to the acoustic SRR, ρ_{ac}, through the array gain ($10\log_{10}A_R$). In other words

$$\rho_{\mathrm{mf}} = A_R \rho_{\mathrm{ac}} . \tag{7}$$

The array gain against reverberation for the broadside beam, and assuming all reverberation arrives horizontally, is [5, p273] $A_R = kL/2$, where $k = 2\pi f/c$. For the specified (approximately $\lambda/2$) hydrophone spacing, this gives $A_R \approx \pi/2\, n_{\mathrm{hp}}$ at all three frequencies (a gain of 20.1 dB). It follows that

$$\rho_{\mathrm{mf}} = \frac{n_{\mathrm{hp}} \eta^2 \sigma_{\mathrm{back}}}{\mu h^2} \left[1 - \exp(-Qr)\right]^{-2} . \tag{8}$$

The SRR before and after processing is plotted in decibels in Figure 2. An alternative calculation is possible using the total energy in the pulse instead of the peak intensity. If one assumes that the duration of the received echo is the same as that of the transmitted pulse [13], it follows that the echo can be written [10]

$$E_0(r) = \frac{S_E}{T_{\mathrm{eq}}} \frac{\sigma_{\mathrm{back}}}{4\eta r^3} \left[\mathrm{erf}\left(\sqrt{Qr}\right)\right]^2 \exp(-4\alpha r) . \tag{9}$$

If the ratio $E_0(r)/R_{\mathrm{ZH}}(r)$ is used instead of $E_{\mathrm{HN}}(r)/R_{\mathrm{ZH}}(r)$, the SRR so calculated is overestimated by between 5 dB (at 250 Hz) and 16 dB (3500 Hz). The influence of ambient noise on signal to background ratio (SBR) is discussed in the next section.

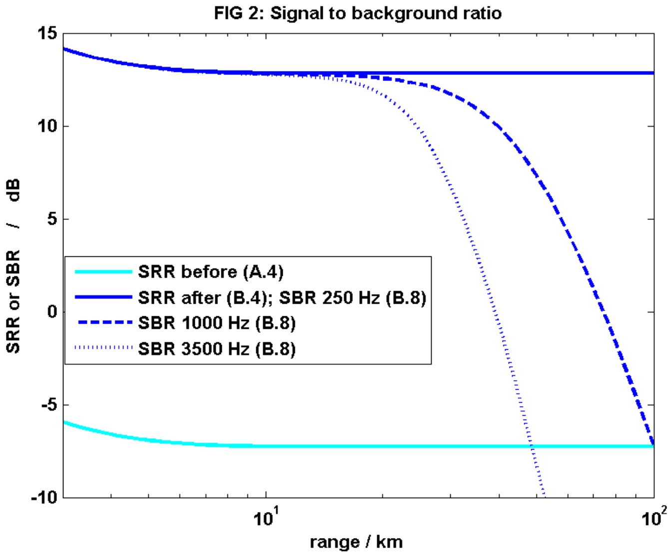

Fig.2: SRR vs range, before and after processing; and SBR vs range after processing.

5. EFFECT OF AMBIENT NOISE

The ambient noise spectral density is obtained by integrating over all contributions from the sea surface with the areic dipole source factor described in Sec. 2.2. Contributions associated with direct path propagation from the sheet source at the sea surface to the receiver are denoted n_{D}. Also considered are multiple reflections via the seabed and sea surface (n_{BL}). The sum of both can be written

$$N_f = 2\pi K_f \left(n_{\mathrm{BL}} + n_{\mathrm{D}} \right), \tag{10}$$

with K_f from (3) for rain noise (1 mm/h).

The direct path contribution at depth d is [5, p39; 6]

$$n_{\mathrm{D}} = E_3 \left(2\alpha d \right), \tag{11}$$

where $\mathrm{E}_3(x)$ is the exponential integral of third order [12].

The waveguide contribution for seabed critical angle θ_{c} (derived from [14], Eq. (8), correcting here for surface decoupling) is

$$\eta n_{\mathrm{BL}} = \sin\theta_{\mathrm{c}} - (a-b)^{-1} \left[a^{3/2} \arctan\left(a^{-1/2} \sin\theta_{\mathrm{c}} \right) - b^{3/2} \arctan\left(b^{-1/2} \sin\theta_{\mathrm{c}} \right) \right] , \tag{12}$$

where $a = 2\alpha h/\eta$ and $b = 1/[2(kd)^2]$. The noise spectral density N_f follows by substituting (11) and (12) in (10). The rain noise spectral density level is calculated in decibels and plotted in Figure 3.

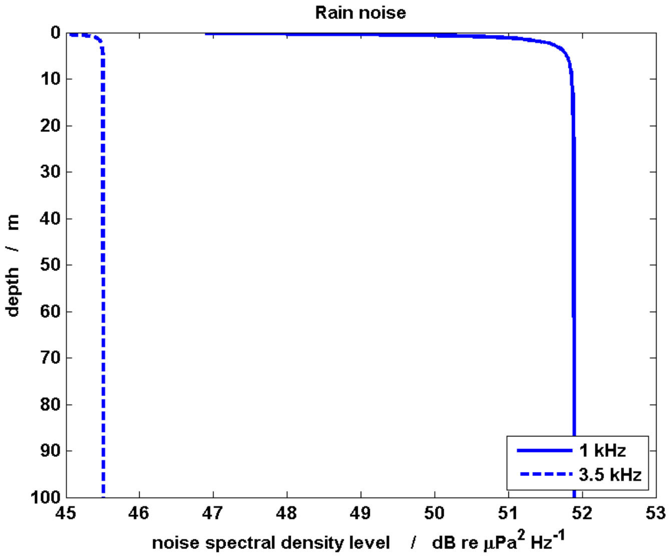

Fig.3: Rain noise spectral density level vs depth; rain rate is 1 mm/h.

The acoustic signal to noise ratio (SNR) (i.e., SNR before any processing) $\nu_{\mathrm{ac}}(\mathrm{r})$ is

$$\nu_{\mathrm{ac}} \approx \frac{c}{4\pi} \frac{S_E}{\Delta B\, K_f} \frac{\sigma_{\mathrm{back}}}{h^2} \frac{\eta r^{-3} \exp(-4\alpha r)}{\sin\theta_{\mathrm{c}} - a^{1/2} \arctan\left(a^{-1/2} \sin\theta_{\mathrm{c}} \right) + \eta E_3 \left(2\alpha d \right)} \tag{13}$$

Notice the introduction of a bandwidth ΔB in this equation. The value of ΔB is assumed to be large enough to include the entire signal, and small enough for K_f to be approximated as a linear function of frequency within ΔB, but is otherwise arbitrary.

The specification requires processing with a replica of the transmitted pulse. The SNR after matched filtering is related to ν_{ac} via

$$\nu_{\mathrm{mf}} = M_N A_N \nu_{\mathrm{ac}} , \tag{14}$$

where M_N is the matched filter gain, which for a CW pulse is

$$M_N = 2T_{\mathrm{eq}} \Delta B \tag{15}$$

and A_N is the array gain against noise, which can be estimated using the knowledge that the contribution from n_{BL} is approximately horizontal (we assume precisely $\lambda/2$ spacing again and neglect the small effect of absorption on the ambient noise),

$$A_N \approx n_{\mathrm{hp}} \left\{ \frac{\pi}{4} \frac{2+\eta \operatorname{cosec}\theta_{\mathrm{c}}}{1+\eta \operatorname{cosec}\theta_{\mathrm{c}}} \right\} . \tag{16}$$

The final signal to background ratio (SBR), after all processing, is

$$\frac{1}{\nu_{\mathrm{mf}}{}^{-1} + \rho_{\mathrm{mf}}{}^{-1}} = \frac{n_{\mathrm{hp}} \sigma_{\mathrm{back}}}{h^2} \left[\frac{8}{c} \frac{\delta B\, K_f}{S_E} \left(\frac{\sin\theta_{\mathrm{c}}}{\eta} + 1 \right) r^3 \exp(4\alpha r) + \frac{\mu}{\eta^2} \left[1 - \exp(-Qr) \right]^2 \right]^{-1} . \tag{17}$$

The SBR is calculated in decibels and plotted in Figure 2 using (17). At the lowest of the three frequencies (250 Hz), the ambient noise is specified to be zero so the SBR at 250 Hz is expected to (and does) approximately reproduce the SRR (all frequencies) of the same figure.

6. CONCLUSIONS

According to the model derived by Harrison and Nielsen (HN) [9] for the echo pulse shape, the echo level, defined as the maximum value of the sound pressure level of the target echo, is independent of seabed parameters. Combining the HN expression for the echo with the Zhou-Harrison [10, 11] formula for reverberation, we obtain a signal to reverberation ratio that is independent of target range from about 5 km. The picture is completed by evaluating the ambient noise level using the method of Ref. [14]. The resulting signal to total background ratio is influenced by noise from 20 km at 1 kHz and from 10 km at 3.5 kHz.

The results presented are not benchmarks. Rather, they are simple solutions, intended to contain enough physics to describe the essence of the problem, in particular regarding the effect of time dispersion on the signal to background ratio. While the question of accuracy is not addressed explicitly, it is shown that a calculation that neglects time dispersion has the potential of overestimating the signal to reverberation ratio by as much as 16 dB. In time, more precise calculations for each of the terms are bound to become available and replace the solution presented herein.

7. ACKNOWLEDGEMENTS

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